SuSy & Random Matrices

In Honor of Tom Spencer

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Institut Henri Poincaré
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- David Brydges
University of British Columbia, Vancouver, Canada

  The Ising Model and the Theory of Heaps

  I will explain some recent work by Tyler Helmuth who has found a simple derivation of the solution of the Ising model in two dimensions along the lines of Kac and Ward (1952) and recent work by Cimasoni. His derivation uses the combinatorial theory of heaps, which I will explain, to write certain correlations as sums over non-backtracking walks with complex weights.

- François David
Institut de Physique Théorique, CEA, Saclay

  Random matrix ensembles for quantum spins and decoherence

  I present a class of random matrix ensembles relevant for the study of quantum decoherence for quantum spins. These ensembles generalize the standard GUE ensemble. For a single spin j, they lead to exact solutions for the dynamics of decoherence and for quantum diffusion, including non-markovian regimes. Generalizations to interacting spins will be discussed.

- Margherita Disertori
Université de Rouen, Rouen

  A supersymmetric non linear sigma model for quantum diffusion

  We study a lattice model which is expected to reflect the Anderson localization and delocalization transition for real symmetric band matrices in 3 dimensions. In this statistical mechanics model, the field takes values in a supermanifold based on the hyperbolic plane. In this context we prove the existence of a diffusive phase in 3 dimensions at low temperatures and localization at high temperatures for any dimension $d \geq 1$. Our analysis uses Ward identities coming from internal supersymmetry. This is joint work with Tom Spencer and Martin Zirnbauer.

- Konstantin Efetov
Ruhr-Universität Bochum, Bochum, Allemagne

  Localization and critical diffusion of quantum dipoles in two dimensions

  Quantum propagation of dipole excitations in two dimensions is discussed. This problem differs from the conventional Anderson localization due to existence of long range hops. We found that the critical wavefunctions of the dipoles always exist which manifest themselves by a scale independent diffusion constant. If the system is T-invariant the states are critical for all values of the parameters. Otherwise, there can be a "metal-insulator" transition between this "ordinary" diffusion and the Lévy-flights (the diffusion constant logarithmically increasing with the scale). These results follow from the two-loop analysis of the modified non-linear supermatrix σ-model.

- Laszlo Erdös
University of Munich, Allemagne

Universality of local spectral statistics of random matrices.

The Wigner-Gaudin-Mehta-Dyson conjecture asserts that the local eigenvalue statistics of large random matrices exhibit universal behavior depending only on the symmetry class of the matrix ensemble. For invariant matrix models, the eigenvalue distributions are given by a statistical mechanical model with logarithmic interaction. Special values of the inverse temperature $\beta = 1, 2, 4$ correspond to the orthogonal, unitary and symplectic ensembles. For other values of beta there is no matrix model behind this model, but the statistical physics interpretation of the log-gas is still valid for all positive beta and universality holds. We demonstrate that the strong local ergodicity of the Dyson Brownian motion is the the intrinsic mechanism behind universality for both invariant and non-invariant ensembles.
This is a joint work with H.-T. Yau, B. Schlein, J. Yin and Paul Bourgade.

- Jürg Fröhlich
ETH, Zürich, Suisse

Localization induced by random exchange interactions

Some fairly recent mathematically rigorous results on electron localization induced by random exchange interactions —due to Bourgain, Egli and the lecturer— are reviewed. Applications of these results to some concrete systems of condensed-matter physics (the hexaboride system $\mathrm{Eu}_x\mathrm{Ca}_{1-x}\mathrm{B}_6$) are sketched. (The lecturer has reached an age that makes it impossible for him to present proofs in public. But he is happy to explain the main physical ideas.)

- Yan Fyodorov
Queen Mary, University of London, London, UK

Freezing Transition: from $1/f$ landscapes to Characteristic Polynomials of Random Matrices and the Riemann zeta-function.

In the talk (based on a joint work with Jon Keating and Ghaith Hiary) I will argue that the freezing transition scenario, previously conjectured to take place in the statistical mechanics of $1/f$-noise random energy models, governs, after reinterpretation, the value distribution of the maximum of the modulus of the characteristic polynomials of large random unitary (CUE) matrices. I then conjecture that the results extend to the large values taken by the Riemann zeta-function over stretches of the critical line $s = 1/2 + \mathrm{i}t$ of constant length, and present the results of numerical computations of the large values of $\zeta(1/2 + \mathrm{i}t)$. The main purpose is to draw attention to possible connections between the statistical mechanics of random energy landscapes, random matrix theory, and the theory of the Riemann zeta function.

- Thomas Guhr
  Universität Duisburg--Essen, Duisburg, Allemagne

  Supersymmetry and Random Matrices: avoiding the saddle-point approximation.

  In mesoscopic physics, Efetov derived the supersymmetric nonlinear sigma model for the motion of an electron in a (stochastic) disorder potential. It has a very broad range of applications. Mathematically, it results from a saddle-point approximation, i.e. it involves an asymptotic expansion. In the random matrix context, one can, under certain conditions, avoid the saddle-point approximation and solve the supersymmetric model exactly. This leads to integrals over supergroups (super-Itzykson-Zuber, Harish-Chandra, Gelfand), not cosets. The eigenvalues of the supermatrices acquire a prominent role. In particular, the Jacobians (Berezinians) directly reflect the determinant structure of the correlations. The mathematical physics aspects, advantages and limitations, as well as some of the physics applications will be discussed.

- Eugene Kanzieper
  HIT, Holon, Israël

  Integrability and The Replica Approach in Random Matrix Theory

  In this talk, I shall present an overview of the exact approach to zero-dimensional replica field theories. Both fermionic, bosonic and supersymmetric variations of replicas will be considered, with a particular emphasis placed on the phenomenon of fermionic-bosonic factorisation of random-matrix-theory correlation functions. The latter will be discussed for all three Dyson's symmetry classes.

- Vladimir Kazakov
  École Normale Supérieure, Paris

  Classical integrability for quantum integrable systems.

  Integrable systems, such as random matrix models, (super)spin chains and two-dimensional field theories, quantum or classical, share very similar mathematical structures. An efficient way to analyze quantum integrable systems is to reduce the problem (for some physical quantities) to classical discrete integrable equations, such as the bilinear finite difference Hirota equation. I will review the applications of Hirota dynamics for exact analysis of rational quantum (super)spin chains and 2D QFT's at a finite volume. Recently, these methods allowed to find the exact system of equations, the so called AdS/CFT Y-system, for the spectrum of anomalous dimensions in the 4-dimensional maximally supersymmetric super Yang-Mills theory, the $N = 4$ SYM theory. I will sketch out the physical and mathematical origins of the AdS/CFT Y-system.

- Sasha Sodin
  IAS, Princeton, États-Unis

  A gradient model with non-convex interaction

  TBA.

- Oleg Yevtushenko
  University of Munich, Allemagne

  Critical almost diagonal RMT: from anomalous scaling to Lévy flights

  Almost diagonal Random Matrix Theory (ADRTM) is defined as the RMT model where off-diagonal matrix elements are parametrically smaller than diagonal ones. The ADRTMs are usually not exactly solvable and cannot be explored by using standard field-theoretical methods, for example, the supersymmetric (SuSy) σ-model. On the other hand, the ADRTMs are useful toy models for many physical problem. Critical Gaussian ADRTM allows one to study the localization transition in large space dimension. I will briefly review an alternative supersymmetric method, the SuSy virial expansion, which yields an expansion of correlation functions of the ADRTM in a number of independent (almost) localized eigenstates. The main part of the talk will be addressed to the application of the SuSy virial expansion to the theory of the critical ADRTM, including scaling behavior of a return probability and of a generalized diffusion propagator of wave packets. I will show that the latter reveals an interrelation between correlations of critical eigenstates at small space distances and Lévy flights in the critical ADRMT.

- Jean Zinn-Justin
  Institut de Physique Théorique, CEA, Saclay

  Matrix models and renormalization group approach

  The statistical properties of large size matrices have been studied extensively, but the main problem is that our knowledge depends on specific classes for which exact solutions can be found. In particular, no quantum field theory with matrix fields has been solved in the large size limit. Several years ago, Brézin and myself have proposed exploring approximate methods based on renormalization group ideas, which I will review.

- Martin Zirnbauer
  Köln University, Allemagne

  Weyl symmetry of an orbital integral transform for symmetric superspaces.

  Recent numerical and experimental studies on disordered quantum Hamiltonian systems have revealed a surprising symmetry property of the distribution function of the local density of states. I will argue that the observed symmetry is related to the Weyl group invariance of an orbital integral transform for the Iwasawa decomposition of a noncompact symmetric space. More precisely, it results from an adaptation of the Iwasawa decomposition to the case of Riemannian symmetric superspaces.