Constant delay enumeration for FO queries over databases with local bounded expansion

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Outline

Introduction, Query evaluation

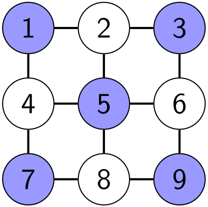
- Query q *small*
- Database D *huge*
- Compute $q(D)$ *gigantic*

Examples :

query q

$$q(x,y) := \exists z(B(x) \wedge E(x,z) \wedge \neg E(y,z))$$

database D



solutions $q(D)$

$\{(1,2)\ (1,3)\ (1,4)$
$(1,6)\ (1,7)\ \cdots$
$(3,1)\ (3,2)\ (3,4)$
$(3,6)\ (3,7)\ \cdots$
$\cdots\ \}$

Enumeration and properties

Input :

- Database : **D**
- Query : $q(\overline{x})$

Enumeration

- Compute an index to speed up the enumeration
- Compute the set of solutions with minimal *delay* between to output

  Ideally : Preprocessing: $O(\|D\|)$ delay: $O(1)$

We talk about *Constant delay enumeration after linear preprocessing*

Properties

- First solution computed in time $O(\|D\|)$
- Overall computation in time $O(\|D\| + |q(D)|)$
- Regularity in the computation

Example

Input :

- Database $D := \langle \{1, \cdots, n\}; E \rangle$ $\qquad \|D\| = |E| \quad (E \subseteq D \times D)$
- Query $q(x, y) := \neg E(x, y)$

D

$$\begin{array}{c} (1,1) \\ (1,2) \\ \boxed{\begin{array}{c} (1,6) \\ \vdots \\ (2,3) \end{array}} \\ \vdots \\ (i,j) \\ (i,j+1) \\ (i,j+3) \\ \vdots \\ (n,n) \end{array}$$

Example

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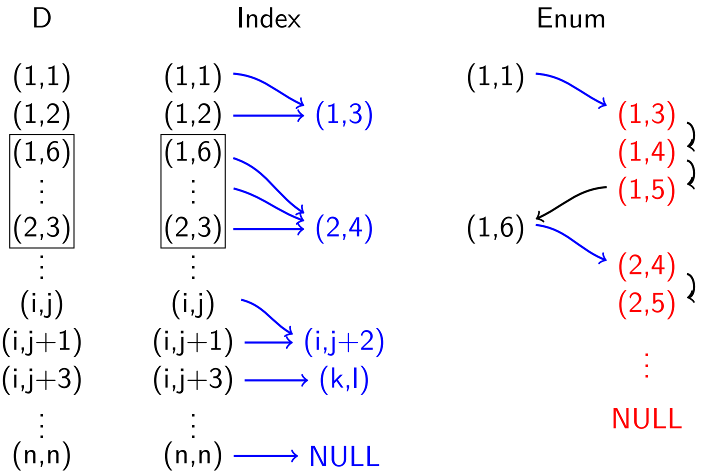
- Database $D := \langle \{1, \cdots, n\}; E \rangle \qquad \|D\| = |E| \quad (E \subseteq D \times D)$
- Query $q(x, y) := \neg E(x, y)$

D	Index	
(1,1)	(1,1) $\rightarrow$	(1,3)
(1,2)	(1,2) $\rightarrow$	(1,3)
(1,6)	(1,6) $\rightarrow$	(2,4)
$\vdots$	$\vdots$ $\rightarrow$	(2,4)
(2,3)	(2,3) $\rightarrow$	(2,4)
$\vdots$	$\vdots$	
(i,j)	(i,j) $\rightarrow$	(i,j+2)
(i,j+1)	(i,j+1) $\rightarrow$	(i,j+2)
(i,j+3)	(i,j+3) $\rightarrow$	(k,l)
$\vdots$	$\vdots$	
(n,n)	(n,n) $\rightarrow$	NULL

Example

Input :

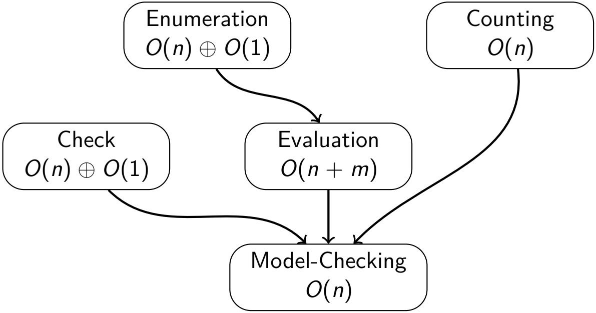
- Database $D := \langle \{1, \cdots, n\}; E \rangle \qquad \|D\| = |E| \quad (E \subseteq D \times D)$
- Query $q(x, y) := \neg E(x, y)$



Other problems

For **FO** queries over a class $\mathcal{C}$ of databases.

Model-Checking	:	Is there a solution ?	$O(n)$
Counting	:	How many solutions ?	$O(n)$
Check	:	Is this tuple a solution ?	$O(n) \oplus O(1)$
Evaluation	:	Compute the entire set	$O(n+m)$



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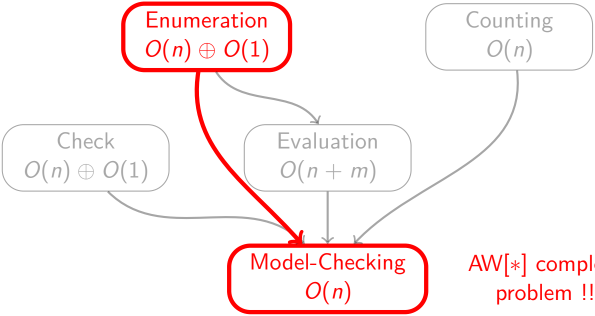
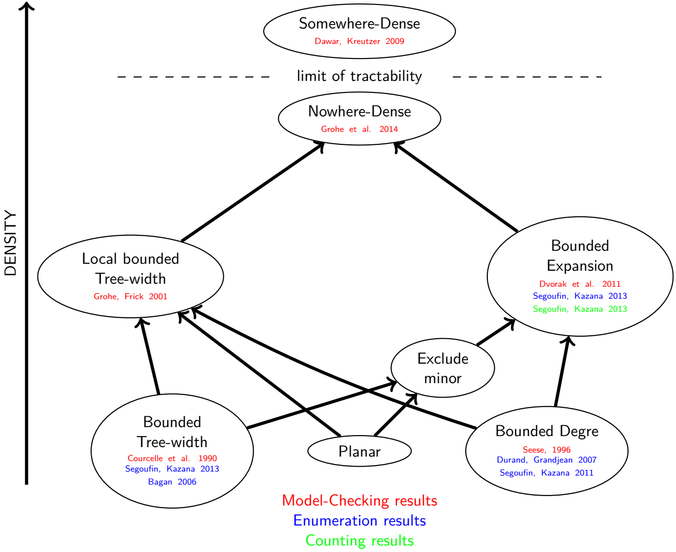


AW[$*$] complete problem !!

Hereditary classes of graphs and FO queries



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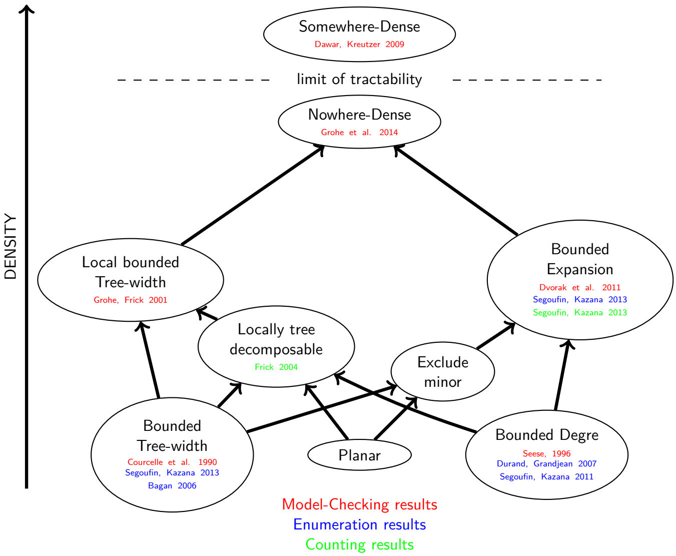

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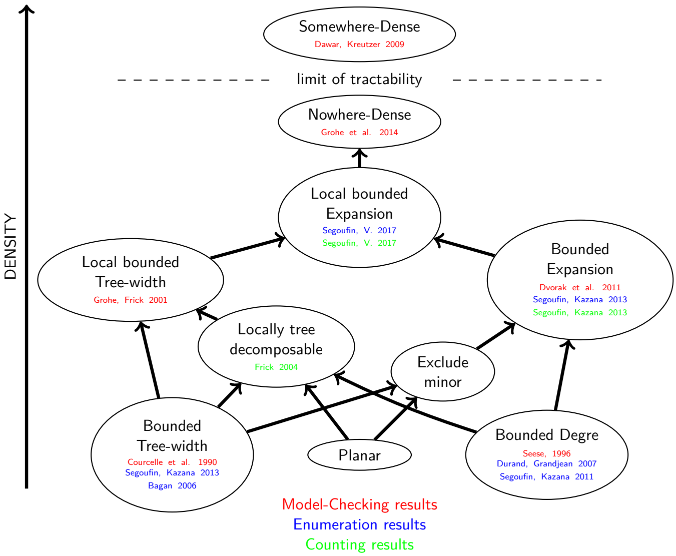

Definition bounded expansion

Introduced by Nešetřil an Ossona de Mendez in '08

Definition: Bounded expansion

$\mathcal{C}$ has bounded expansion if:

$$\exists f : \mathbb{N} \mapsto \mathbb{R} \quad \forall G \in \mathcal{C} \quad \forall r \in \mathbb{N} \quad \nabla_r(G) \leq f(r)$$

Examples:

- Graphs with bounded degree
- Planar graphs
- Graphs with bounded tree width

Properties:

Bounded in-degree, linear number of edges, ...

Definitions local bounded expansion

Definition : Class of r-neighbourhoods

Let $\mathcal{C}$ be a class of graphs, $r \in \mathbb{N}$. $\mathcal{C}_r := \{N_r^G(a) \mid G \in \mathcal{C},\ a \in G\}$

Definition : Local bounded expansion

$\mathcal{C}$ has locally bounded expansion if and only if for all integer r, $\mathcal{C}_r$ has bounded expansion.

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Differences

The number of edges can be non-linear

Our results

Theorem (Segoufin, V. '17)

Over classes of graphs with local bounded expansion, every ***FO*** *queries can be enumerate with constant delay after a pseudo-linear preprocessing.*

Theorem (Segoufin, V. '17)

Over classes of graphs with local bounded expansion, the counting problem for ***FO*** *queries is pseudo-linear.*

Theorem (Segoufin, V. '17)

Over classes of graphs with local bounded expansion, every ***FO*** *queries can be tested in constant time after a pseudo-linear preprocessing.*

Pseudo-linear ?

A function f is pseudo linear if and only if:

$$\forall \epsilon > 0,\ \exists N_\epsilon \in \mathbb{N},\ \forall n \in \mathbb{N},\ n > N_\epsilon \implies f(n) \leq n^{1+\epsilon}$$

$$n \ll n\log^i(n) \ll \text{pseudo-linear} \ll n^{1,0001} \ll n\sqrt{n}$$

"Pseudo linear $\approx n\log^i(n)$"

"Pseudo constant $\approx \log^i(n)$"

Tools

- Gaifman theorem.
- Neighbourhood cover.[1]
- Enumeration for graphs with **Bounded expansion**.[2]
- Short-cut pointers dedicated to the enumeration.

[1] Grohe et al. STOC '14
[2] Kazana, Segoufin PODS '13

Sketch of proof (1/4) : Gaifman theorem

Theorem (Gaifman)

Every first order query is a combination of sentences and local queries.

$$q(x,y) := q_1(x) \wedge q_2(y) \wedge \mathrm{dist}(x,y) > 2r$$

Where q_1 and q_2 are r-local queries.

$$G \models q_1(a) \Longleftrightarrow N_r^G(a) \models q_1(a)$$

$$\rightsquigarrow q(x,y) := Red(x) \wedge Blue(y) \wedge \mathrm{dist}(x,y) > r$$

Sketch of proof (2/4) : Neighbourhood cover

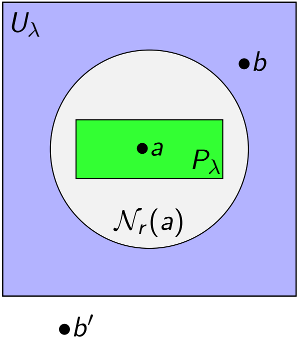
A neighbourhood cover is a set of "representative" neighbourhood.
$T := U_1, \ldots, U_\omega$ with the following properties:

- $\forall a \in G, \quad \exists U_\lambda \in T, \quad N_r(a) \subseteq U_\lambda$
- $\forall U_\lambda \in T, \quad \exists a \in G, \quad U_\lambda \subseteq N_{2r}(a)$ *(The bags have bounded expansion !)*

$\vdots$

Sketch of proof (3/4) : Using bounded expansion

$P_\lambda := \{a \in G \mid N_r(a) \subseteq U_\lambda\}$



2 kind of solutions

close enough	far enough
exemple : (a,b)	*exemple : (a,b')*
$x \in P_\lambda \wedge y \in U_\lambda$	$x \in P_\lambda \wedge y \notin U_\lambda$
Studied locally (within U_λ)	No need to check the distance

Sketch of proof (4/4) : The short-cut pointers

Let $a \in Red(G)$, let λ such that $a \in P_\lambda$ we want to enumerate :

- $b \in Blue(G) \cap U_\lambda$ Easy using the enumeration procedure within U_λ.
- $b \in Blue(G) \setminus U_\lambda$ Here we need something else.

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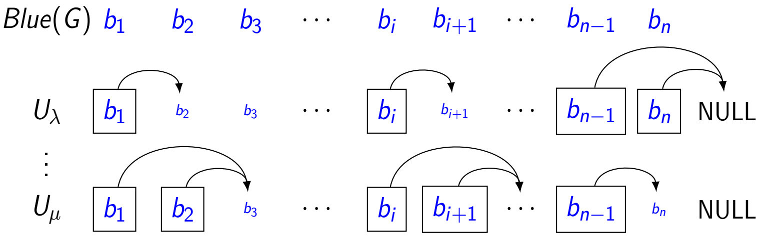

Sketch of proof conclusion

By carefully mixing the two algorithms (bounded expansion / short-cut pointers) we can enumerate FO queries.

Moreover:

- We have lexicographical enumeration.
- Given a tuple $\overline{a}$ we can compute in $O(1)$ the smallest solution $\overline{b} \geq \overline{a}$.
- We can test in time $O(1)$ if a tuple is a solution.

Future work

- Generalize to the Nowhere-Dense case.
- Enumeration with update.
  What happens if a small change occurs after the preprocessing ?

Thank you !

Any Question ?