

Dynamics and Geometry of Moduli Spaces

Homework assignment (based on Lectures 1–4)

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March 16, 2023

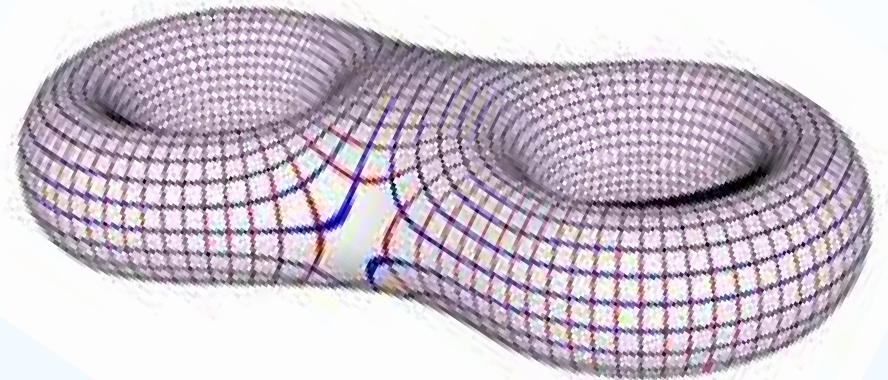
Homework assignment

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- Homework assignment
- What is the ambient stratum?
- Exercise

Homework assignment 1

Homework assignment

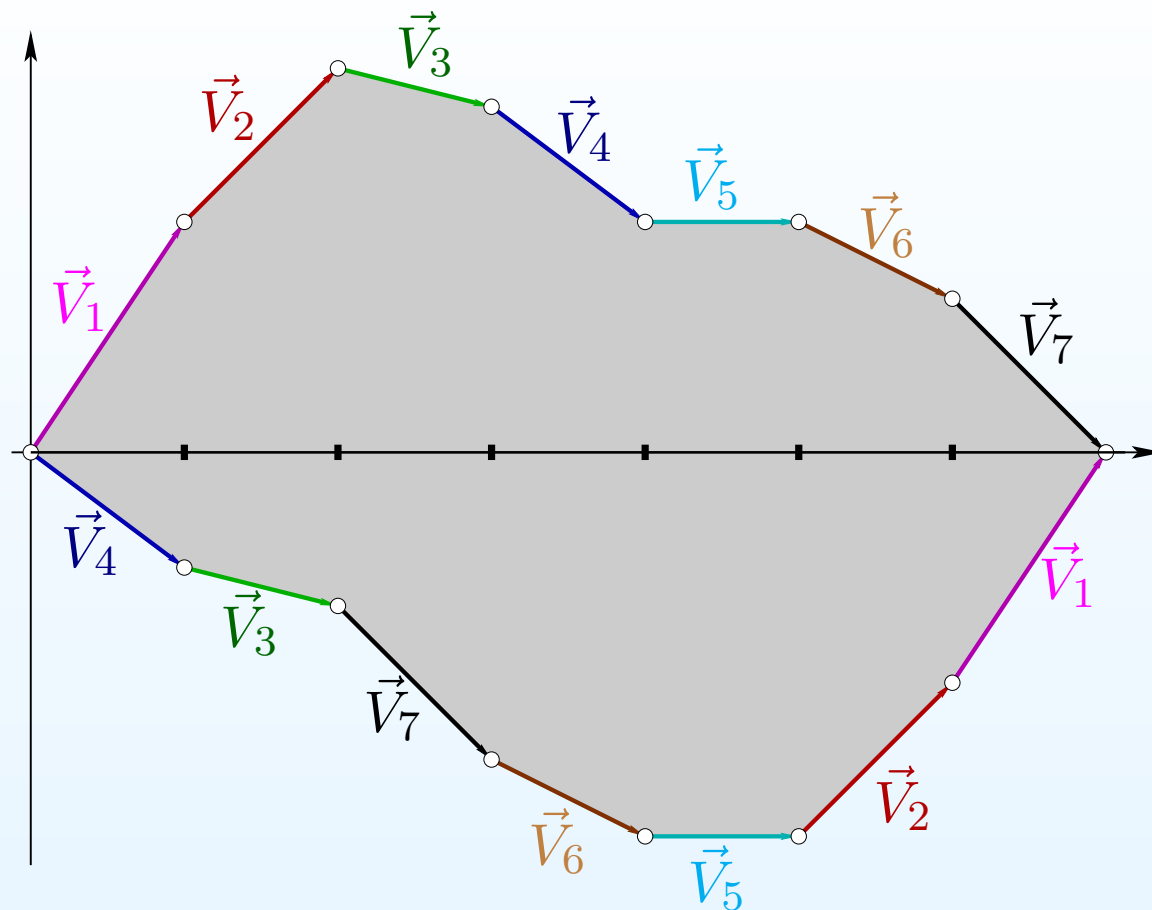


Picture created by Jian Jiang

Questions.

- *To what stratum belongs this square-tiled surface?*
- *Find all realizable separatrix diagrams for this stratum.*
- *To which of the found diagrams corresponds the square-tiled surface from the picture?*

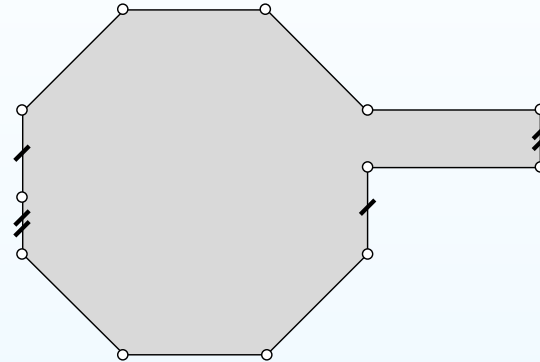
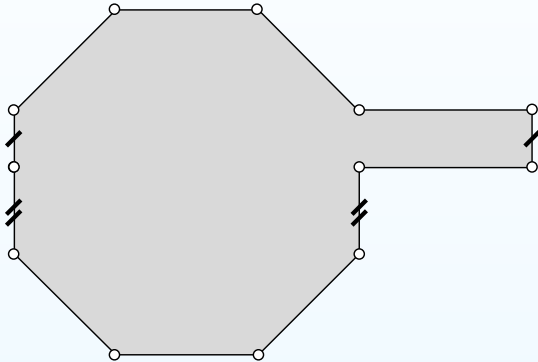
What is the ambient stratum?



What is the ambient stratum $\mathcal{H}(d_1, \dots, d_n)$ for the translation surface obtained by identifying the pairs of sides corresponding to the same vectors $\vec{V}_1, \dots, \vec{V}_7$ by parallel translations?

Exercise

- Check that the following two flat surfaces belong to the stratum $\mathcal{H}(4)$.



- Present a collection of closed curves representing a basis of cycles in the first homology of the surfaces. Compute their intersection numbers.

A *hyperelliptic involution* is a holomorphic involution of a Riemann surface such that the quotient over the involution is a Riemann sphere.

- Compute the parity of the spin structure for these surfaces (and notice that it is not the same).
- Determine which of the two surfaces is hyperelliptic and find the hyperelliptic involution in geometric terms. Find the Weierstrass points (the fixed points of the hyperelliptic involution). Check that there are $2g + 2$ such points.