Book review of "From Stein to Weinstein and Back. Symplectic Geometry of Affine Complex Manifolds" by Kai Cieliebak and Yakov Eliashberg, AMS Colloquium Publications vol. 59, AMS, Providence, RI, 2012, 364 pp., Hardcover, ISBN-10: 0-8218-8533-2, ISBN-13: 978-0-8218-8533-8, List price US$78.[1]

The book under review is a landmark piece of work that establishes a fundamental bridge between complex geometry and symplectic geometry. It is both a research monograph of the deepest kind and a panoramic companion to the two fields. The main characters are Stein manifolds on the complex side, respectively Weinstein manifolds on the symplectic side. The connection between the two is established by studying the corresponding geometric structures up to homotopy, or deformation, a context in which the h-principle plays a fundamental role.

Stein manifolds are fundamental objects in complex analysis of several variables and in complex geometry. There are several equivalent definitions, but the most intuitive one is the following: they are the complex manifolds that admit a proper holomorphic embedding in some $\mathbb{C}^n$ (they are in particular noncompact). Stein manifolds can be thought of as being a far reaching generalization of domains of holomorphy in $\mathbb{C}^n$. Other important classes of examples are noncompact Riemann surfaces, and complements of hyperplane sections of smooth algebraic manifolds. The latter are actually examples of affine (algebraic) manifolds, meaning that they are described by polynomial equations in $\mathbb{C}^n$. While affine manifolds are open analogues of smooth algebraic varieties, Stein manifolds form a much larger class within the analytic category. They are sometimes thought of as being natural domains for holomorphic maps with values in arbitrary complex manifolds [26].

In the wake of Cartan's foundational theorems A and B and Serre's fundamental GAGA principle [3, 37], much of the literature on Stein manifolds dealt with their analytic properties and used coherent sheaves as a primary tool. This well-established and rich subject is taken up in several classical monographs, for example [23, 16]. One upshot is that the biholomorphism type of a Stein manifold is very rigid: for example, there are uncountably many non-biholomorphic Stein manifolds that are C^∞-small perturbations of the open ball in $\mathbb{C}^n$ [25].

Here we come upon the *first shift in point of view* operated by the present book: Stein manifolds are not studied up to biholomorphism, but rather up to *deformation, or homotopy* of the Stein structure.

Such a shift in point of view may seem entirely unnatural from the perspective of complex analysis, but it is natural from the topological perspective of the Oka principle for Stein manifolds, which we now describe. As early as 1939 Oka [33] had discovered that the second Cousin problem on the existence of globally defining functions for positive analytic divisors inside domains of holomorphy of $\mathbb{C}^n$ admits a holomorphic solution if and only if it admits a continuous solution (see also [34, pp. 24–35]). This discovery gradually evolved into one of the most powerful tools for the study of Stein spaces, called the *Oka principle*, formulated as follows: *"on a reduced Stein space X, problems which can be cohomologically formulated have only topological obstructions. In other words, such problems are holomorphically solvable if and only if they are continuously solvable"* [16, p. 145], see also [36].

[1]MSC (2010): 53D05, 53D10, 53D99, 32E10, 32Q28, 55P10, 57R42.

The Oka principle appears nowadays to have been a precursor for an entirely new strand of mathematics known under the name of *homotopy principle, or h-principle.* The h-principle was formulated as a broad unifying concept by Gromov in his 1970 ICM address [20]. As we understand it now, it refers to a class of theorems whose content is the following: the existence of a "formal" solution of a PDE (of geometric origin, and usually heavily underdetermined) implies the existence of a "genuine" solution (see below for a discussion of "formal" solutions *vs.* "genuine" solutions). Formal solutions are always objects of a topological nature, while genuine solutions are always objects of an analytic nature. It is often the case that the correspondence between formal solutions and genuine solutions holds for arbitrary finite-dimensional families, in which case we speak of the *parametric h-principle.* The parametric h-principle can be succinctly formulated as saying that the inclusion of the space of genuine solutions into the space of formal solutions is a (weak) homotopy equivalence (the adjective "weak" refers to the property that the inclusion induces isomorphisms on all homotopy groups; the brackets signify that the stronger statement about homotopy equivalence always holds, though weak homotopy equivalence is the only thing that is used in practice, see the discussion in [12, §6.2]). In the case of 1-dimensional families, the h-principle becomes a statement about homotopies: the existence of a formal homotopy between two genuine solutions implies the existence of a genuine homotopy. The classical reference for the h-principle in its various incarnations is the book by Gromov [18]. A recent and friendly treatment is given by Eliashberg and Mishachev in [12].

To grasp the meaning of genuine solutions *vs.* formal solutions, it is instructive to examine the case of smooth immersions. This was historically the first instance of the h-principle other than Oka's work, and is known as the Smale-Hirsch theorem [39, 24]. Though slightly weaker, the original statement proved by Smale and Hirsch is essentially equivalent to the following: the parametric h-principle holds for smooth immersions of a k-dimensional manifold M^k into an n-dimensional manifold N^n for $n > k$. In other words, the inclusion $\mathrm{Imm}(M^k, N^n) \hookrightarrow \mathrm{FImm}(M^k, N^n)$ of the space of immersions into the space of formal immersions, consisting of injective bundle maps from $TM \to M$ to pull-backs $f^*TN \to M$ by arbitrary smooth maps $f : M \to N$, is a homotopy equivalence. The inclusion associates to a smooth immersion $f : M^k \to N^n$ the formal immersion $df : TM \to f^*TN$. The particular case $M = S^1$, $N = \mathbb{R}^2$ was established much earlier by Whitney and Graustein [42]. Smale proved his famous sphere eversion theorem [38] by showing that the space of formal immersions $S^2 \to \mathbb{R}^3$ is connected, a statement that reduces to the vanishing of the second homotopy group of $\mathbb{R}P^3$. This is symptomatic of the fact that formal solutions to a problem governed by the h-principle are classified by algebraic invariants.

The h-principle also holds in certain holomorphic setups, most notably for holomorphic immersions of Stein manifolds into $\mathbb{C}^n$ (Gromov-Eliashberg [22, 21]). Using the same kind of methods Gromov and Eliashberg proved that any Stein manifold of dimension n embeds holomorphically into $\mathbb{C}^{\lfloor \frac{3n}{2} \rfloor + 1}$, an optimal result [11]. Underlying these developments is the classical Oka principle stated above, and we refer to the recent monograph [13] for a self-contained treatment of this circle of ideas. It is worth emphasizing that, in this context, although the methods are very much of a topological flavor, the point of view is reminiscent of the classical one in that the complex structure is fixed. By contrast, deformations of Stein structures as considered in the book under review allow the complex structure to change.

Let us now discuss the notion of *Stein structure* and that of *deformation of a Stein structure*. These use the following alternative definition of Stein manifolds due to Grauert [15]: a complex manifold (W, J) is Stein if and only if it admits an exhausting, i.e. proper and bounded from below, *J-convex* function $\varphi : W \to \mathbb{R}$. The condition of J-convexity means that the 2-form $\omega_\varphi := -dd^c\varphi$, with $d^c\varphi := d\varphi \circ J$, satisfies $\omega_\varphi(v, Jv) > 0$ for all tangent vectors $v \neq 0$. A *Stein structure* on an even-dimensional smooth manifold W is a pair (J, φ) consisting of a complex structure J and of a J-convex exhausting smooth function $\varphi : W \to \mathbb{R}$. Note that the condition of J-convexity is open in the C^2-topology, so that the function φ can, and often will be assumed to be Morse, meaning that its critical points are nondegenerate. If the function φ can be chosen to have only finitely many (nondegenerate) critical points we speak of a *Stein manifold of finite type*. A *deformation, or homotopy of Stein structures* is a continuous 1-parameter family (J_t, φ_t), $t \in [0, 1]$ of generalized Morse functions subject to the additional requirement that critical points "do not escape to infinity" during the homotopy. Here by *generalized Morse function* we mean a function whose critical points are either nondegenerate or of birth-death type (these functions form the codimension 1 stratum in the space of all functions, the open stratum being that of Morse functions). The condition of no escape to infinity is subtle, and without imposing it the theory would be void: any two Stein structures on $\mathbb{C}^n$ would be homotopic! This is carefully explained in §11.6 of the book, see in particular Remark 11.24. In formal terms, it means that on any small time subinterval of $[0, 1]$ the manifold W can be written as a countable union $W = \cup_{k\geq 1} W_t^k$ over families W_t^k which are smooth in the time parameter t and consist of regular sublevel sets for φ_t. In the case of Stein manifolds of finite type the relevant notion of deformation requires that the union over $t \in [0, 1]$ of the critical sets of the functions φ_t be compact.

The fact that the J-convexity condition is open in the C^2-topology, so that the J-convex function can be assumed to be (generalized) Morse, has the following fundamental topological consequence : a Stein manifold of complex dimension n – hence of real dimension $2n$ – retracts onto a CW-complex of dimension $\leq n$, and if the manifold is of finite type it retracts onto a CW-complex which is finite, and in particular compact (this last fact echoes the above assumption on homotopies of Stein structures of finite type). Indeed, a direct computation essentially due to Milnor [31, §7] shows that all critical points of a Morse function which is J-convex have index $\leq n$. The resulting obvious homological conditions $H_i(W; \mathbb{Z}) = 0$, $i > n$ are actually so strong in the case of affine manifolds that they imply the Lefschetz hyperplane theorem. We will see below that there is a symplectic reason why the index is $\leq n$, namely that an isotropic submanifold of a symplectic manifold of dimension $2n$ has dimension $\leq n$.

Indeed, the *second shift in point of view* operated by the book under review is the one *from complex geometry to symplectic geometry.*

A *symplectic manifold* is a pair (M^{2n}, ω) consisting of a smooth manifold of even real dimension $2n$ and of a closed and nondegenerate 2-form $\omega \in \Omega^2(M)$. An immersion $f : N^k \to M^{2n}$ is called *isotropic* if $f^*\omega = 0$. We then necessarily have $k \leq n$, and if $k = n$ we speak of a *Lagrangian immersion.* The existence of symplectic forms in a fixed cohomology class on open manifolds is governed by the h-principle (see [19] and [12, §10.2]). Lagrangian immersions into a given symplectic manifold are also governed by the h-principle (see [20, 27], [12, §16.3], and §7.2). Similarly, the h-principle governs phenomena in contact geometry, the odd-dimensional counterpart of symplectic geometry [12, §§12, 16], [14].

A first connection between symplectic geometry and complex geometry is the following: given a symplectic manifold (W, ω), there always exists an almost complex structure J that is compatible with the symplectic form ω in the sense that $\omega(\cdot, J\cdot)$ is a Riemannian metric. Here by "almost complex structure" we mean a smooth endomorphism of the tangent bundle $J \in \mathrm{End}(TM)$ such that $J^2 = -\mathrm{Id}$. Symplectic manifolds are thus naturally *almost Kähler manifolds*. It turns out that the space of such almost complex structures is contractible, so that the "moduli space up to deformation" of the resulting almost Kähler manifold with fixed symplectic structure is reduced to a point.

At a much deeper conceptual level, the geometry of Stein manifolds connects to symplectic geometry via the h-principle, which governs essential phenomena in both worlds. The book under review turns this philosophy into a solid theorem. Indeed, the authors establish a precise

correspondence between Stein manifolds and Weinstein manifolds.

Weinstein manifolds are open symplectic objects. It appears that the ultimate reason why the h-principle holds for Stein immersions is because Stein manifolds are open symplectic objects, and open symplectic objects tend to be flexible.

Weinstein manifolds were first introduced by Weinstein [41] and their name was coined by Eliashberg and Gromov [10]. A *Weinstein manifold* is a tuple (W, ω, X, φ) where (W, ω) is a symplectic manifold, X is a complete vector field which is *Liouville* for ω, meaning that $\mathcal{L}_X \omega = \omega$, or $d(\iota_X \omega) = \omega$, and $\varphi : W \to \mathbb{R}$ is an exhausting smooth generalized Morse function which is *Lyapunov for* X. That φ is a Lyapunov function for X means that $X \cdot \varphi \geq c(|X|^2 + |d\varphi|^2)$ for some Riemannian metric on W and for some constant $c > 0$ (we also say that X is a *pseudo-gradient for* φ). The key class of examples – coming from classical mechanics, the historical ancestor of symplectic geometry – consists of phase spaces T^*M of smooth manifolds M, endowed with the canonical symplectic form given locally by $d\mathbf{p} \wedge d\mathbf{q}$ in coordinates $\mathbf{q}$ on the base and in dual coordinates $\mathbf{p}$ in the fiber. In this case one can take φ to be a $\mathbf{q}$-dependent Morse perturbation of the function $(\mathbf{q}, \mathbf{p}) \mapsto |\mathbf{p}|^2$, the norm being considered with respect to some Riemannian metric on the base, and X to be a suitable Hamiltonian perturbation of the radial vector field $\mathbf{p}\frac{\partial}{\partial \mathbf{p}}$ (see §11.4 in the book).

Stein manifolds are Weinstein manifolds. Indeed, given a Stein structure (J, φ) on a manifold W such that φ is a generalized Morse function, we define a Weinstein structure $(\omega_\varphi, X_\varphi, \varphi)$ as follows. The 2-form $\omega_\varphi := -dd^c\varphi$, with $d^c\varphi := d\varphi \circ J$, is symplectic by the assumption of J-convexity for φ. It then turns out that J is compatible with ω_φ and the gradient $X_\varphi := \nabla^{g_\varphi}\varphi$ of φ with respect to the Riemannian metric $g_\varphi := \omega_\varphi(\cdot, J\cdot)$ is a Liouville vector field for ω_φ. Obviously φ is a Lyapunov function for X_φ.

This is the key correspondence of the book. Given a manifold W of even dimension the authors denote $\mathfrak{Stein}$ the space of Stein structures (J, φ) on W such that φ is generalized Morse, and by $\mathfrak{Weinstein}$ the space of Weinstein structures (ω, X, φ) on W (such that φ is generalized Morse), with a canonical map

$$\mathfrak{Stein} \longrightarrow \mathfrak{Weinstein}, \qquad (J, \varphi) \longmapsto (\omega_\varphi, \nabla^{g_\varphi}\varphi, \varphi).$$

We obtain in particular a canonical diagram

$$\begin{tikzcd} \mathfrak{Stein} \arrow[rr] \arrow[dr] & & \mathfrak{Weinstein} \arrow[dl] \\ & \mathfrak{Morse} & \end{tikzcd} \tag{1}$$

where the vertical arrows are the canonical projections $(J, \varphi) \mapsto \varphi$, respectively $(\omega, X, \varphi) \mapsto \varphi$ over the space $\mathfrak{Morse}$ of generalized Morse functions on W. Theorem 1.2 in the Introduction of the book encompasses most of its main results and states the following.

Theorem. (Theorem 1.2. in §1, see also §13) *The fibers of the two vertical arrows at a generalized Morse function φ are (weakly) homotopy equivalent.*

The authors make the explicit conjecture (Conjecture 1.4 in §1) that the map

$$\mathfrak{Stein} \to \mathfrak{Weinstein}$$

is a (weak) homotopy equivalence.

The above theorem is phrased in the spirit of the h-principle, though its content is *not* that of an h-principle. Rather, its message is that the classification of Stein manifolds up to homotopy of the Stein structure is not a problem of a complex analytic nature, but of a symplectic nature. That the symplectic geometry of Weinstein manifolds is much more flexible than the complex geometry of Stein manifolds is illustrated by the fact that homotopic Weinstein structures are symplectomorphic (§11.2 in the book). There is no such statement in the Stein category.

So how flexible is the symplectic world of Weinstein manifolds? It turns out that this is one of the core questions of symplectic geometry.

Following again the h-principle philosophy, let us call a symplectic phenomenon "flexible" if it only depends on differential geometric properties of the underlying manifold, and "rigid" otherwise. We have already mentioned the example of Lagrangian immersions, which obey the h-principle: a manifold of dimension n admits a Lagrangian immersion into an exact symplectic manifold of dimension $2n$ if and only if it admits a formal Lagrangian immersion therein, meaning an immersion covered by an isotropic injective homomorphism from its tangent bundle to the tangent bundle of the target. Two Lagrangian immersions are homotopic through Lagrangian immersions if and only if they are homotopic through formal Lagrangian immersions. The existence of Lagrangian immersions is thus a problem of a "flexible" nature. In contrast, each of the previous assertions fails if we replace "immersions" by "embeddings". Lagrangian embeddings do not obey an h-principle, there are obstructions that go beyond smooth ones (e.g. S^n, $n \geq 2$ has no Lagrangian embedding into standard symplectic space $\mathbb{R}^{2n}$ [17]). The existence and classification of Lagrangian embeddings is thus a problem of a "rigid" nature.

Drawing the line between flexibility and rigidity is at the heart of the subject called *symplectic topology.* The category of Weinstein manifolds features both flexibility and rigidity properties, and we now discuss an example of each.

One of the corollaries of the techniques developed in the book is the following theorem, originally due to Cieliebak [4]. We explain below that, given a Weinstein manifold (W, ω, X, φ) of dimension $2n$, all the critical points of the Morse function φ have index $\leq n$. Call a Weinstein manifold of dimension $2n$ *subcritical* if it admits a Weinstein structure (ω, X, φ) with φ a Morse function without critical points of index n. Obvious examples are products $V \times \mathbb{C}$, where V is a Weinstein manifold. Any subcritical Weinstein manifold is diffeomorphic, by Morse theory, to such a product. Cieliebak's theorem states that any subcritical Weinstein manifold is also deformation equivalent, and hence symplectomorphic to such a product (see §14.4). This is an instance of flexibility: a phenomenon that is present at the smooth level is replicated at the symplectic level.

On the other hand, many rigidity results concerning Weinstein manifolds have been proved recently, starting with a groundbreaking paper by McLean [30] who, in the wake of Seidel and Smith [35], constructed infinitely many distinct Weinstein structures on $\mathbb{R}^{2n}$, $n \geq 4$. This was extended to $n = 3$ by Abouzaid and Seidel [1]. Other constructions of "exotic" Weinstein manifolds have been given by Maydanskiy [28], Maydanskiy-Seidel [29], and Bourgeois-Ekholm-Eliashberg [2]. All these results rely in some form or another on handle presentations of Stein manifolds, either directly or through the bias of Lefschetz fibrations. In order to distinguish the various Weinstein manifolds constructed in this way most of the literature uses an invariant called *symplectic homology.* This is a variant of Hamiltonian Floer homology adapted to open J-convex manifolds, first defined by Viterbo [40] (see also [7] for a related earlier construction). The final outcome is the following (Abouzaid-Seidel [1, Corollary 1.2] for the affine case, Theorem 17.2 in the book under review for the general case): *for any Weinstein manifold of finite type, there are infinitely many mutually non-symplectomorphic, hence non-deformation equivalent, Weinstein manifolds diffeomorphic to it.*

The above two examples of flexibility and rigidity illustrate the constant tension between these two aspects of symplectic topology.

Let us recall the example of non-biholomorphic Stein structures on small C^∞ perturbations of the ball in $\mathbb{C}^n$ mentioned at the beginning of this review. Such Stein structures are all deformation equivalent and subcritical. As such, they become doubly indistinguishable in the homotopic setup of the book and in the symplectic category. However, some form of "discrete" rigidity does persist in the world of Weinstein manifolds, since the moduli space of Weinstein structures carried by a given diffeomorphism type has at least countably many components, as seen above.

"Modern Symplectic Geometry was born in a long battle for establishing the borderline between the areas where the h-principle holds and where it fails. Since the beginning of the eighties the Symplectic Rigidity army scored a lot of victories which brought to life the whole new area of Symplectic Topology*. However, there were also several amazing unexpected breakthroughs on the Flexibility side [...]. In fact, it is still possible that in spite of great recent successes of Symplectic*

Topology, the world of Symplectic Rigidity is just a small island floating in the Flexible Symplectic Ocean." (Eliashberg and Mischachev [12, §6.1, p. 61]*).*

This is a quotation from 2002, which today sounds prophetic: ten years later, in 2012, Emmy Murphy made a fundamental discovery [32] that allowed to extend flexibility phenomena for Weinstein manifolds well beyond the subcritical range.

In order to explain Murphy's discovery in our context we need to describe symplectic handle presentations of Weinstein manifolds. Recall that, on a smooth manifold, a Morse function together with a choice of smooth pseudo-gradient vector field determines a handle presentation as follows: given a critical level c containing a unique critical point of index λ, for $\epsilon > 0$ small enough the sublevel set $\leq (c+\epsilon)$ is diffeomorphic to the sublevel set $\leq (c-\epsilon)$ with a handle of index λ attached to it. The core of the handle is a ball inside the unstable manifold of the critical point with respect to the negative pseudo-gradient vector field, and the attaching sphere is the intersection of this unstable manifold with the level set $c-\epsilon$. Note that the critical points of the Morse function coincide by assumption with the zeroes of the pseudo-gradient vector field.

In the symplectic setup of a Weinstein manifold $(W^{2n}, \omega, X, \varphi)$ the handle decomposition determined by φ and X has the following special features: (i) the unstable manifolds of the negative pseudo-gradient vector field $-X$ are isotropic, meaning that ω restricts to the zero form (§11.3); (ii) the primitive $\theta := \iota_X \omega$ of ω restricts to a contact form on any regular level set of φ, meaning that $\theta \wedge (d\theta)^{n-1} \neq 0$. Thus every regular level set is a *contact manifold*; (iii) each attaching sphere is a *Legendrian submanifold* inside the corresponding regular level set, meaning that θ restricts to the zero form. A handle attachment featuring these properties is called a *Weinstein handle attachment*. Note in particular that the isotropic nature of the unstable manifolds implies that all the critical points of a Morse function φ defining a Weinstein structure have index $\leq n$, a fact that has been mentioned previously. This provides a symplectic perspective on the fact that Stein manifolds have the homotopy type of CW-complexes of dimension $\leq n$.

Any finite type Weinstein manifold can be built in a finite number of steps by such a sequence of handle attachments. The necessary piece of data for a Weinstein handle attachment is an isotropic sphere inside the contact boundary of a sublevel set (together with a framing of its normal bundle). In the spirit of our previous flexibility example of subcritical Weinstein manifolds, it turns out that one can first attach all the subcritical handles, and then simultaneously attach the critical handles along a Legendrian link.

Now comes into play Murphy's discovery in [32]: inside *any* contact manifold of *dimension* ≥ 5 there is an *explicit* class of embedded Legendrian spheres (called Legendrian knots), and, more generally, of embedded Legendrian links, which obey the h-principle. Murphy calls them *loose Legendrian knots*, respectively *loose Legendrian links* (see §7.7 in the book). Building on this, Cieliebak and Eliashberg define in §11.8 a finite type Weinstein manifold of dimension ≥ 6 to be *flexible* if it can be realized by handle attachment along a loose Legendrian link. Since any Legendrian knot/link can be C^0-approximated by a loose Legendrian knot/link (lying actually in the same formal Legendrian isotopy class), it follows in particular that any diffeomorphism type of Weinstein manifold of dimension ≥ 6 carries a flexible

Weinstein structure! Chapter 14 of the book is dedicated to proving that *flexible Weinstein structures are indeed "flexible"*, meaning that they satisfy appropriate versions of the h-principle.

The description of Weinstein manifolds via handle attachments plays a central role throughout the book. The authors develop in particular in Chapter 10 a theory of cancellation of critical points in the J-convex setting, similar to the cancellation theory one encounters in the proof of the smooth h-cobordism theorem. This is in turn a direct offspring of a groundbreaking theorem of Eliashberg [8], which states that *there exists a Stein structure on any almost complex manifold of dimension* $2n$, $n > 2$ *which admits a Morse function without critical points of index* $> n$. The original proof of this theorem used in a crucial way handle attachments and appropriate versions of the h-principle, an old friend.

As the authors mention in the introduction to the book, their "original goal was a complete and detailed exposition of the existence theorem for Stein structures in [8]". The outcome is a fascinating exploration and recasting of half a century of mathematics, embedded into a landmark research monograph. We can only be grateful to the authors, and perhaps the best way to express this gratitude is to read the book. Here is a tip borrowed from Chris Wendl's thorough review on MathSciNet: the two expository articles by the authors [5, 6] can serve as points of entry. Finally, we refer to the very recent survey paper by Eliashberg [9] for a much broader perspective on the topic of symplectic flexibility.

Acknowledgements. The author is grateful to Mihai Damian, Janko Latschev, and Maksim Maydanskiy, for having read a preliminary version of this text and for having made substantial comments.

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