Dynamics and Geometry of Moduli Spaces

Homework assignment
(based on Lectures 6–7)

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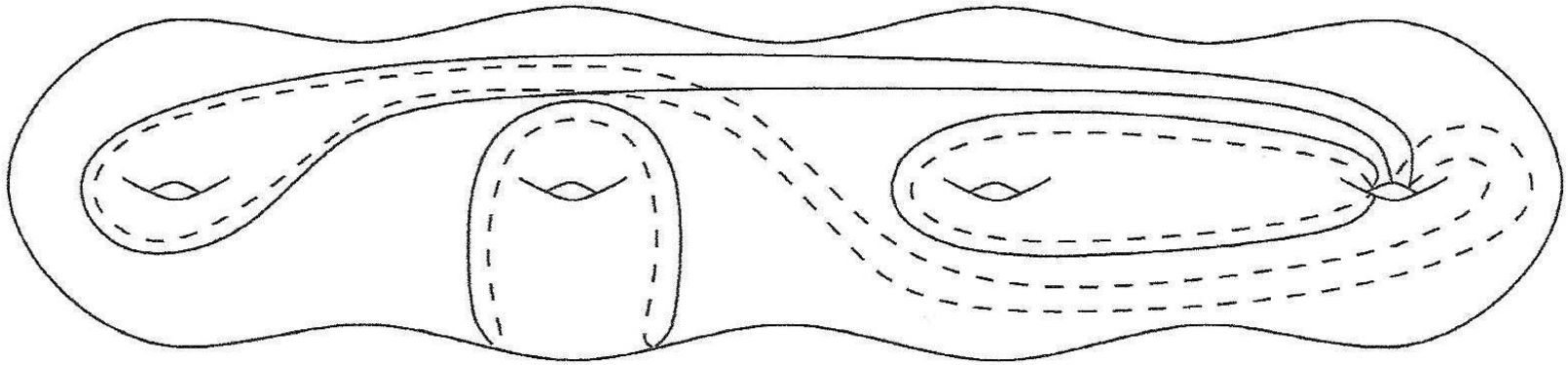
Homework assignment 2

Separating curves

Exercise. *Prove that all curves presented at the picture are separating.*

Hint: choose an appropriate basis of cycles and verify that intersection numbers of each curve with all basic cycles are zero.



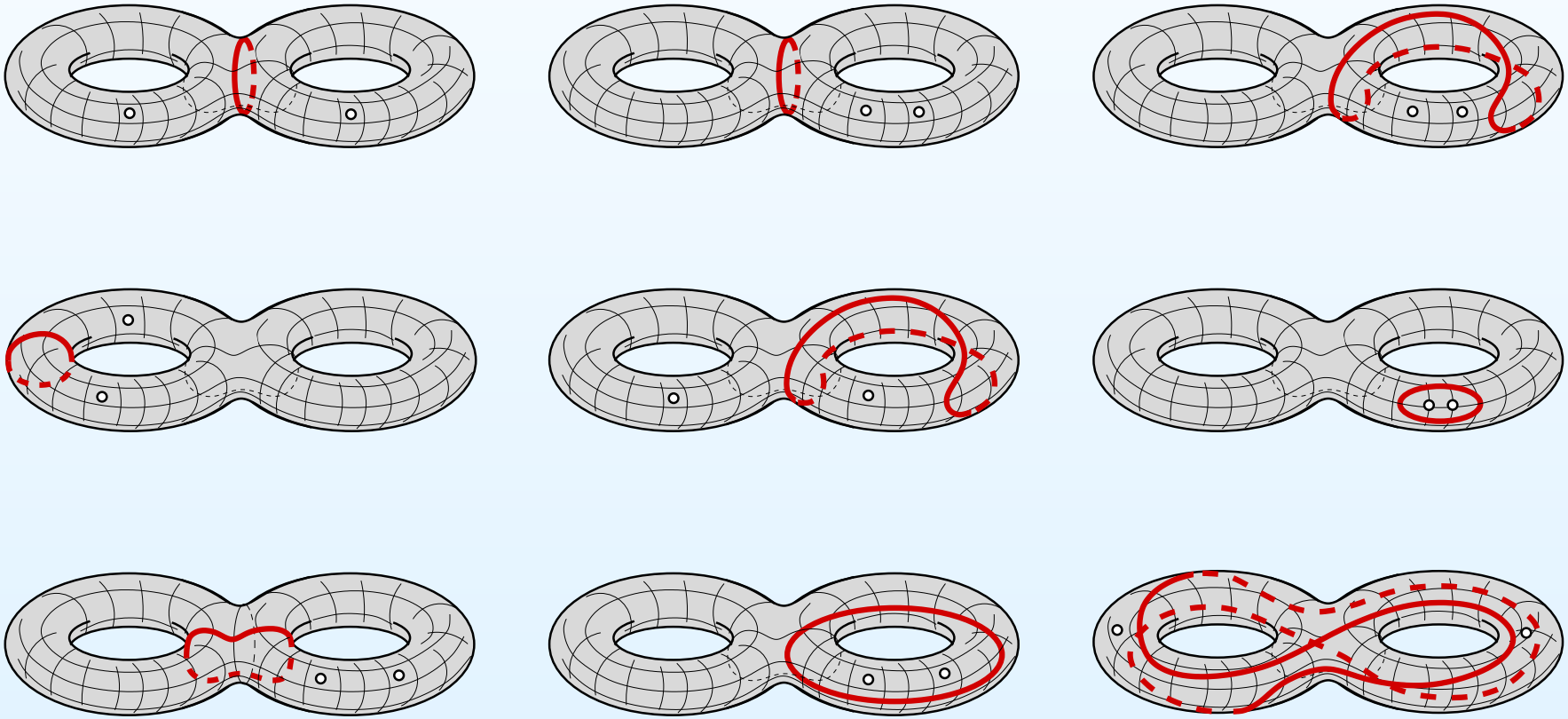
The picture is taken from the book of B. Farb and D. Margalit “A Primer on Mapping Class Groups”.

Exercise. *Detect which curves are essential and which essential curves belong to the same orbit of the mapping class group.*

Orbits of the mapping class group

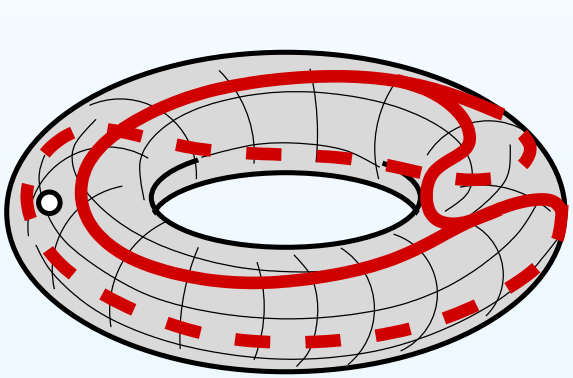
Select all simple closed curves in the picture below which might be isotopic to simple closed hyperbolic geodesics on a twice-punctured surface of genus two.

How many distinct orbits of $\mathrm{Mod}_{2,2}$ they represent? Indicate which curves correspond to which orbit.

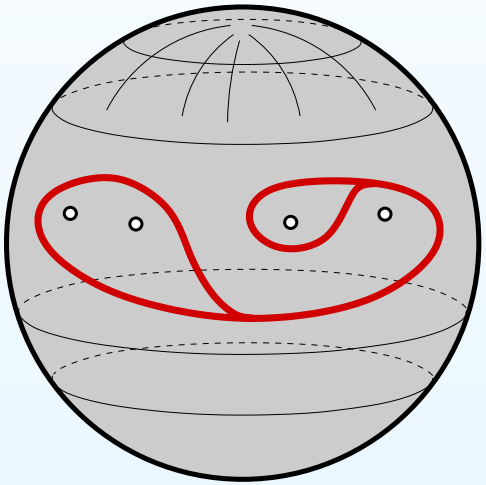


Train-tracks

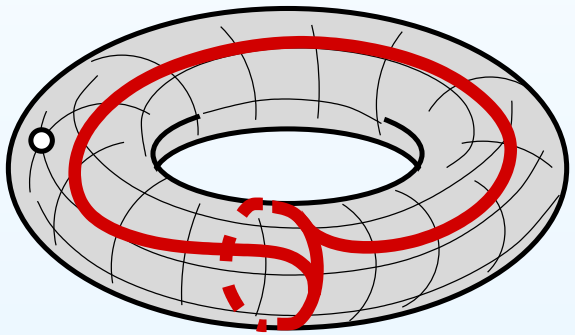
Which of the given train-tracks τ_1, τ_2, τ_3 might carry a simple closed hyperbolic geodesic? Indicate some legitimate weights if you claim that the train track carries a simple closed hyperbolic geodesic.



τ_1



τ_2



τ_3

Can any of the given train-tracks τ_1, τ_2, τ_3 carry *different* simple closed hyperbolic geodesic? Indicate the corresponding different legitimate collections of weights if you claim that.

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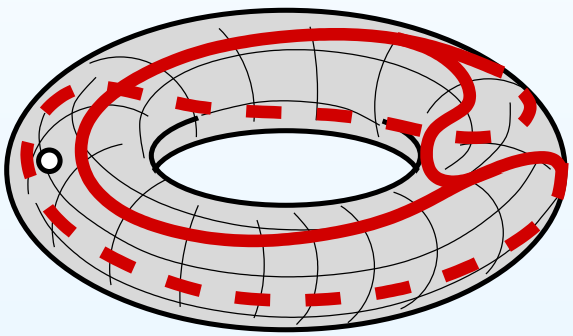
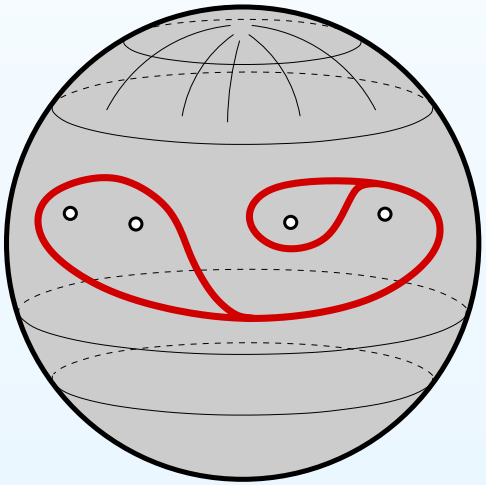
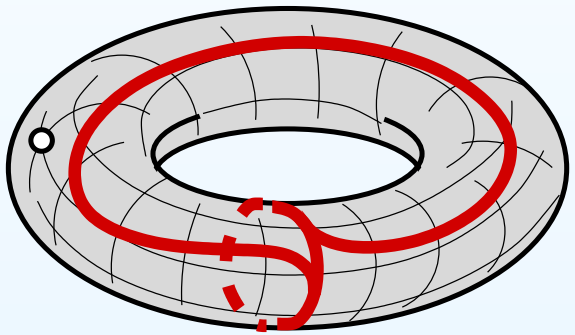


τ_1



τ_2



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