High-Dimensional Probability (M16)

Non-Examinable (Graduate Level)

Alexandros Eskenazis

This course is an introduction to nonasymptotic methods for the study of random structures in high dimensions. Emphasis will be put on developing a common set of tools that has proved to be useful in different areas of mathematics, statistics and computer science. The key phenomenon explored in this course is that of *concentration of measure*. Informally, the phenomenon asserts that if $X_1, \ldots, X_n$ are independent random variables, then any random variable of the form $f(X_1, \ldots, X_n)$ does not deviate much from its mean $\mathbf{E}f(X_1, \ldots, X_n)$, provided that the function $f(x_1, \ldots, x_n)$ is not too sensitive to any of its coordinates x_i. In the core part of this course we shall make this statement rigorous via the *semigroup method*, which amounts to investigating concentration properties of measures μ via the study of Markov processes having μ as their stationary distribution. After developing some general principles we shall investigate more refined concentration properties of specific distributions that are primarily motivated by geometric applications, which will constitute the final part of the course.

The main part of the course will cover a selection of topics including:

- Isoperimetry and concentration of measure
- Classical functional inequalities (Poincaré, logarithmic Sobolev) for Markov semigroups
- Lipschitz concentration and transportation cost inequalities
- Sharp thresholds and superconcentration
- Infimum convolution inequalities and two-level concentration

Depending on the interests of the audience, the following additional topics may be discussed:

- Concentration on product spaces à la Talagrand
- Suprema of Gaussian processes
- Refined isoperimetric-type inequalities in Gauss space

The final part of the course will be devoted to geometric applications of the tools developed, primarily in convexity and metric geometry.

Prerequisites

A solid understanding of graduate level probability would be useful.

Literature

1. M. Ledoux, *The Concentration of Measure Phenomenon*. American Mathematical Society, 2001.
2. R. van Handel, *Probability in High Dimensions*. Available at https://web.math.princeton.edu/~rvan/APC550.pdf.
3. R. Vershynin, *High Dimensional Probability*. Cambridge University Press, 2018, also available at https://www.math.uci.edu/~rvershyn/papers/HDP-book/HDP-book.pdf.