

# Geometry and dynamics in moduli spaces

## Homework assignment 2

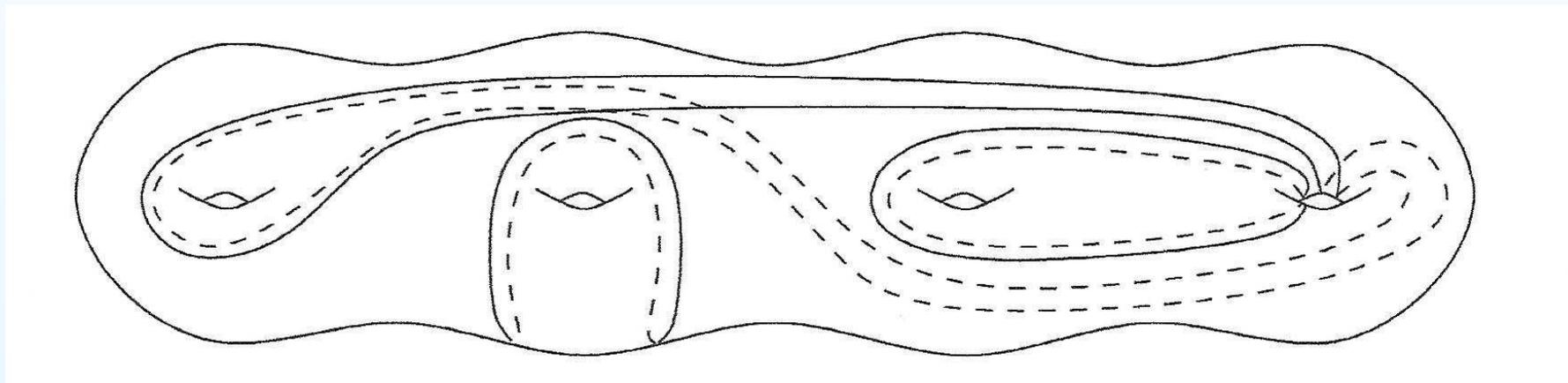
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## Separating curves

**Exercise.** *Prove that all curves presented at the picture are separating.*

Hint: choose an appropriate basis of cycles and verify that intersection numbers of each curve with all basic cycles are zero.



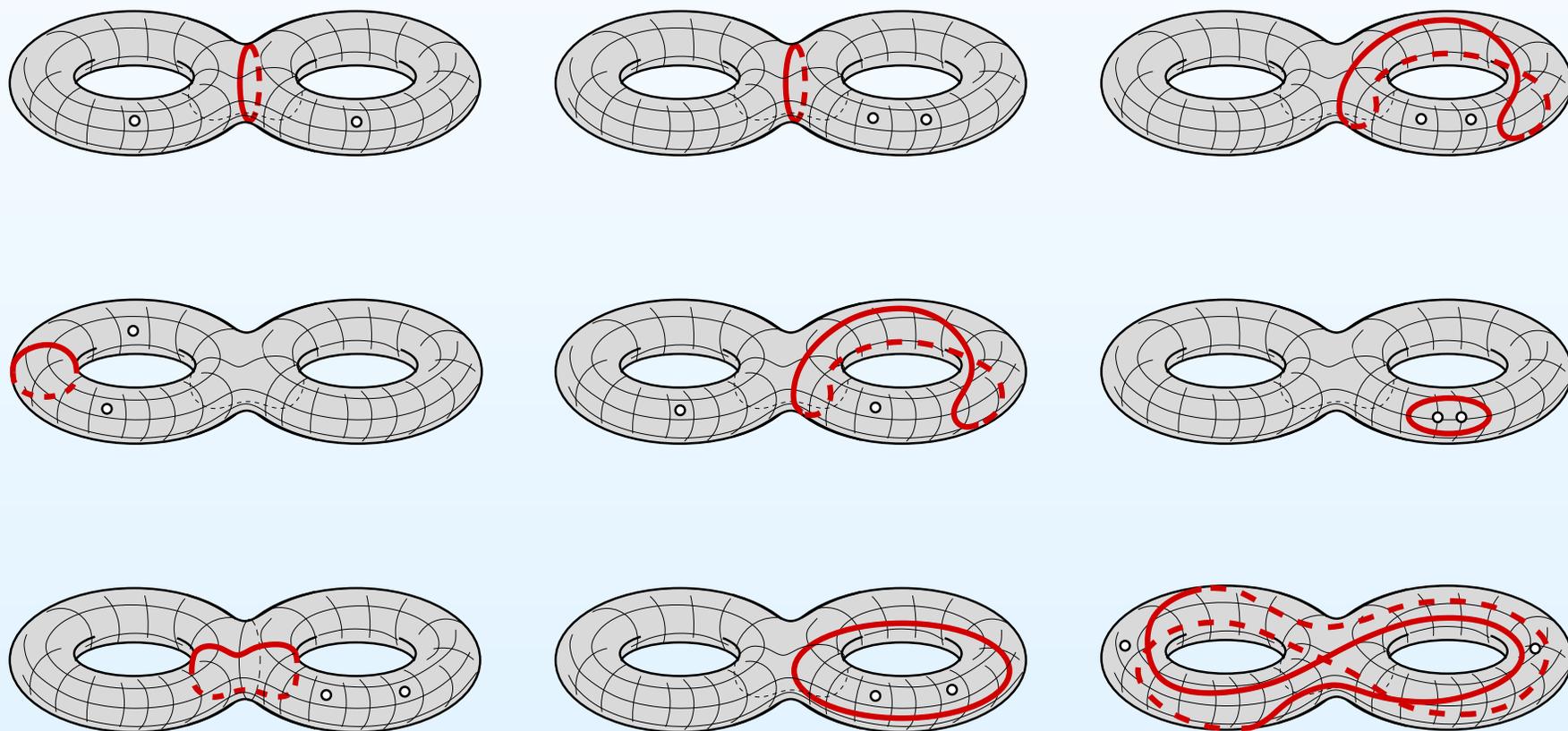
The picture is taken from the book of B. Farb and D. Margalit “A Primer on Mapping Class Groups”.

**Exercise.** *Detect which curves are essential and which essential curves belong to the same orbit of the mapping class group.*

## Orbits of the mapping class group

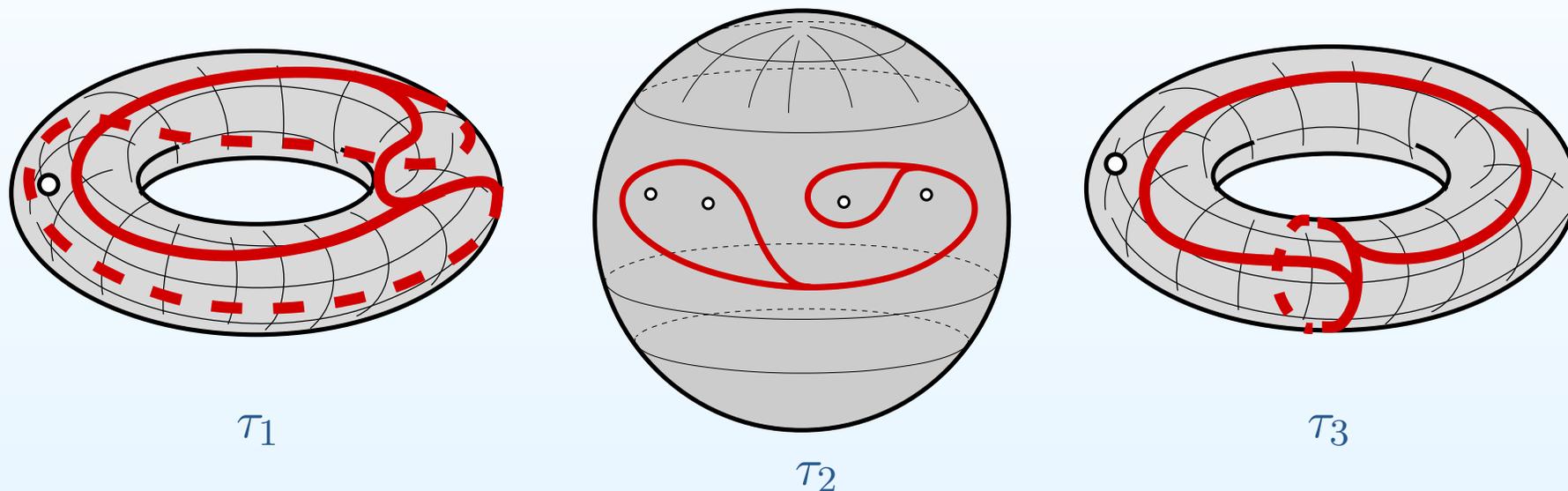
Select all simple closed curves in the picture below which might be isotopic to simple closed hyperbolic geodesics on a twice-punctured surface of genus two.

How many distinct orbits of  $\text{Mod}_{2,2}$  they represent? Indicate which curves correspond to which orbit.



## Train-tracks

Which of the given train-tracks  $\tau_1, \tau_2, \tau_3$  might carry a simple closed hyperbolic geodesic? Indicate some legitimate weights if you claim that the train track carries a simple closed hyperbolic geodesic.



Can any of the given train-tracks  $\tau_1, \tau_2, \tau_3$  carry *different* simple closed hyperbolic geodesic? Indicate the corresponding different legitimate collections of weights if you claim that.

## Playing with train tracks

The picture on the left presents the four “basic” train tracks  $\tau_1, \dots, \tau_4$  on a four-punctured sphere.

a) Find which of these four basic train-tracks (if any) carries the simple closed curve  $\gamma$  on top of the right picture. Indicate the resulting weights.

b) Find a sequence of “zips” and/or “unzips” which deform the given train track  $\tau$  on the bottom of the right picture to one of the four “basic” train tracks  $\tau_1, \dots, \tau_4$ . Indicate the weights of all branches of the initial and of the final train tracks.

