Arithmetic Geometry
Feb. 28th- March 4th 2022

Bruno Klingler: On abelian differentials and their periods

(Joint work with Leonardo Lerer) An abelian differential is a smooth projective complex curve endowed with an algebraic 1-form. I will discuss the arithmetic and functional transcendence properties of abelian differentials, as seen in the format of "bi-algebraic geometry" developed in Hodge theory.

Lenny Taelman: Deformations of ordinary Calabi-Yau varieties

(Joint work with Lukas Brantner). Let X be a variety with trivial canonical bundle over a perfect field k of characteristic p. We show that if X is ordinary, then its deformation space is a formal torus over the Witt vectors of k. In particular, deformations are unobstructed and these varieties have a canonical lift to characteristic zero. We also show that this canonical lift is algebraizable.

Special cases of this were known by results of Serre-Tate, Deligne, Nygaard, Ward, and Achinger-Zdanowicz. Our proof uses derived deformation theory in an essential way, and one aim of the talk is to show how techniques from derived algebraic geometry can be used in proving results in classical deformation theory.

Wiesia Nizioł and Pierre Colmez: On p-adic pro-etale cohomology of analytic spaces, I and II

We will discuss properties of p-adic pro-etale cohomology of p-adic analytic spaces (comparison with de Rham type cohomologies, duality). This is based on a joint work of Pierre Colmez, Sally Gilles, and Wieslawa Niziol.

Sarah Zerbes: Euler systems and the Birch—Swinnerton-Dyer conjecture for abelian surfaces

Euler systems are one of the most powerful tools for proving cases of the Bloch–Kato conjecture, and other related problems such as the Birch and Swinnerton-Dyer conjecture.

I will recall a series of recent works (variously joint with Loeffler, Pilloni, Skinner) giving rise to an Euler system in the cohomology of Shimura varieties for GSp(4), and an explicit reciprocity law relating the Euler system to values of L-functions. I will then explain recent work with Loeffler, where we use this Euler system to prove new cases of the BSD conjecture for modular abelian surfaces over Q, and for modular elliptic curves over imaginary quadratic fields.

Kestutis Cesnavicius: The Bass–Quillen phenomenon for reductive group torsors

For a regular ring R, the Bass–Quillen conjecture predicts that every vector bundle on the relative affine space $\mathbb{A}^d_R$ descends to R. I will discuss the generalization of this conjecture to torsors under more general reductive groups.

Piotr Achinger: Fundamental groups of rigid spaces

I will survey different notions of (non-Tannakian) fundamental groups of rigid spaces, including variants of de Jong's fundamental group recently developed in joint work with Lara and Youcis, as well as the Betti fundamental group (for rigid spaces over C((t))) defined using joint work with Talpo.

Alberto Vezzani: Relative de Rham cohomology for adic spaces

(Joint work with A.-C. Le Bras.) We show how to define and study a relative overconvergent de Rham cohomology for adic spaces in mixed characteristic, using the 6-functor formalism and the language of solid quasi-coherent modules. We also extend this construction to the equi-characteristic p case, taking values on quasi-coherent sheaves over the relative Fargues-Fontaine curve, as conjectured by Fargues and Scholze. Finiteness results and the relation to classical cohomology theories are also discussed.

Olivier Benoist: On sums of squares of real meromorphic functions

Hilbert's 17th problem (solved by Artin in 1927) asks whether all positive semidefinite real polynomials are sums of squares of real rational functions. In this talk, I will present some positive results on a complex-analytic variant of this question.

Andrei Yafaev: Point counting, Lower bounds for degrees of special points and the Andre-Oort conjecture

(Joint work with Gal Biniyamini and Harry Schmidt) The outstanding problem in the full and unconditional proof of the Andre-Oort conjecture was that of bounding below the degrees of special points.

We will present a work in which we formulate a conjecture on heights of special points and deduce from it (via a point counting theorem) the required lower bounds. The proof of our conjecture on heights has recently been announced by Pila, Shankar and Tsimerman, (with a contribution by Esnault and Groechenig).

Anna Cadoret: Etale cohomology with ultraproduct coefficients

(Joint work with Weizhe Zheng) I will explain how (and why !) to define constructible sheaves with ultraproduct coefficients.

Jakob Stix: A π_1-obstruction to lifting to characteristic 0

(joint work with Hélène Esnault and Vasudevan Srinivas) We will explain the notion of a p'-discretely finitely generated pro-finite group. The question of whether the étale fundamental group of a variety in characteristic $p > 0$ admits such a "discrete structure" is related to the possibility of lifts to characteristic 0. The Roquette curve with an automorphism group exceeding the Hurwitz bound allows to construct examples with a π_1-obstructed lifting problem.

Fabrizio Andreatta: Ekedahl-Oort stratifications and the Hodge-Tate period map

In this talk I will show how to use the mod p Hodge-Tate period map for Hodge type Shimura varieties to prove to prove that Ekedahl-Oort strata are non empty.

Dimitri Wyss: Geometric applications of non-archimedean integration

(Joint work with Francesca Carocci and Giulio Orecchia) Non-archimedean and motivic integration can be seen as theories that produce well-behaved invariants of the arc-space of a (singular) algebraic variety X. In special cases one can relate these invariants to more classical invariants (e.g. Betti or Hodge numbers) of X, giving new ways of manipulating them. I will try to give an overview of the main ideas and then report on a recent project with Francesca Carocci and Giulio Orecchia, where we express so called BPS-numbers for local del Pezzo and K3 surfaces, a certain curve counting theory, as non-archimedean integrals.

François Charles: Formal-analytic geometry in dimension 2

(Joint work with Jean-Benoît Bost) We will describe the geometry of so-called formal-analytic arithmetic surfaces, which are an arithmetic analogue of neighborhoods of curves embedded in complex surfaces. We will study those under some positivity assumptions, proving a simple inequality that will allow us to prove an algebraization theorem that generalizes a result of Calegari-Dimitrov-Tang, as well, as bounds on fundamental groups.

Giuseppe Ancona: Algebraic classes in mixed characteristic and André's p-adic periods

(Joint work with D. Fratila) Motivated by the study of algebraic classes in mixed characteristic, we define a countable subalgebra of $\mathbb{Q}_p$ which we call the algebra of "André's p-adic periods". We will explain the analogy and the difference between these p-adic periods and the classical complex periods. For instance, they both contain several examples of special values of classical functions (logarithm, gamma function,...) and they share transcendence properties. On the other hand, the classical tannakian formalism which is used to bound the transcendence degree of complex periods has to be modified in order to be used in the p-adic setting. We will discuss concrete exemples of all these instances though elliptic curves and Kummer extensions.

Benjamin Schraen: The action of GL2 on the cohomology of Shimura curves

(Joint work with Christophe Breuil, Florian Herzig, Yongquan Hu and Stefano Morra) The p-adic Langlands correspondence for the group $\mathbf{GL}_2(\mathbb{Q}_p)$ allows us to describe the action of the group $\mathbf{GL}_2(\mathbb{Q}_p)$ on the completed p-adic cohomology of the tower of modular curves. If F is a strict finite extension of $\mathbb{Q}_p$, the situation is much less understood. In this talk I'll describe some results concerning the description of this action of $\mathbf{GL}_2(F)$ on the completed cohomology of a tower of Shimura curves.

Peter Jossen: E-functions and Geometry

Siegel introduced E-functions in 1929 with the goal of generalising the Lindemann-Weierstrass Theorem on the algebraic independence of exponentials of algebraic numbers. E-functions are power series with algebraic coefficients which behave in many respects like the exponential function. Last year, Javier and myself gave a negative answer to Siegel's question whether all E-functions are expressible in terms of hypergeometric functions. In my talk, I will try to amend Siegel's question: I will explain how E-functions can be produced from geometry (from exponential motives), and we can then ask whether, or in fact conjecture that, all E-functions come from geometry.

Javier Fresán: Quantitative sheaf theory

The applications of the Weil conjectures to problems of analytic number theory, such as the equidistribution of exponential sums over finite fields, often rely on the existence of uniform bounds for Betti numbers. Except in very simple cases, the uniformity of such estimates is not a formal feature of étale cohomology. Recently, Will Sawin introduced a notion of complexity of a complex of ℓ-adic sheaves on a quasi-projective variety that is well-behaved with respect to the six operations, in the sense that the complexity of the output objects is bounded solely in terms of the input objects. In many interesting cases, it provides bounds for the sum of Betti numbers that can be made uniform in the characteristic of the base field. I will give a friendly introduction to the main ideas of the construction (based on Beilinson and Saito's theory of characteristic cycles), and explain a few applications from a joint work with Arthur Forey and Emmanuel Kowalski.

Yves André: L'application de Betti dans tous ses états

L'application de Betti est une certaine fonction réelle-analytique multiforme associée aux sections d'un schéma abélien, qui apparaît en géométrie diophantienne sous de nombreuses formes. Elle est devenue un outil fondamental dans le domaine des " intersections atypiques " et aussi, plus tard, dans les questions d'uniformité concernant les points rationnels. L'exposé fera le point sur la notion, ses origines, son lien subtil avec Kodaira-Spencer, et certaines de ses nombreuses applications.