Linear and Nonlinear type for Normed Spaces

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1 Introduction

In this lecture, we will prove that a Banach space having Rademacher type is equivalent to a Banach space having Enflo type[IvHV20]. In the following, we donate ε is uniformly distributed on the discrete cube $\{-1,1\}^n$.

Definition 1 (Rademacher type)**.** *Let* $(X, \|\cdot\|)$ *be a Banach space. We say that* X *has* Rademacher type $p \in [1,2]$ *if there exists* $C \in (0,\infty)$ *such that for all* $n \geq 1$ *and* $x_1, \ldots, x_n \in X$

$$\mathbb{E}\left\|\sum_{j=1}^{n} \varepsilon_j x_j\right\|^p \leq C^p \sum_{j=1}^{n} \|x_j\|^p.$$

We denote by $T_p^{\mathrm{R}}(X)$ *the smallest possible constant* C *in this inequality.*

Definition 2 (Enflo type)**.** *A Banach space* X *has* Enflo type p *if there exists* $C \in (0,\infty)$ *such that for all* $n \geq 1$ *and* $f : \{-1,1\}^n \to X$

$$\mathbb{E}\left\|\frac{f(\varepsilon) - f(-\varepsilon)}{2}\right\|^p \leq C^p \sum_{j=1}^{n} \mathbb{E}\|D_j f(\varepsilon)\|^p,$$

and we denote by $T_p^{\mathrm{E}}(X)$ the smallest possible constant C in this inequality. Here we define the discrete partial derivatives on the cube $\{-1,1\}^n$ as

$$D_j f(\varepsilon) := \frac{f(\varepsilon_1,\dots,\varepsilon_j,\dots,\varepsilon_n) - f(\varepsilon_1,\dots,-\varepsilon_j,\dots,\varepsilon_n)}{2}.$$

Taking $f(\varepsilon) = \sum_{j=1}^n \varepsilon_j x_j$, we see that the defining inequality of Enflo type reduces to the inequality of Rademacher type. Hence, Enflo type p implies Rademacher type p with $T_p^{\mathrm{R}}(X) \le T_p^{\mathrm{E}}(X)$. In the end of the lecture, we will prove the following theorem.

Theorem 3. *We have*

$$T_p^{\mathrm{R}}(X) \le T_p^{\mathrm{E}}(X) \le \frac{\pi}{\sqrt{2}} T_p^{\mathrm{R}}(X)$$

for every $p \in [1,2]$ and Banach space X.

2 History of the Problem

2.1 Pisier's Inequality

In the following, we donate $p \ge 1$, $f : \{-1,1\}^n \to X$ and ε, δ are independent random vectors that are uniformly distributed on the discrete cube $\{-1,1\}^n$.

In order to prove the above theorem, one of the attempt is to research on following Pisier's inequality[Pis86].

$$\mathbb{E}\,\|f(\varepsilon) - \mathbb{E}f(\varepsilon)\|^p \le C^p \mathbb{E} \sum_{j=1}^n \|\delta_j D_j f(\varepsilon)\|^p$$

If we could find a constant C such that the above inequality holds for any n. Then, if a Banach space is Rademacher type with $T_p^R(X)$.

$$\mathbb{E}\,\|f(\varepsilon) - \mathbb{E}f(\varepsilon)\|^p \le C^p \mathbb{E} \sum_{j=1}^n \|\delta_j D_j f(\varepsilon)\|^p \le (C T_p^R(X))^p \sum_{j=1}^n \mathbb{E}\|D_j f(\varepsilon)\|^p.$$

Therefore, $T_p^E(X) \le C T_p^R(X)$.

However, the above conjecture is not true in general case. By the Theorem of Pisier and Talagrand, we find that $C \sim \log n$.

$$\mathbb{E}\,\|f(\varepsilon) - \mathbb{E}f(\varepsilon)\|^p \le (2e \log n)^p \mathbb{E} \sum_{j=1}^n \|\delta_j D_j f(\varepsilon)\|^p$$

2.2 Pisier's Inequality for Special Banach Space

Another way to solve the problem is to establish Pisier's inequality not for a general Banach space but for a special class of Banach spaces. In particular, we have the following theorem.

Definition 4 (UMD space)**.** *[Esk20] A Banach space* $(X, \|\cdot\|_X)$ *is called a* UMD space *if for every* $p \in (1, \infty)$*, there exists a constant* $\beta_p \in (0, \infty)$ *such that for every* $n \in \mathbb{N}$*, every probability space* $(\Omega, \mathcal{F}, \mu)$ *and every filtration* $\{\mathcal{F}_i\}_{i=0}^n$ *of sub-*σ*-algebras of* $\mathcal{F}$*, every martingale* $\{M_i : \Omega \to X\}_{i=0}^n$ *adapted to* $\{\mathcal{F}_i\}_{i=0}^n$ *satisfies*

$$\max_{\delta=(\delta_1,\ldots,\delta_n)\in\{-1,1\}^n} \left\| \sum_{i=1}^n \delta_i (M_i - M_{i-1}) \right\|_{L_p(\Omega,\mu;X)} \leq \beta_p \|M_n - M_0\|_{L_p(\Omega,\mu;X)}.$$

The least constant $\beta_p \in (0, \infty)$ *is called the UMD*$_p$ *constant of* X *and is denoted by* $\beta_p(X)$*.*

Then, we have the following Pisier's inequality for UMD Banach space[+2002].

$$\mathbb{E}\,\|f(\varepsilon) - \mathbb{E}f(\varepsilon)\|^p \leq \beta_p^p \mathbb{E} \sum_{j=1}^n \|\delta_j D_j f(\varepsilon)\|^p$$

2.3 A Dimension-Free Pisier Inequality

In order to give a dimension-free Pisier inequality, we could somehow change the distribution of the right side. For this purpose, we give some notaions.[IvHV20]

Let ε be a random vector that is uniformly distributed on the cube $\{-1, 1\}^n$. Given $t > 0$, we let $\xi(t)$ be a random vector in the cube, independent of ε, whose coordinates $\xi_i(t)$ are independent and identically distributed with

$$\mathbf{P}\{\xi_i(t) = 1\} = \frac{1 + e^{-t}}{2}, \quad \mathbf{P}\{\xi_i(t) = -1\} = \frac{1 - e^{-t}}{2}.$$

We also define the standardized vector $\delta(t)$ by

$$\delta_i(t) := \frac{\xi_i(t) - \mathbb{E}\xi_i(t)}{\sqrt{\operatorname{Var} \xi_i(t)}} = \frac{\xi_i(t) - e^{-t}}{\sqrt{1 - e^{-2t}}}.$$

Then, we could prove the following dimension-free Pisier inequality.

Theorem 5. *For any linear space* X*, function* $f : \{-1, 1\}^n \to X$*, and convex function* $\Phi : X \to \mathbb{R}$*, we have*

$$\mathbb{E}[\Phi(f(\varepsilon) - \mathbb{E}f(\varepsilon))] \leq \int \mathbb{E}\left[\Phi\left(\frac{\pi}{2}\sum_{j=1}^n \delta_j(t) D_j f(\varepsilon)\right)\right] \mu(dt)$$

where μ *is the probability measure on* $\mathbb{R}_+$ *with density*

$$\mu(dt) := \frac{2}{\pi} \frac{1}{\sqrt{e^{2t} - 1}}\, dt.$$

For the case $\Phi(x) = \|x\|^p$, the above theorem may be slightly improved.

Theorem 6. *Let μ be as in above theorem. Then for any Banach space $(X, \|\cdot\|)$, function $f : \{-1,1\}^n \to X$, and $1 \leq p < \infty$, we have*

$$\left(\mathbb{E}\|f(\varepsilon) - \mathbb{E}f(\varepsilon)\|^p\right)^{1/p} \leq \frac{\pi}{2} \int \left(\mathbb{E}\left\|\sum_{j=1}^{n} \delta_j(t) D_j f(\varepsilon)\right\|^p\right)^{1/p} \mu(dt). \tag{1.4}$$

3 Proof of Dimension-Free Pisier Inequality

Let $f : \{-1,1\}^n \to X$ be a function defined on the discrete cube, where X is a Banach space. For each $j \in \{1, \ldots, n\}$, recall the discrete partial derivative

$$D_j f(\varepsilon) := \frac{f(\varepsilon_1, \ldots, \varepsilon_j, \ldots, \varepsilon_n) - f(\varepsilon_1, \ldots, -\varepsilon_j, \ldots, \varepsilon_n)}{2}.$$

The *discrete Laplacian* of f is then defined by

$$\Delta f := -\sum_{j=1}^{n} D_j f.$$

We denote by P_t the standard heat semigroup on the cube, that is,

$$P_t := e^{t\Delta},$$

where Δ is the discrete Laplacian defined above. Equivalently, P_t acts on a function $f : \{-1,1\}^n \to X$ by

$$P_t f := \sum_{k=0}^{\infty} \frac{t^k}{k!} \Delta^k f.$$

Then, we know that Δ is self-adjoint.

$$-\mathbb{E}[f(\varepsilon)\Delta g(\varepsilon)] = -\sum_{j=1}^{n} \mathbb{E}[f(\varepsilon) D_j g(\varepsilon)]$$

$$= -\frac{1}{2^n} \sum_{j=1}^{n} \sum_{\varepsilon \in \{-1,1\}^n} f(\varepsilon) \frac{(g(\varepsilon_j) - g(-\varepsilon_j))}{2}$$

$$= -\frac{1}{2^n} \sum_{j=1}^{n} \sum_{\varepsilon \in \{-1,1\}^n} \frac{(f(\varepsilon_j) - f(-\varepsilon_j))}{2} \cdot \frac{(g(\varepsilon_j) - g(-\varepsilon_j))}{2}$$

$$= \sum_{j=1}^{n} \mathbb{E}[D_j f(\varepsilon) D_j g(\varepsilon)].$$

Now, we begin to prove the Theorem 5 and the Theorem 6. We first prove the following lemma to characterize the heat operator P_j.

Lemma 7. *We have*

$$P_t f(x) = \mathbb{E}[f(x_1\xi_1(t), \ldots, x_n\xi_n(t))] \quad \text{for } t \geq 0,$$

and

$$D_j P_t f(x) = \frac{1}{\sqrt{e^{2t}-1}} \mathbb{E}[\delta_j(t) f(x_1\xi_1(t), \ldots, x_n\xi_n(t))] \quad \text{for } t > 0.$$

Proof. Because

$$\begin{aligned} &\mathbb{E}[f(x_1\xi_1(t), \ldots, x_n\xi_n(t))] \\ &= \sum_{\xi \in \{-1,1\}^n} \left[\prod_{i=1}^n \frac{1+e^{-t}\xi_i}{2}\right] f(x_1\xi_1, \ldots, x_n\xi_n) \end{aligned}$$

So,

$$\mathbb{E}[f_{x_j \to -x_j}(x_1\xi_1(t), \ldots, x_n\xi_n(t))] = \sum_{\xi \in \{-1,1\}^n} \left[\prod_{i=1}^n \frac{1+e^{-t}\xi_i}{2}\right] \frac{1-e^{-t}\xi_j}{1+e^{-t}\xi_j} f(x_1\xi_1, \ldots, x_n\xi_n).$$

Because

$$\frac{1}{2}\left(1 - \frac{1-e^{-t}\xi_j}{1+e^{-t}\xi_j}\right) = \frac{e^{-t}\xi_j}{1+e^{-t}\xi_j} = \frac{e^{-t}}{1-e^{-2t}}(\xi_j - e^{-t}).$$

Therefore,

$$D_j \mathbb{E}[f(x_1\xi_1(t), \ldots, x_n\xi_n(t))] = \frac{1}{\sqrt{e^{2t}-1}} \mathbb{E}[\delta_j(t) f(x_1\xi_1(t), \ldots, x_n\xi_n(t))].$$

So, we only need to prove the first equation. We need to show $\mathbb{E}[f(x_1\xi_1(t), \ldots, x_n\xi_n(t))]$ satisfies the same differential equation and initial condition as $P_t f$.

$$(\frac{\partial}{\partial t} - \Delta)\mathbb{E}[f(x_1\xi_1(t), \ldots, x_n\xi_n(t))] = 0$$

and

$$\mathbb{E}[f(x_1\xi_1(0), \ldots, x_n\xi_n(0))] = f$$

Because

$$\begin{aligned} \frac{\partial}{\partial t}\mathbb{E}[f(x_1\xi_1(t), \ldots, x_n\xi_n(t))] &= -\sum_{j=1}^n \sum_{\xi \in \{-1,1\}^n} \left[\prod_{i=1}^n \frac{1+e^{-t}\xi_i}{2}\right] \frac{e^{-t}\xi_j}{1+e^{-t}\xi_j} f(x_1\xi_1, \ldots, x_n\xi_n) \\ &= -\sum_{j=1}^n D_j \mathbb{E}[f(x_1\xi_1(t), \ldots, x_n\xi_n(t))] = \Delta \mathbb{E}[f(x_1\xi_1(t), \ldots, x_n\xi_n(t))]. \end{aligned}$$

and

$$\mathbb{E}[f(x_1\xi_1(0), \ldots, x_n\xi_n(0))] = \sum_{\xi \in \{-1,1\}^n} \left[\prod_{i=1}^n \frac{1+e^{-0}\xi_i}{2}\right] f(x_1\xi_1, \ldots, x_n\xi_n) = f.$$

The proof is complete. $\square$

Now, we will give the proof of Theorem 5.

3.1 Proof of Theorem 5

Proof. By Theorem 7 we have

$$P_0 f = f$$

$$\lim_{t\to+\infty} P_t f = \lim_{t\to+\infty} \mathbb{E}[f(x_1\xi_1(t),\ldots,x_n\xi_n(t))] = \lim_{t\to+\infty} \sum_{\xi\in\{-1,1\}^n} \left[\prod_{i=1}^n \frac{1+e^{-t}\xi_i}{2}\right] f(x_1\xi_1,\ldots,x_n\xi_n) = \mathbb{E}f$$

Because Δ is self-adjoint, then for any $g:\{-1,1\}^n \to X^*$

$$\begin{aligned}\mathbb{E}[\langle g(\varepsilon), f(\varepsilon)-\mathbb{E}f(\varepsilon)\rangle] &= -\int_0^\infty \mathbb{E}\left[\left\langle g(\varepsilon), \frac{d}{dt}P_t f(\varepsilon)\right\rangle\right] dt \\ &= -\int_0^\infty \mathbb{E}[\langle g(\varepsilon), \Delta P_t f(\varepsilon)\rangle] dt \\ &= \int_0^\infty \sum_{j=1}^n \mathbb{E}[\langle D_j P_t g(\varepsilon), D_j f(\varepsilon)\rangle] dt,\end{aligned}$$

By Theorem 7

$$\sum_{j=1}^n \mathbb{E}[\langle D_j P_t g(\varepsilon), D_j f(\varepsilon)\rangle] = \frac{1}{\sqrt{e^{2t}-1}} \mathbb{E}\left[\left\langle g(\varepsilon\xi(t)), \sum_{j=1}^n \delta_j(t) D_j f(\varepsilon)\right\rangle\right]$$

For the convex function $\Phi : X \to \mathbb{R}$, define

$$\Phi^*(y) := \sup_{x\in X}\big(\langle y,x\rangle - \Phi(x)\big), \qquad y\in X^*.$$

By the duality, we have Φ^* is convex and

$$\Phi(y) := \sup_{x\in X}\big(\langle y,x\rangle - \Phi^*(x)\big), \qquad y\in X^*.$$

So,

$$\mathbb{E}[\Phi(f(\varepsilon)-\mathbb{E}f(\varepsilon))] = \sup_{g:\{-1,1\}^n\to X^*} \{\mathbb{E}[\langle g(\varepsilon), f(\varepsilon)-\mathbb{E}f(\varepsilon)\rangle] - \mathbb{E}[\Phi^*(g(\varepsilon))]\}$$

In order to finish the proof, we only need to show that for any $g:\{-1,1\}^n \to X^*$.

$$\mathbb{E}[\langle g(\varepsilon), f(\varepsilon)-\mathbb{E}f(\varepsilon)\rangle] - \mathbb{E}[\Phi^*(g(\varepsilon))] \le \int \mathbb{E}\left[\Phi\left(\frac{\pi}{2}\sum_{j=1}^n \delta_j(t) D_j f(\varepsilon)\right)\right] \mu(dt)$$

and by the calculation above

$$\begin{aligned}&\mathbb{E}[\langle g(\varepsilon), f(\varepsilon)-\mathbb{E}f(\varepsilon)\rangle] - \mathbb{E}[\Phi^*(g(\varepsilon))] \\ &= \int_0^\infty \sum_{j=1}^n \mathbb{E}[\langle D_j P_t g(\varepsilon), D_j f(\varepsilon)\rangle] dt - \mathbb{E}[\Phi^*(g(\varepsilon))]\end{aligned}$$

$$= \int_0^\infty \frac{1}{\sqrt{e^{2t}-1}} \mathbb{E}\left[\left\langle g(\varepsilon\xi(t)), \sum_{j=1}^n \delta_j(t) D_j f(\varepsilon)\right\rangle\right] dt - \mathbb{E}[\Phi^*(g(\varepsilon))]$$

$$= \int_0^\infty \mathbb{E}\left[\left\langle g(\varepsilon\xi(t)), \sum_{j=1}^n \delta_j(t) D_j f(\varepsilon)\right\rangle\right] \mu(dt) - \mathbb{E}[\Phi^*(g(\varepsilon))]$$

Because the random vectors $\varepsilon\xi(t)$ and ε have the same distribution, $\mathbb{E}[\Phi^*(g(\varepsilon))] = \mathbb{E}[\Phi^*(g(\varepsilon\xi(t)))]$. Thus

$$= \int \mathbb{E}\left[\left\langle g(\varepsilon\xi(t)), \frac{\pi}{2}\sum_{j=1}^n \delta_j(t) D_j f(\varepsilon)\right\rangle - \Phi^*(g(\varepsilon\xi(t)))\right] \mu(dt)$$

$$\leq \int \mathbb{E}\left[\Phi\left(\frac{\pi}{2}\sum_{j=1}^n \delta_j(t) D_j f(\varepsilon)\right)\right] \mu(dt),$$

Then, the proof is complete. □

3.2 Proof of Theorem 6

Proof. For any $f : \{-1,1\}^n \to X$, then $\|f(\varepsilon) - \mathbb{E}f(\varepsilon)\|$ for all $\varepsilon \in \{-1,1\}^n$ is a vector in $L_p(\{-1,1\}^n, \mathbb{R})$. Here, we use the uniform distribution measure on $\{-1,1\}^n$.

$$\left(\mathbb{E}\|f(\varepsilon) - \mathbb{E}f(\varepsilon)\|^p\right)^{1/p} = \|f(\varepsilon) - \mathbb{E}f(\varepsilon)\|_{L^p}$$
$$= \|f(\varepsilon) - \mathbb{E}f(\varepsilon)\|_{(L^q)^*} = \sup_{\mathbb{E}\|g(\varepsilon)\|^q \leq 1} \mathbb{E}[\langle g(\varepsilon), f(\varepsilon) - \mathbb{E}f(\varepsilon)\rangle]$$

where $\frac{1}{p} + \frac{1}{q} = 1$. Using the result in Theorem 6, then for any $\mathbb{E}\|g(\varepsilon)\|^q \leq 1$

$$\mathbb{E}[\langle g(\varepsilon), f(\varepsilon) - \mathbb{E}f(\varepsilon)\rangle] = \int \mathbb{E}\left[\left\langle g(\varepsilon\xi(t)), \frac{\pi}{2}\sum_{j=1}^n \delta_j(t) D_j f(\varepsilon)\right\rangle\right] \mu(dt)$$

$$\leq \int \left(\mathbb{E}\|g(\varepsilon\xi(t))\|^q\right)^{1/q} \left(\mathbb{E}\left\|\frac{\pi}{2}\sum_{j=1}^n \delta_j(t) D_j f(\varepsilon)\right\|^p\right)^{1/p} \mu(dt)$$

$$\leq \int \left(\mathbb{E}\left\|\frac{\pi}{2}\sum_{j=1}^n \delta_j(t) D_j f(\varepsilon)\right\|^p\right)^{1/p} \mu(dt)$$

Here we use Hölder's inequality and $\mathbb{E}\|g(\varepsilon\xi(t))\|^q = \mathbb{E}\|g(\varepsilon)\|^q$ as the random vectors $\varepsilon\xi(t)$ and ε have the same distribution. □

4 Proof of Theorem 3

Proof. We only need to show $T_p^{\mathrm{E}}(X) \leq \frac{\pi}{\sqrt{2}} T_p^{\mathrm{R}}(X)$. Because ε and $-\varepsilon$ have the same distribution and $x \mapsto \|x\|^p$ is convex.

$$\mathbb{E}\left\|\frac{f(\varepsilon)-f(-\varepsilon)}{2}\right\|^p = \mathbb{E}\left\|\frac{f(\varepsilon)-\mathbb{E}f(\varepsilon)-f(-\varepsilon)+\mathbb{E}f(-\varepsilon)}{2}\right\|^p \leq \mathbb{E}\|f(\varepsilon)-\mathbb{E}f(\varepsilon)\|^p$$

By Theorem 5 with $\Phi(x) = \|x\|^p$

$$\mathbb{E}\left\|\frac{f(\varepsilon)-f(-\varepsilon)}{2}\right\|^p \leq \int \mathbb{E}\left\|\frac{\pi}{2}\sum_{j=1}^n \delta_j(t) D_j f(\varepsilon)\right\|^p \mu(dt).$$

Let $\xi'(t)$ be an independent copy of $\xi(t)$ and ε' be an independent copy of ε. Then, we have

$$\mathbb{E}\left\|\sum_{j=1}^n \delta_j(t) D_j f(\varepsilon)\right\|^p = \mathbb{E}_\xi\left\|\sum_{j=1}^n \frac{\xi_j(t)-\mathbb{E}_{\xi'}\xi'_j(t)}{\sqrt{\operatorname{Var}\xi_j(t)}} D_j f(\varepsilon)\right\|^p$$

By Jensen's inequality

$$\leq \mathbb{E}_\xi \mathbb{E}_{\xi'}\left\|\sum_{j=1}^n \frac{\xi_j(t)-\xi'_j(t)}{\sqrt{\operatorname{Var}\xi_j(t)}} D_j f(\varepsilon)\right\|^p$$

$$= \mathbb{E}\left\|\sum_{j=1}^n \frac{\xi_j(t)-\xi'_j(t)}{\sqrt{\operatorname{Var}\xi_j(t)}} D_j f(\varepsilon)\right\|^p$$

Because $\xi(t)-\xi'(t)$ has the same distribution as $\varepsilon'(\xi(t)-\xi'(t))$

$$= \mathbb{E}\left\|\sum_{j=1}^n \varepsilon'_j \frac{\xi_j(t)-\xi'_j(t)}{\sqrt{\operatorname{Var}\xi_i(t)}} D_j f(\varepsilon)\right\|^p$$

By Rademacher type condition,

$$\leq T_p^{\mathrm{R}}(X)^p \sum_{j=1}^n \mathbb{E}\left\|\frac{\xi_j(t)-\xi'_j(t)}{\sqrt{\operatorname{Var}\xi_i(t)}} D_j f(\varepsilon)\right\|^p,$$

$$\leq T_p^{\mathrm{R}}(X)^p \sum_{j=1}^n \mathbb{E}\left\|\frac{\xi_j(t)-\xi'_j(t)}{\sqrt{\operatorname{Var}\xi_i(t)}}\right\|^p \mathbb{E}\|D_j f(\varepsilon)\|^p,$$

As $p \leq 2$ and by Cauchy's inequality,

$$\mathbb{E}\left|\frac{\xi_j(t)-\xi'_j(t)}{\sqrt{\operatorname{Var}\xi_i(t)}}\right|^p \leq \left(\frac{\mathbb{E}[(\xi_j(t)-\xi'_j(t))^2]}{\operatorname{Var}\xi_i(t)}\right)^{p/2} = 2^{p/2}$$

So

$$\mathbb{E}\left\|\frac{f(\varepsilon)-f(-\varepsilon)}{2}\right\|^p \leq \left(\frac{\pi}{\sqrt{2}}T_p^{\mathrm{R}}(X)\right)^p \sum_{j=1}^{n}\mathbb{E}\|D_j f(\varepsilon)\|^p,$$

which implies $T_p^{\mathrm{E}}(X) \leq \frac{\pi}{\sqrt{2}}T_p^{\mathrm{R}}(X)$. □

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