ANR-DFG PROJECT: SUSPENSIONS

Suspensions consist of many small particles in a fluid, that is a liquid or a gas. They are ubiquitous in nature, for example as dust, mud and blood. Applications include food industry, paints, centrifuges as well as many aspects of climate science, and environmental and medical engineering. Suspensions exhibit very complex phenomena that one wishes to understand in order to predict and tailor their behavior. Typical phenomena include sedimentation of particles, an increase of viscosity of the fluid, collective drag forces and even viscoelastic behavior. Which of these macroscopic effects are visible and how strong they are is highly sensitive to the properties of the particles, for example their number, size and shape.

Mathematical analysis plays a crucial role for the deep understanding of suspensions, in particular, since they are difficult to access by means of experiments and numerical simulations due to the large number of particles as well as the different inherent length and time scales. On the microscopic level, one can model a suspension by individual equations of motion for the particles which are coupled to an equation that describes the fluid. However, not only are the particles influenced by the surrounding fluid, but the fluid is also perturbed by the particles. That way, the particles interact with each other through the fluid in a complex way. For applications, one is typically not interested in the individual particle trajectories, though, but one rather wishes to understand *effective* macroscopic or mesoscopic quantities like the averaged particle density or fluid velocity.

Our goal is therefore the mathematically rigorous derivation of effective equations from the underlying microscopic systems, and their analysis. This objective is addressed by a progressive mathematical approach with three levels.

(1) The first concerns the derivation of the effective properties of the fluid under the artificial assumption that the particle positions are prescribed, which simplifies the analysis.
(2) In a second step, we aim to incorporate the particle dynamics in the model in order to derive fully coupled systems.
(3) Thirdly, we study the obtained effective models, for instance with regard to existence, uniqueness and qualitative properties such as long-time behavior of solutions.

The nature of the interaction in suspensions, which is implicit, long-range, and singular, makes this topic mathematically very interesting and challenging. We pursue the analysis by combining and advancing techniques from mathematical fluid mechanics, homogenization, mean-field analysis and kinetic theory. The last years have seen the beginning of tremendous progress in this research area. This project brings together experts from France and Germany to boost this progress in order to rigorously justify existing models as well as to reveal several new phenomena.