



Institut de Mathématiques
de
Jussieu-Paris Rive Gauche

$$\begin{array}{r}
 x^3 - \sqrt[3]{2}x^2 + 0x + 0 \\
 \hline
 -\sqrt[3]{2}x^2 + 0x - 2 \\
 \hline
 -(\sqrt[3]{2}x^2 - \sqrt[3]{2}^2x + 0) \quad + \sqrt[3]{2}x \\
 \hline
 \sqrt[3]{2}^2x - 2 \\
 \hline
 -(\sqrt[3]{2}^2x - \sqrt[3]{2}^3) \quad + \sqrt[3]{2}^2 \\
 \hline
 0
 \end{array}$$

$$\begin{array}{r}
 -\sqrt[3]{2}x^2 + 0x - 2 \\
 \hline
 -(\sqrt[3]{2}x^2 - \sqrt[3]{2}^2x + 0) \quad + \sqrt[3]{2}x \\
 \hline
 \sqrt[3]{2}^2x - 2 \\
 \hline
 -(\sqrt[3]{2}^2x - \sqrt[3]{2}^3) \quad + \sqrt[3]{2}^2 \\
 \hline
 0
 \end{array}$$

Therefore, $x^3 - 2 = (x - \sqrt[3]{2})(x^2 + \sqrt[3]{2}x + \sqrt[3]{2}^2)$

Ariane Mézard

Professeur des universités UPMC/ENS

Géométrie arithmétique

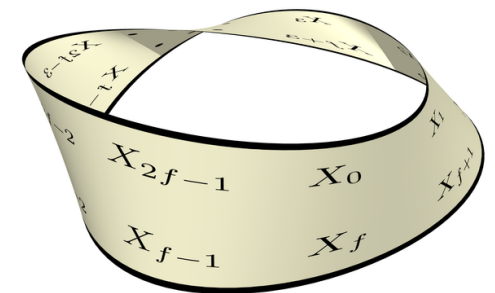
21 décembre 2017

Géométrie arithmétique

Arithmétique = compter, résoudre des problèmes
avec des entiers

Géométrie = étudier et représenter les objets de
l'espace

1. Géométrie
2. Arithmétique
3. Géométrie arithmétique : tore, Moebius
4. Ensembles de Keakeya



1. Géométrie

Questions :

- Quels objets géométriques connaissez-vous ?
- Comment définir ces objets ?
- Sont-ils différents ?
- Sont-ils identiques ?

1. Géométrie

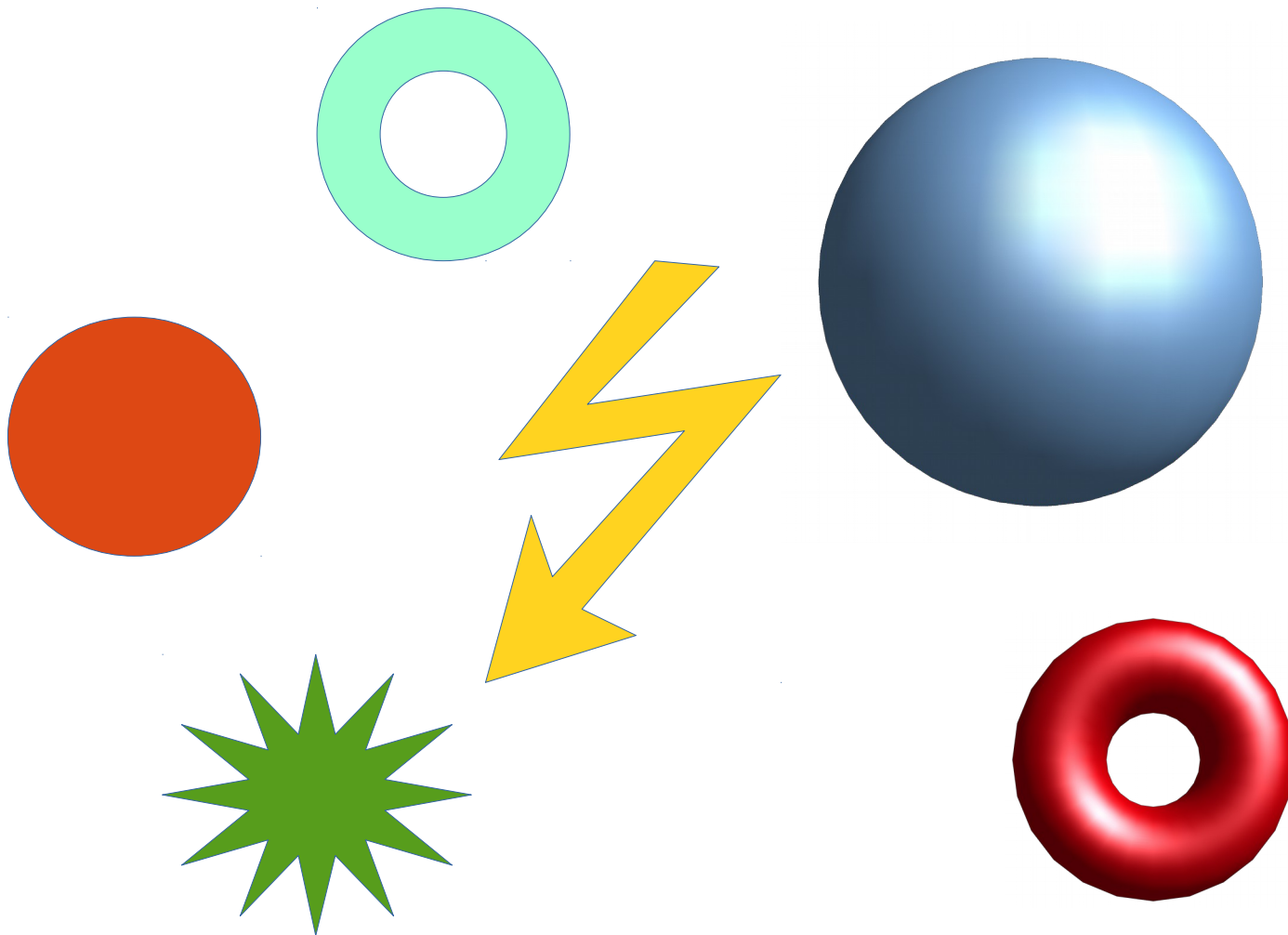
Découpages
Déformations



Et le collage ?

1. Géométrie

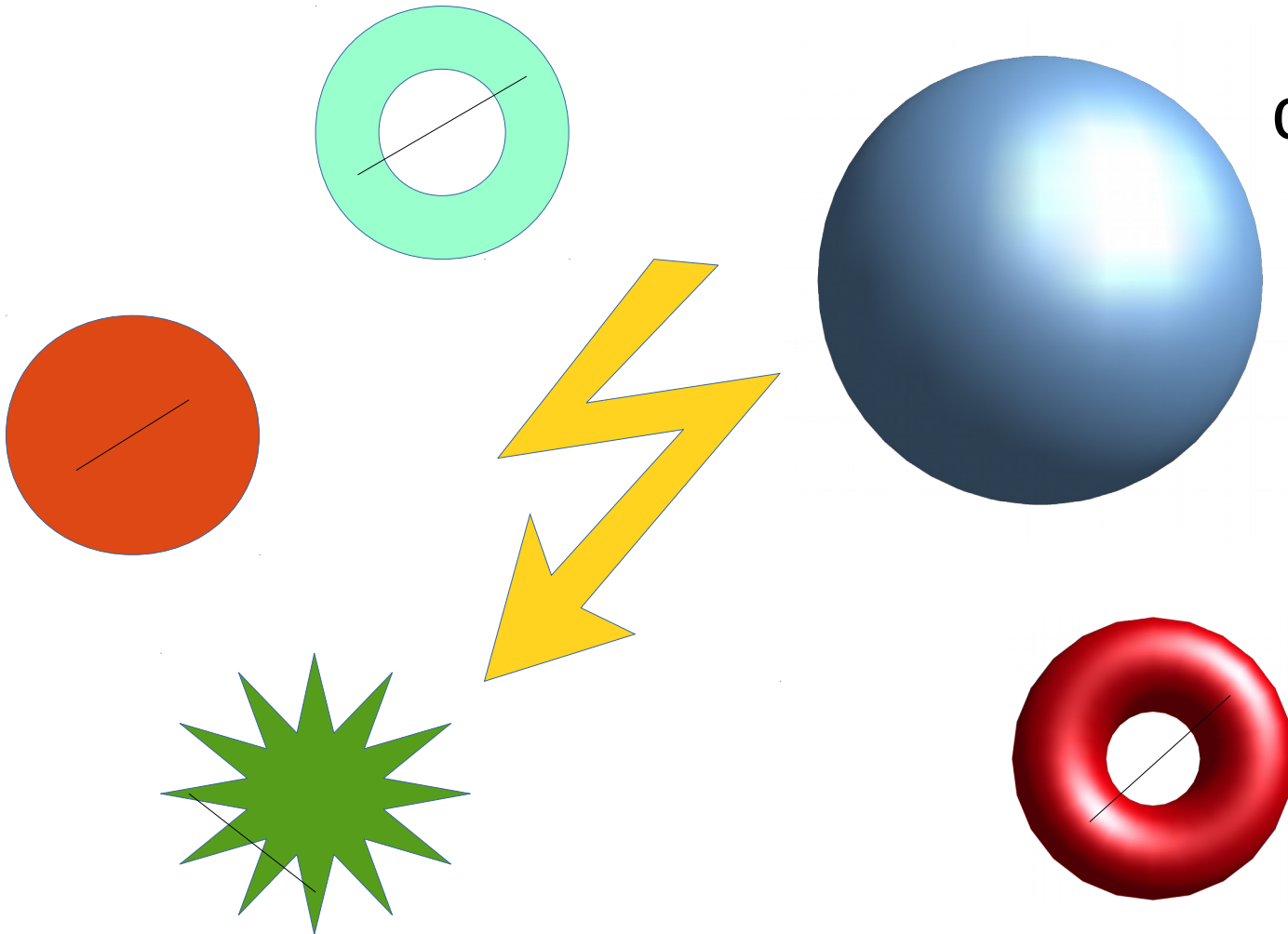
Disque, couronne, boule, tore...



1. Géométrie

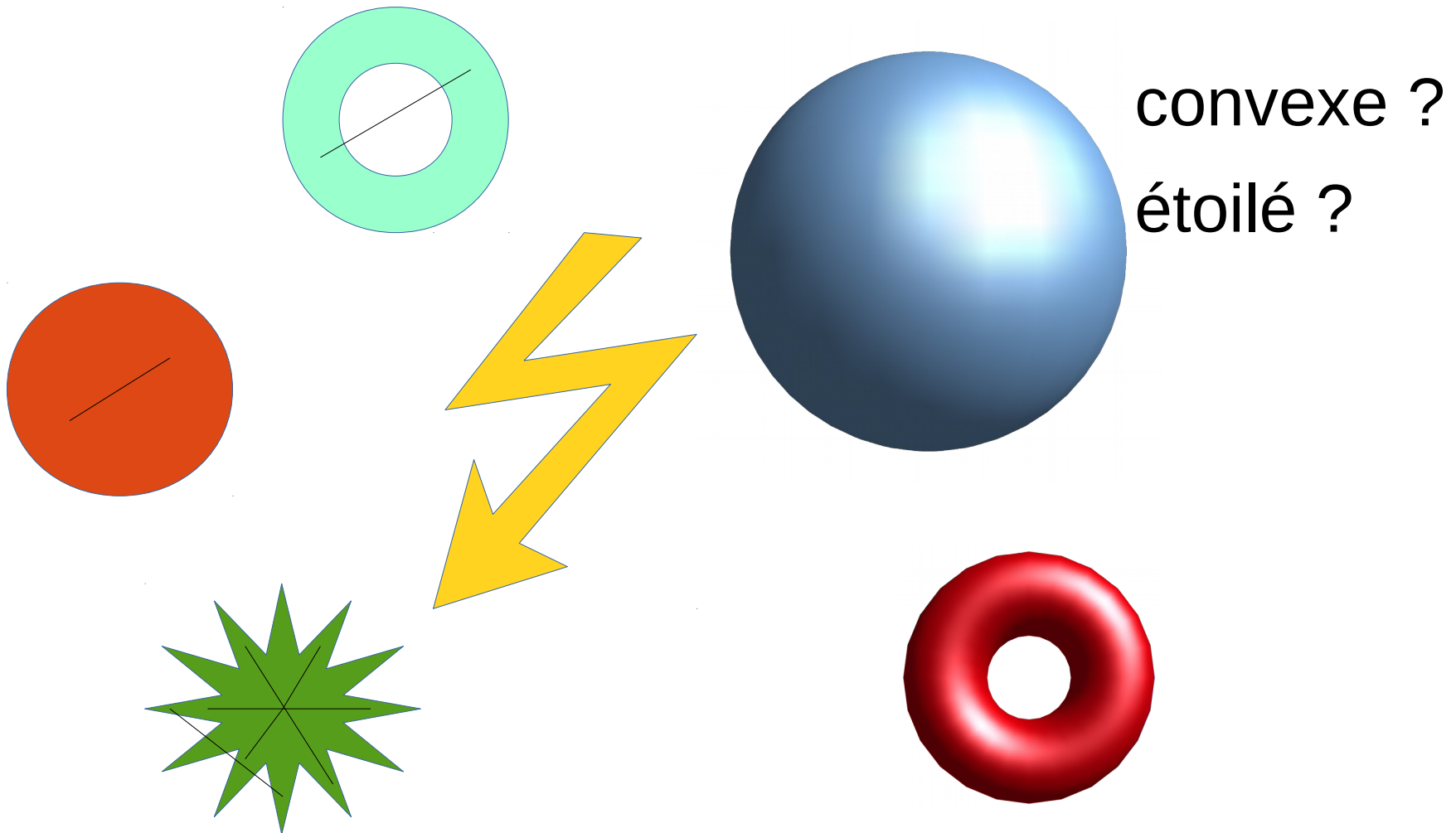
Disque, couronne, boule, tore...

convexe ?



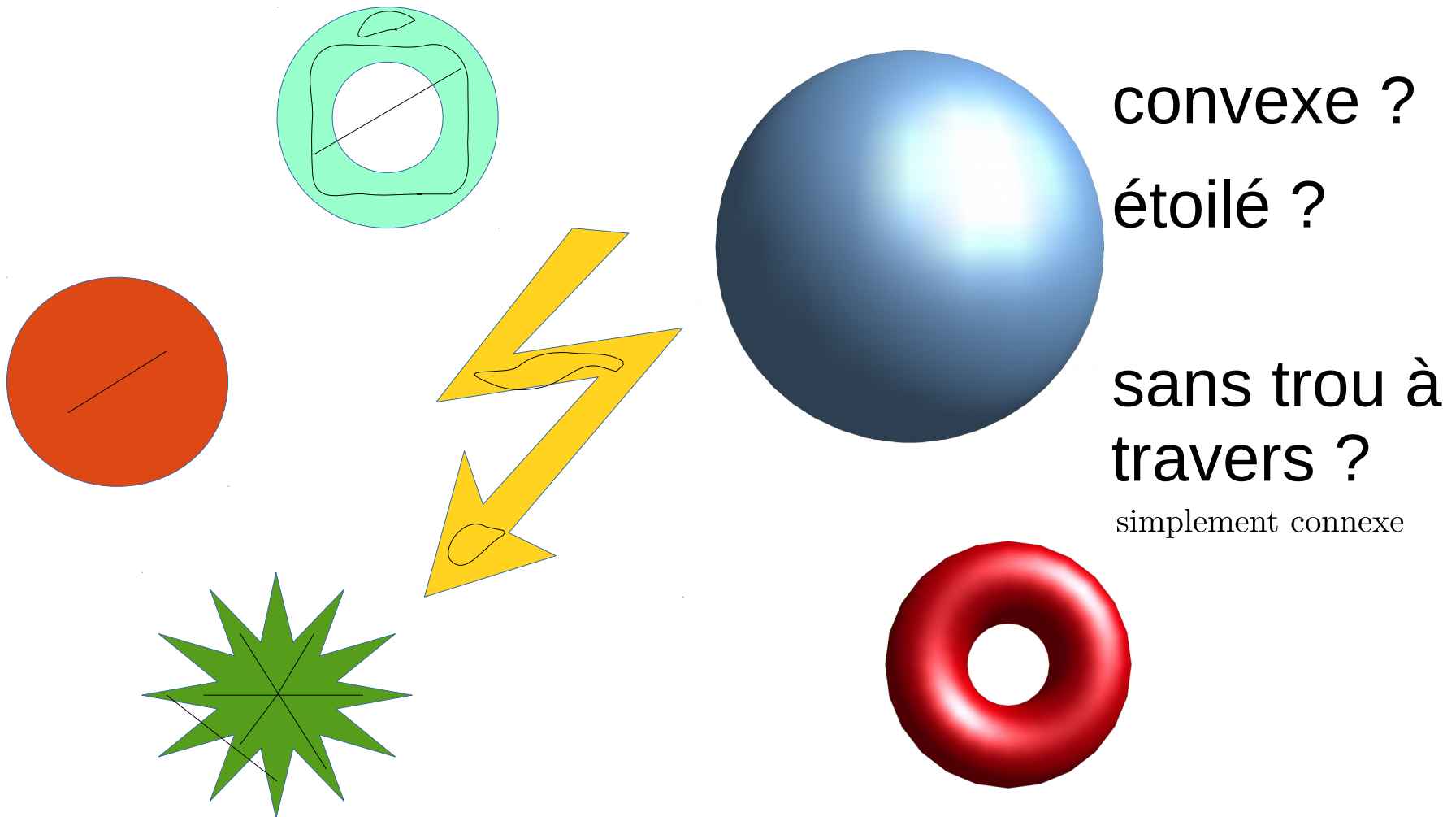
1. Géométrie

Disque, couronne, boule, tore...



1. Géométrie

Disque, couronne, boule, tore...



convexe ?

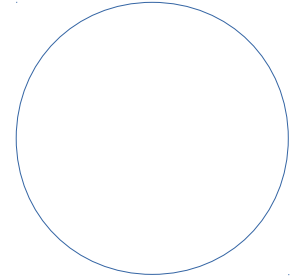
étoilé ?

sans trou à
travers ?

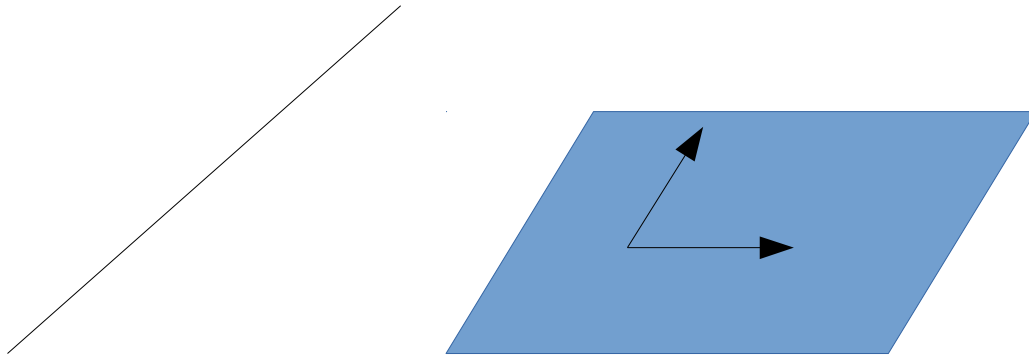
simplement connexe

1. Géométrie

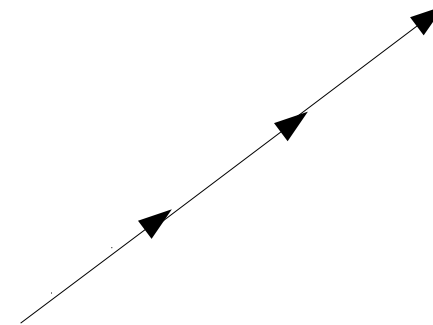
Point, droite, plan, cercle



+



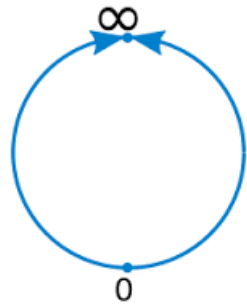
Représentation ? Définition ?



Z, Q, R, C ...

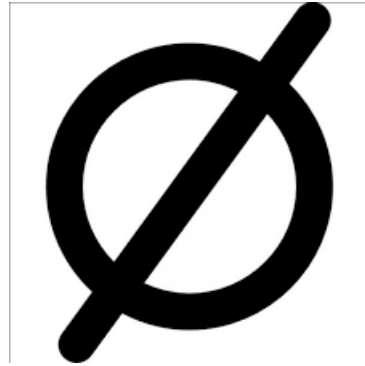
1. Géométrie

L'importance du point



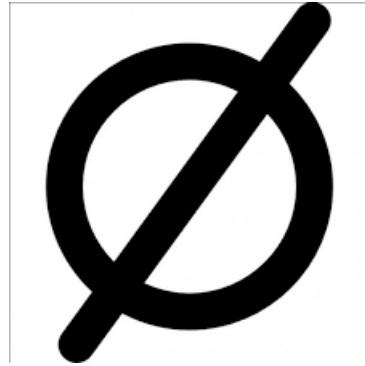
1. Géométrie

Vide



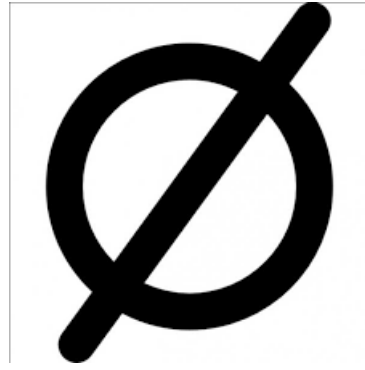
1. Géométrie

Vide



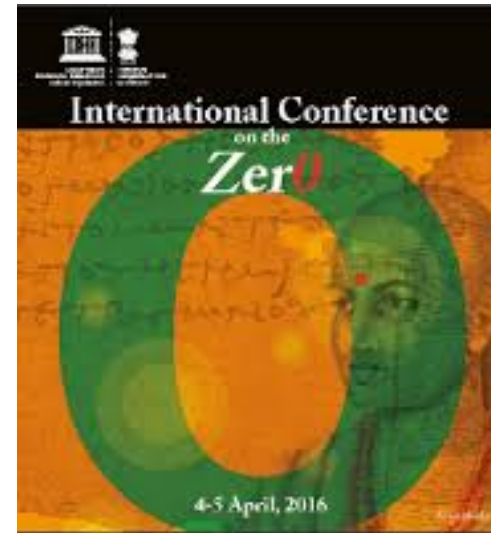
1. Géométrie

Vide

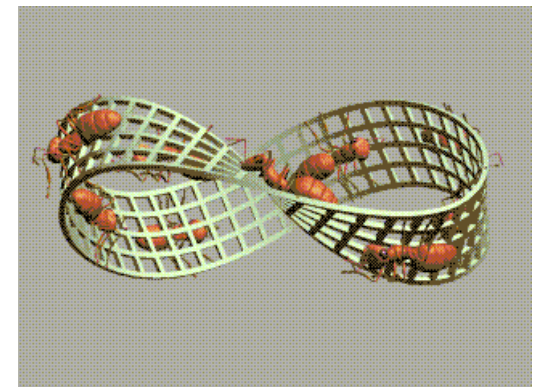
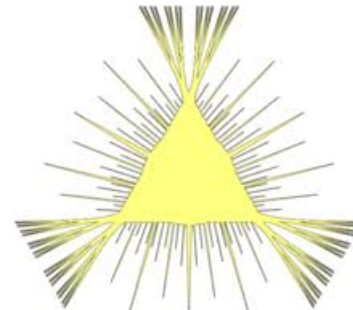


1. Géométrie

Le vide et le zéro

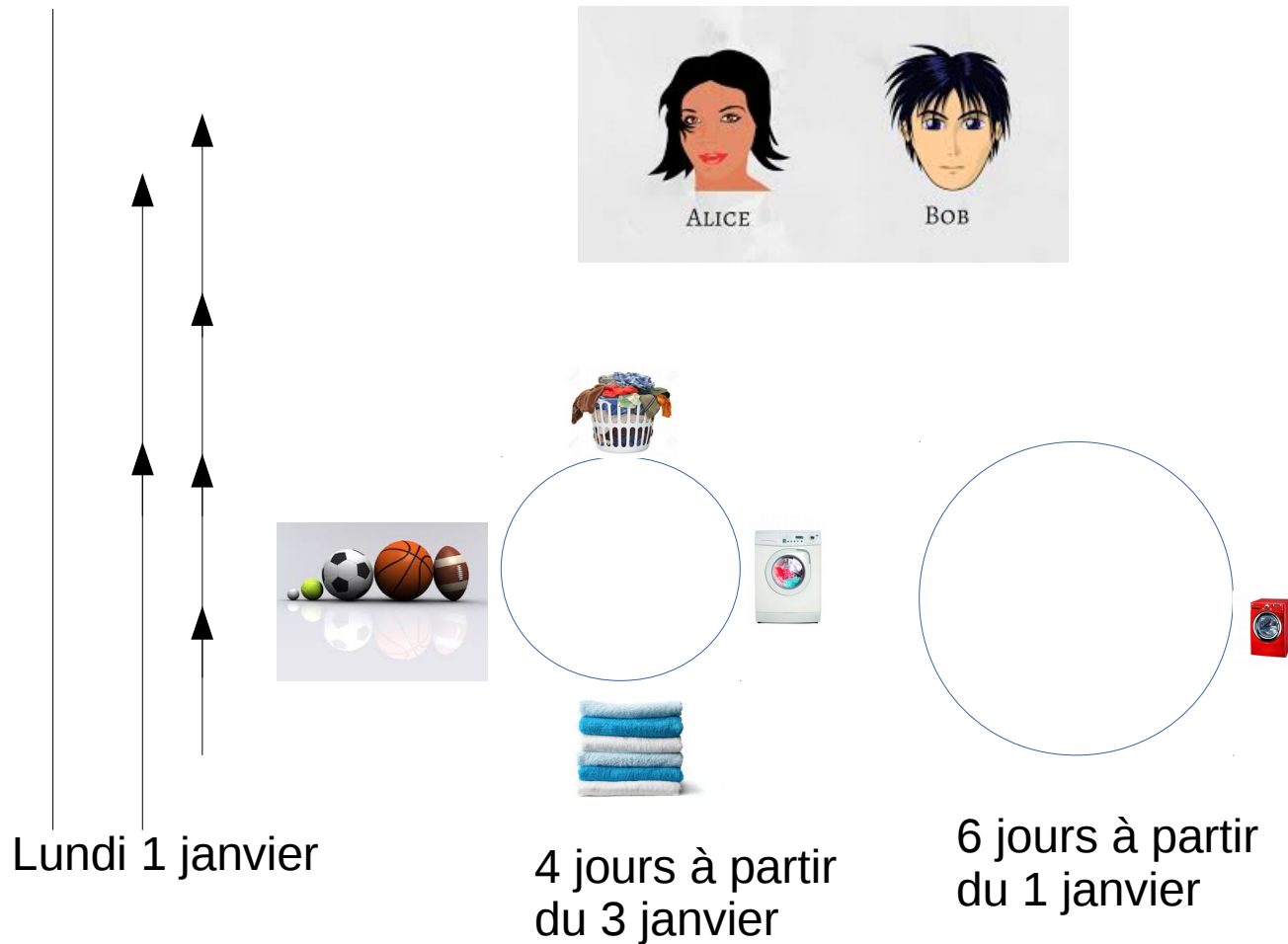


Magie ou sorcellerie ?



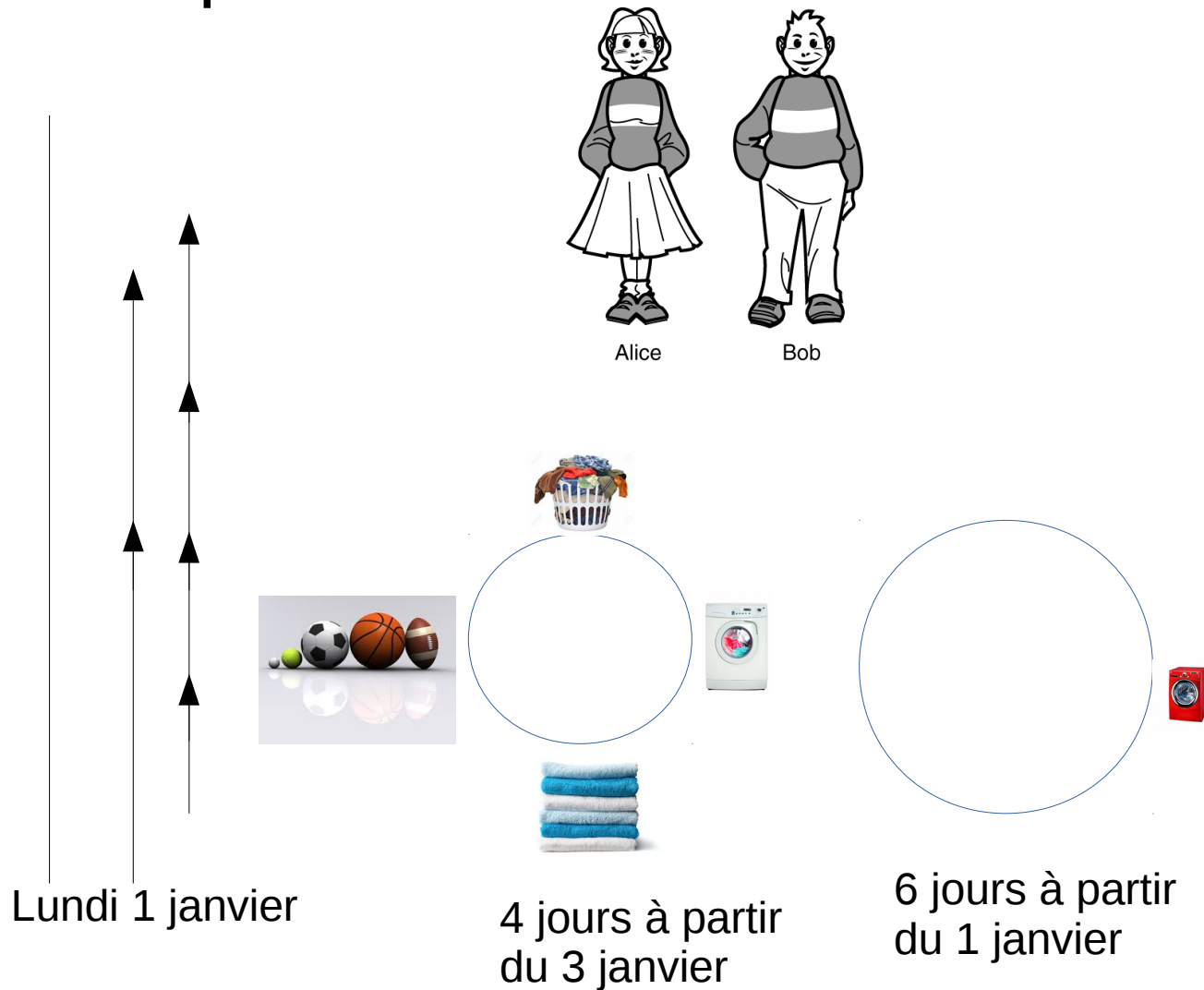
2.Arithmétique

Résoudre des problèmes



2.Arithmétique

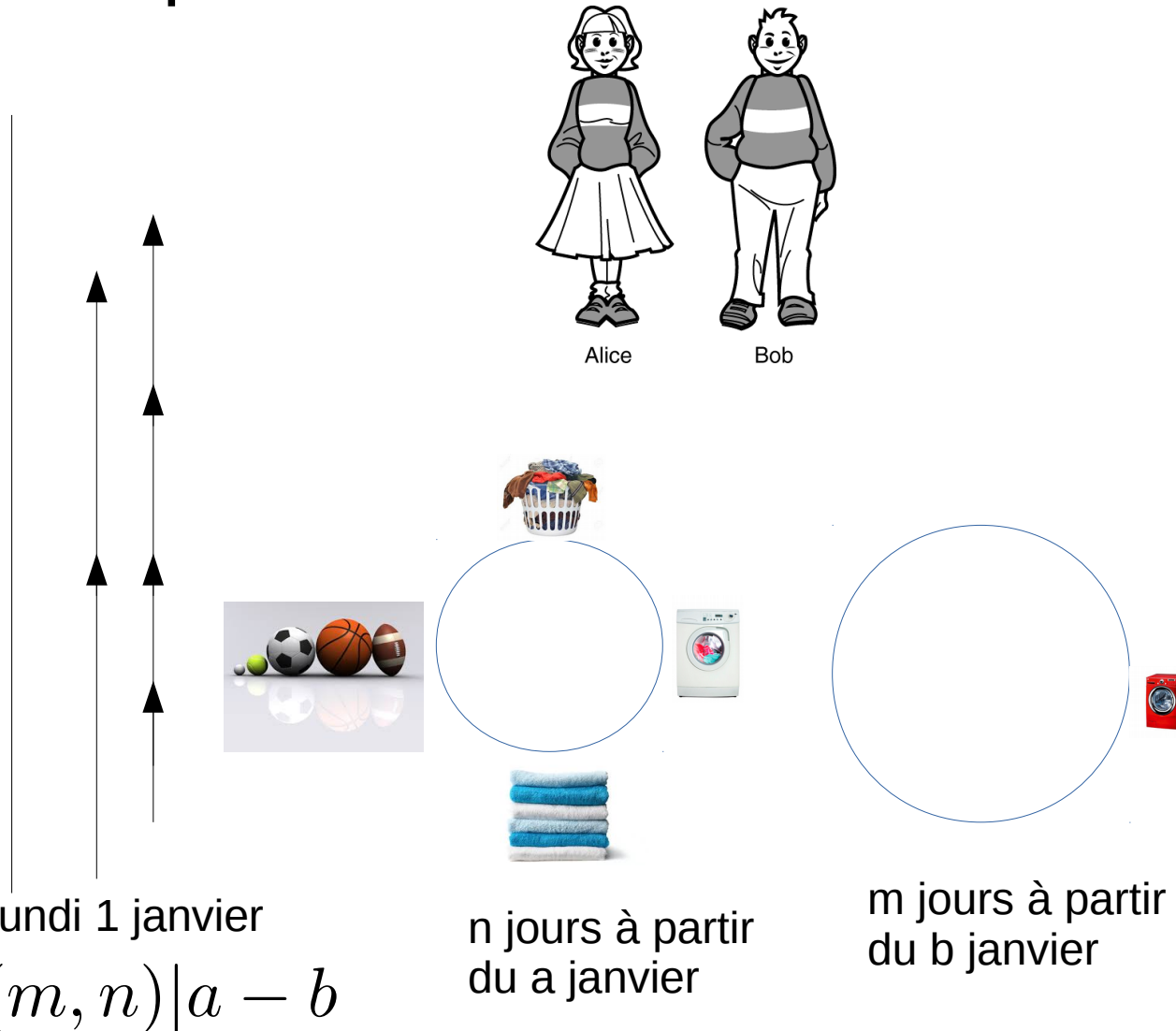
Temps



Jeu 24		
Mer 23		
Mar 22		
Lun 21		
Dim 20		
Sam 19		
Ven 18		
Jeu 17		
Mar 16		
Lun 15		
Dim 14		
Sam 13		
Ven 12		
Jeu 11		
Mer 10		
Mar 9		
Lun 8		
Dim 7		
Sam 6		
Ven 5		
Jeu 4		
Mer 3		
Mar 2		
Lun 1		

2.Arithmétique

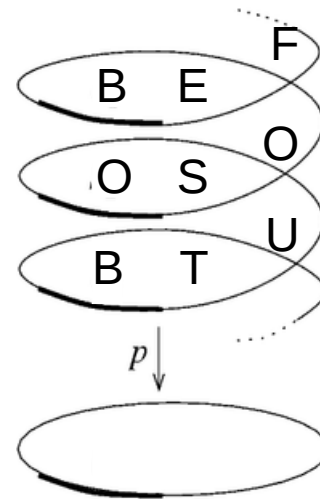
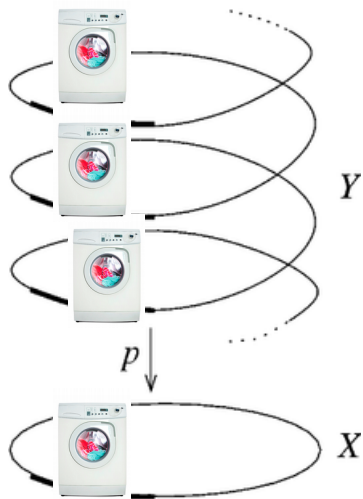
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Dim 7		
Sam 6		
Ven 5		
Jeu 4		
Mer 3		
Mar 2		
Lun 1		

2.Arithmétique

Cryptographie



BTUOSOBEF

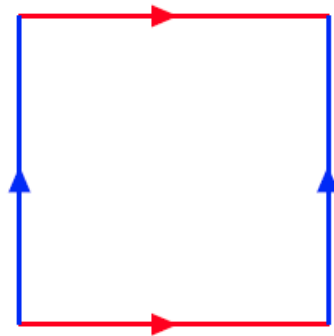
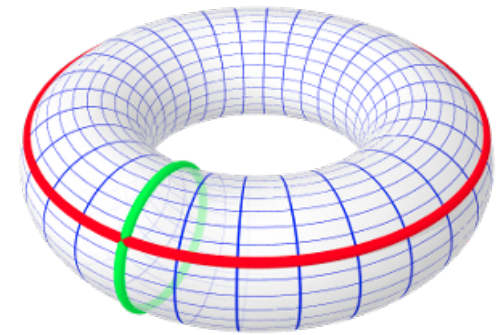
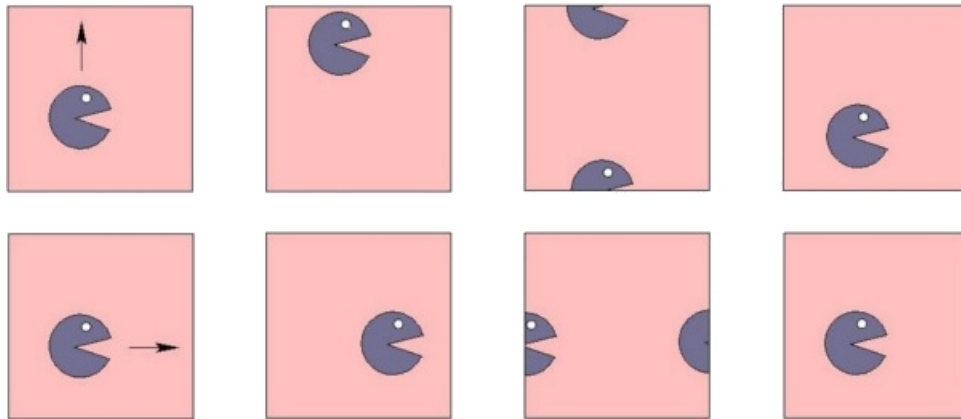


Scytale

revêtement du cercle

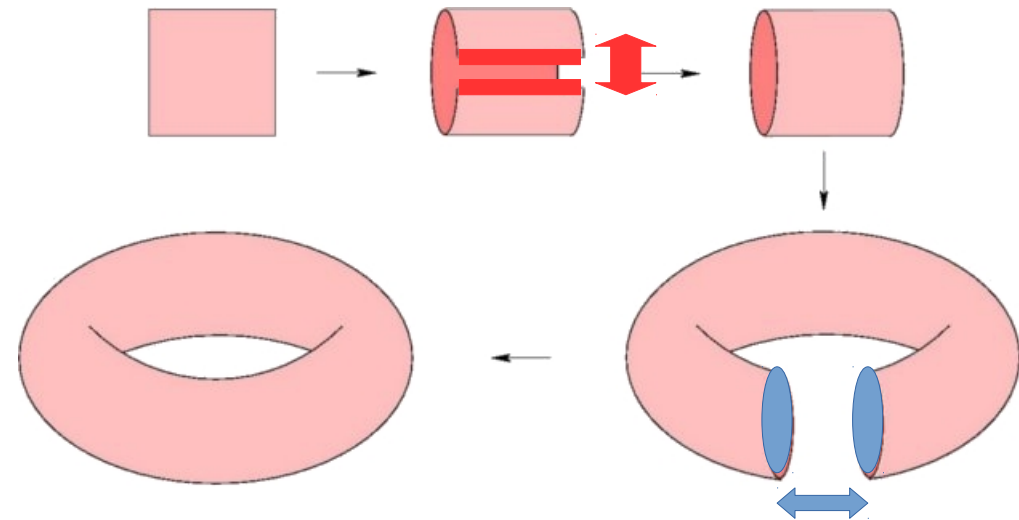
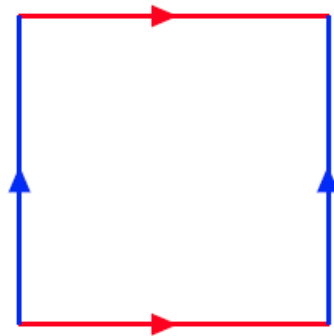
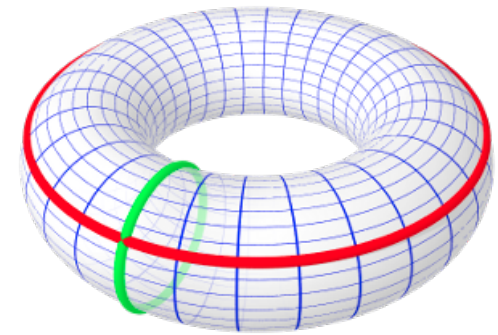
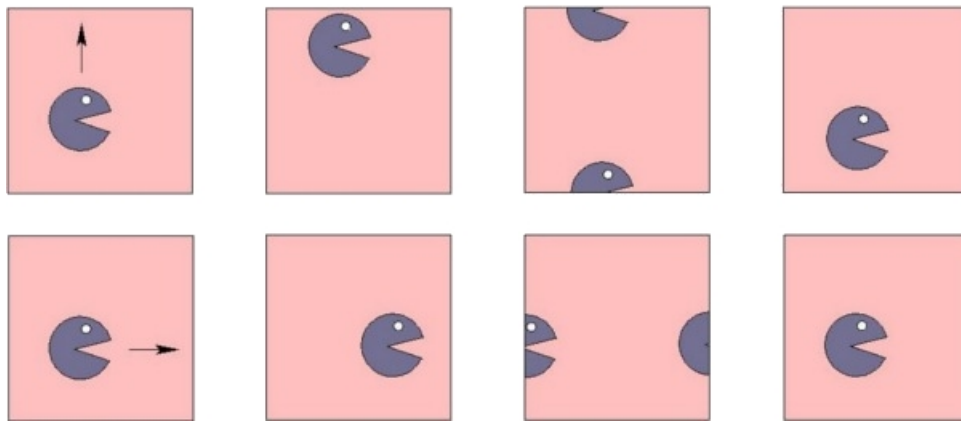
3. Géométrie arithmétique : tore

Représenter un tore



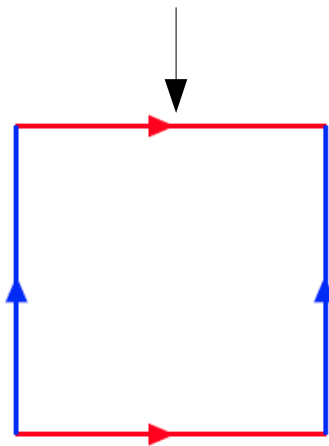
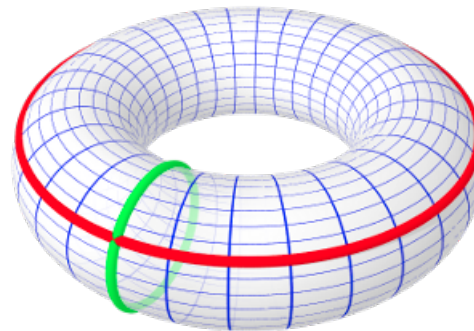
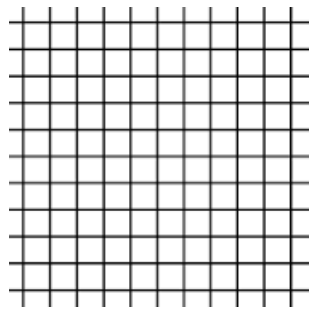
3. Géométrie arithmétique : tore

Représenter un tore



3. Géométrie arithmétique : tore

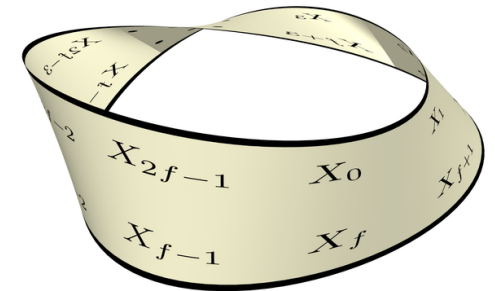
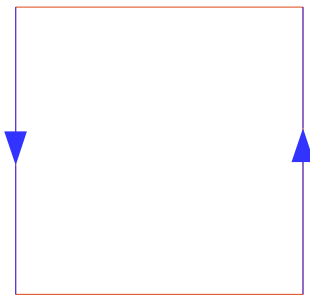
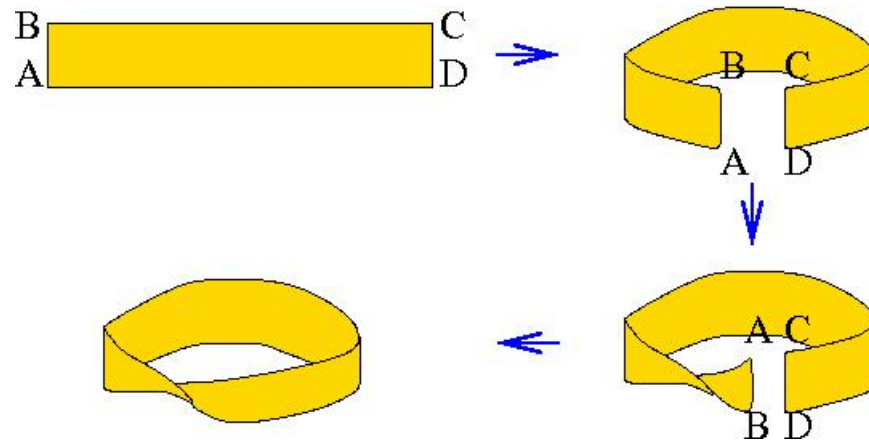
Représenter un tore



revêtement du tore

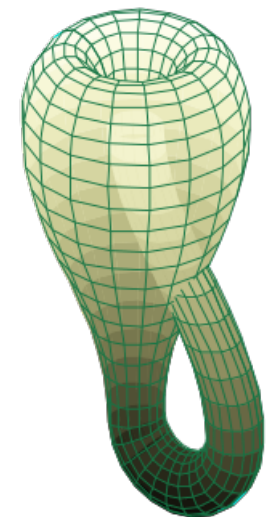
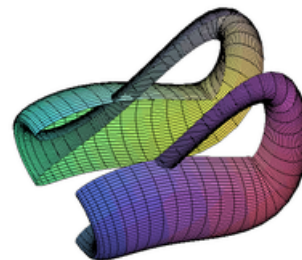
3. Géométrie arithmétique : Moebius

Ruban de Moebius



3. Géométrie arithmétique : Moebius

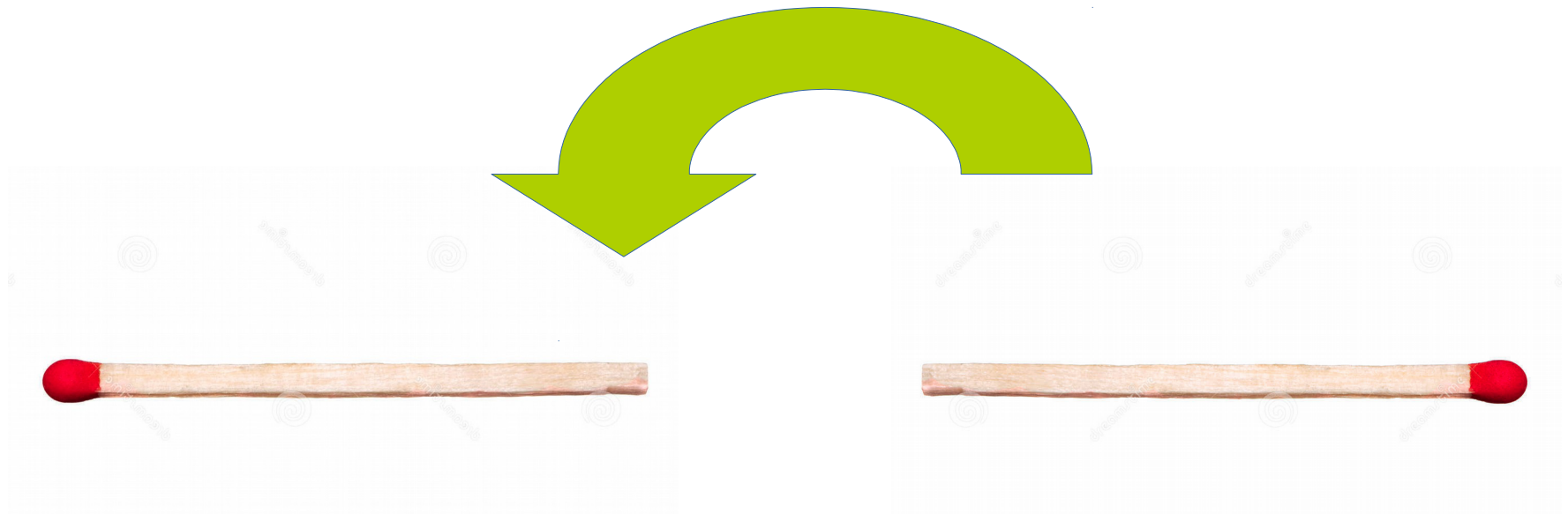
Découper, coller



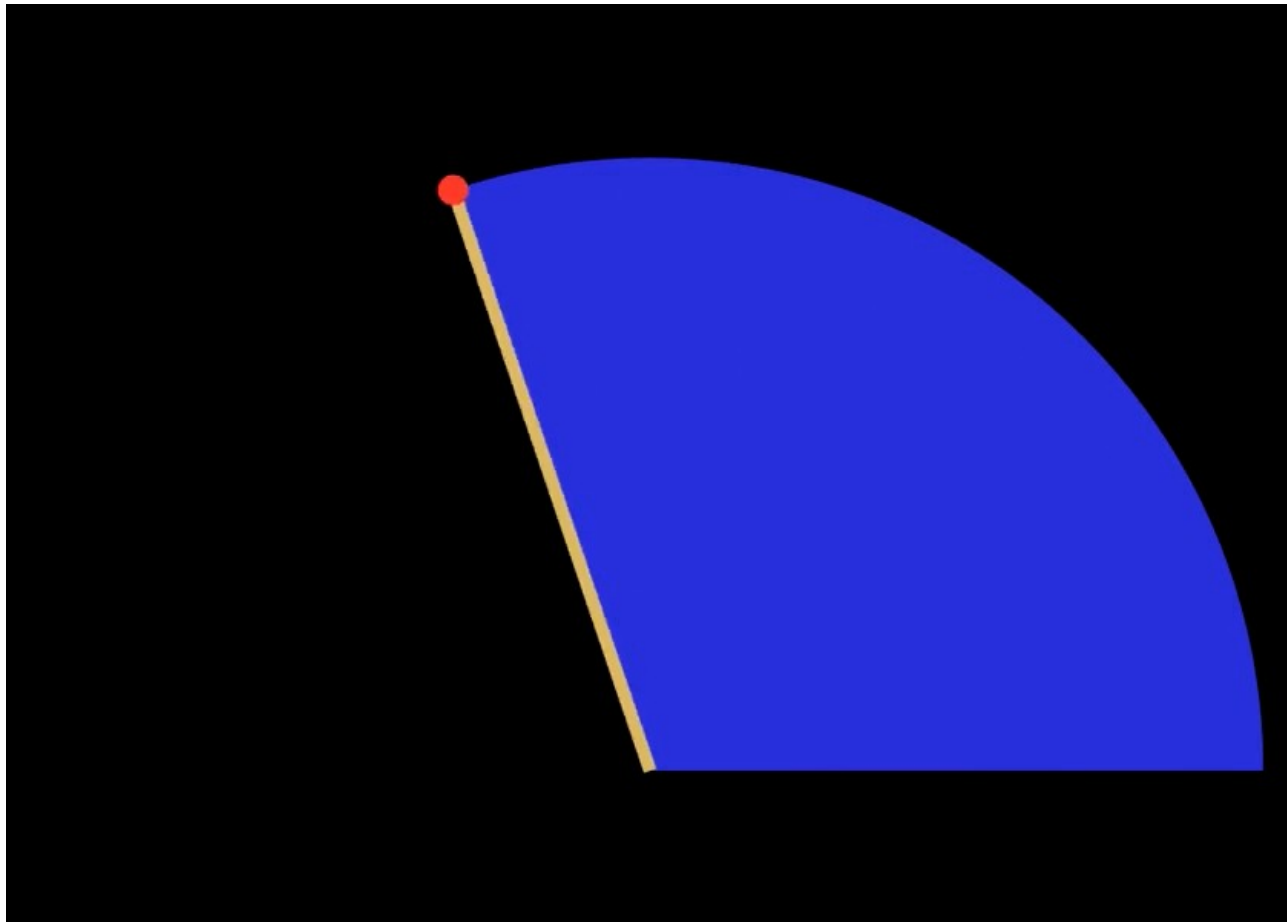
4. Ensembles de Kakeya

Retourner une allumette

Kekaya 1917



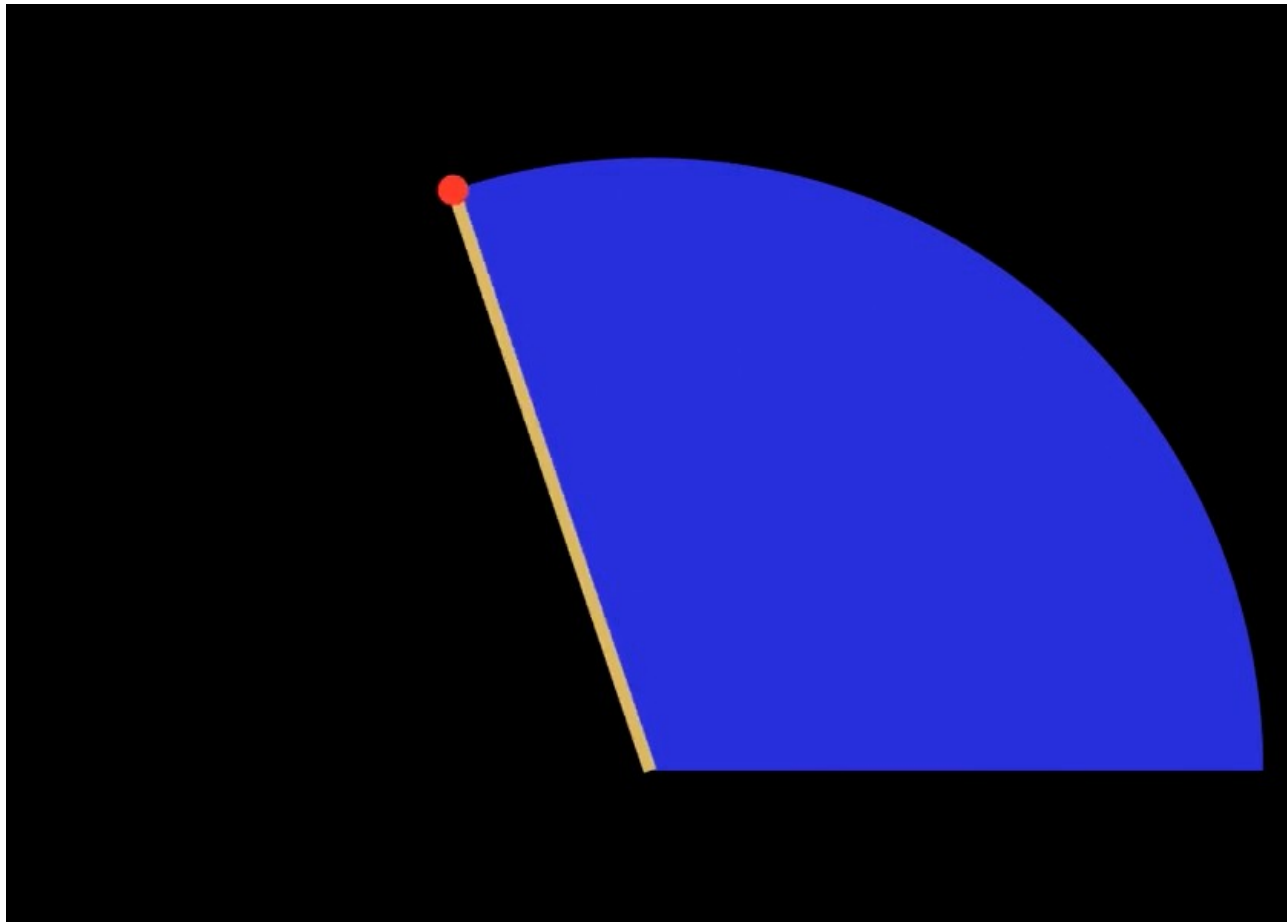
4. Ensembles de Kakeya (Xavier Caruso)



$$\pi r^2 / 2, r = 1$$

$$\pi / 2 \simeq 1,5708$$

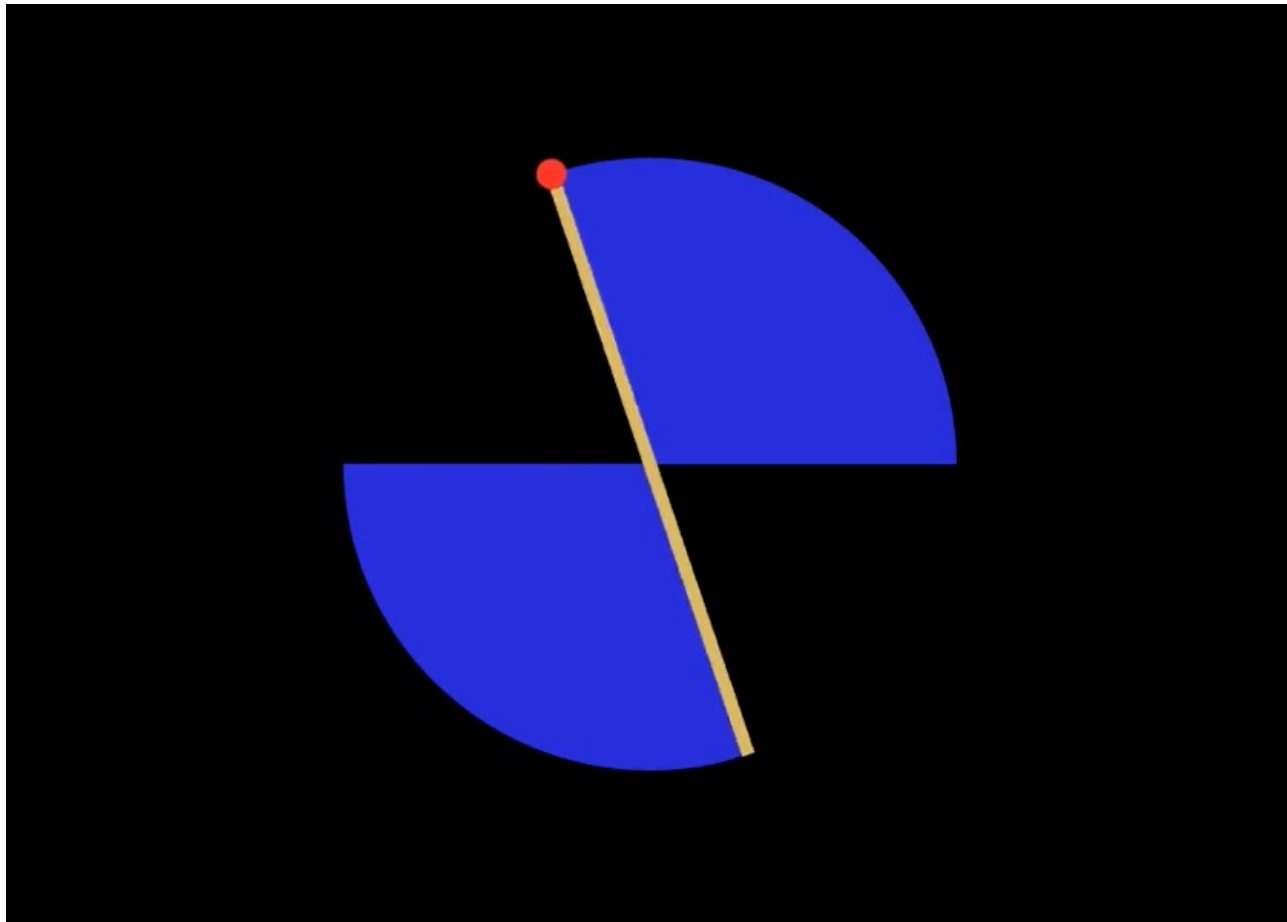
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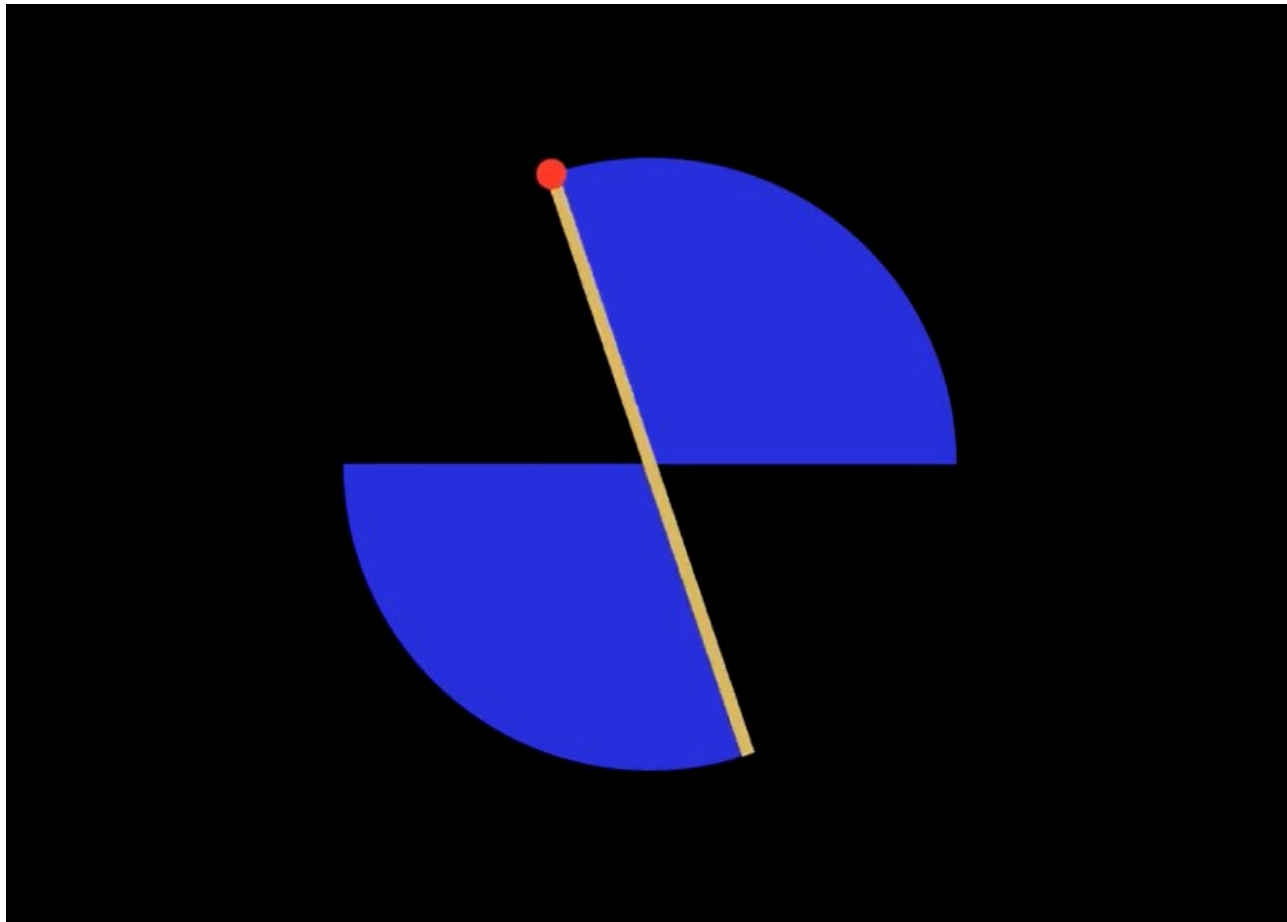
4. Ensembles de Kakeya



$$\pi r^2, r = 1/2$$

$$\pi/4 \simeq 0,7854$$

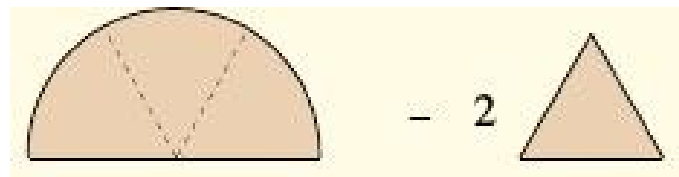
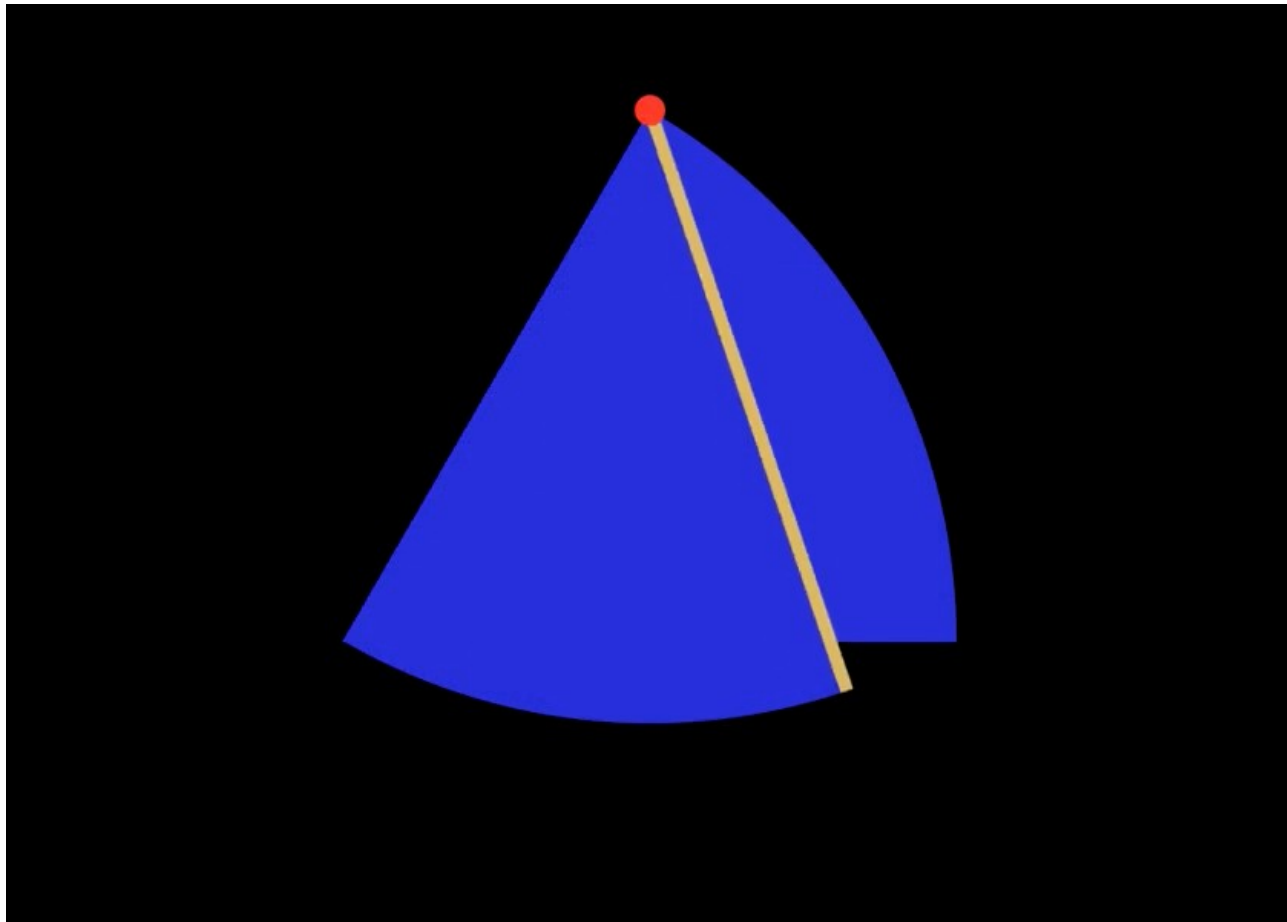
4. Ensembles de Kakeya



$$\pi r^2, r = 1/2$$

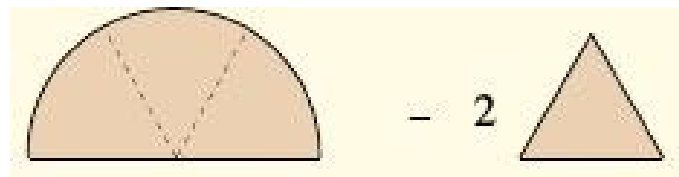
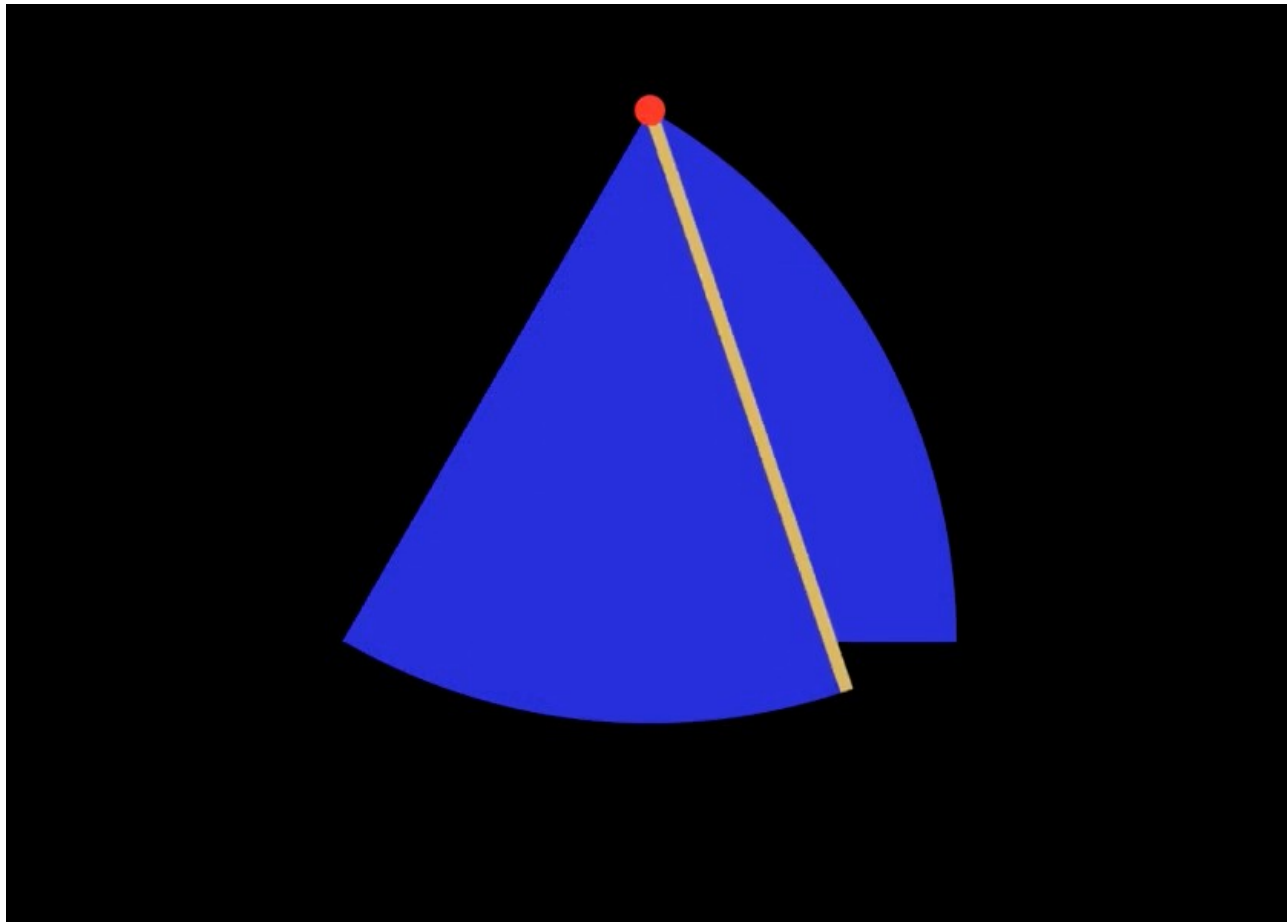
$$\pi/4 \simeq 0,7854$$

4. Ensembles de Kakeya



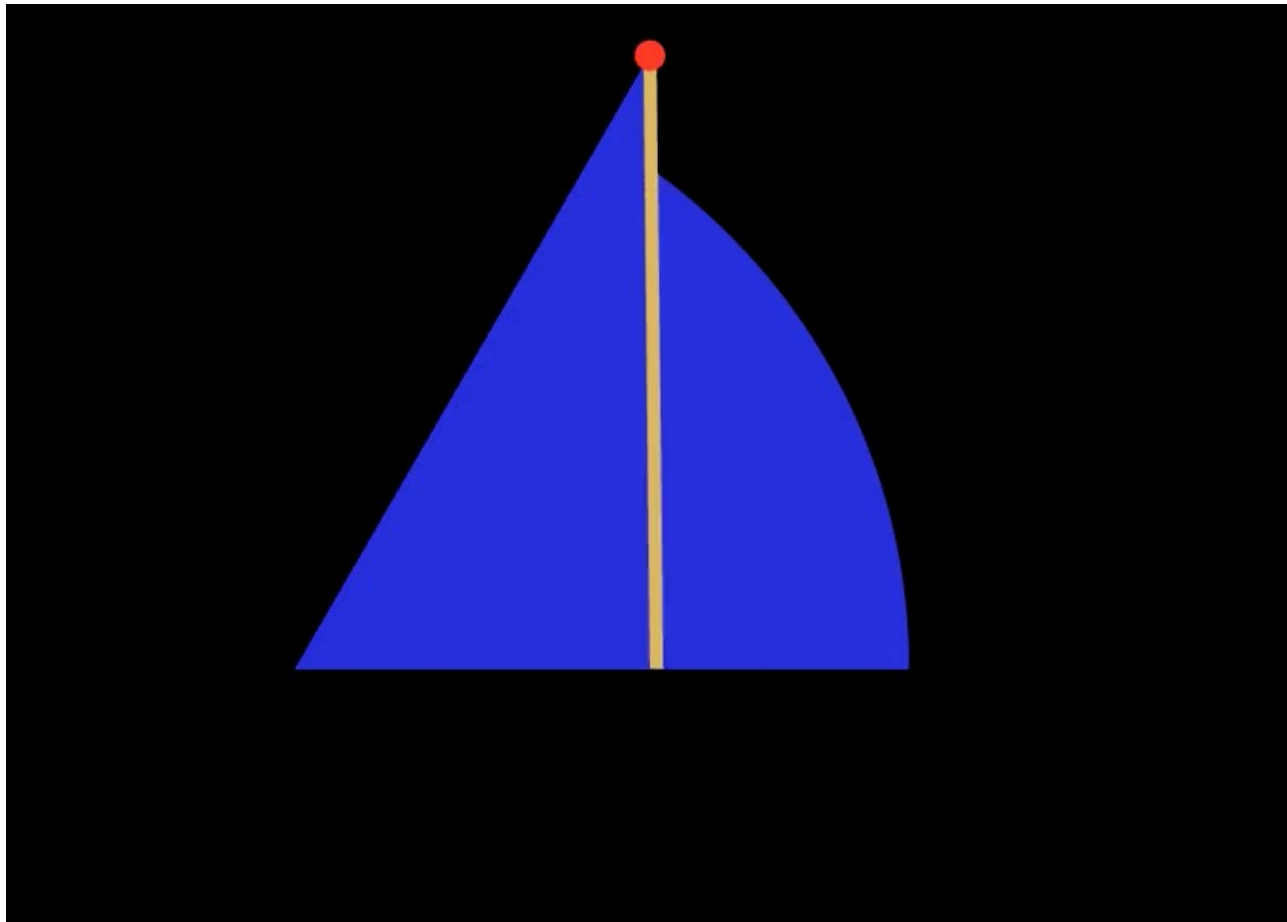
$$\frac{\pi}{2} - 2\frac{\sqrt{3}}{4} \simeq 0,7048$$

4. Ensembles de Kakeya



$$\frac{\pi}{2} - 2\frac{\sqrt{3}}{4} \simeq 0,7048$$

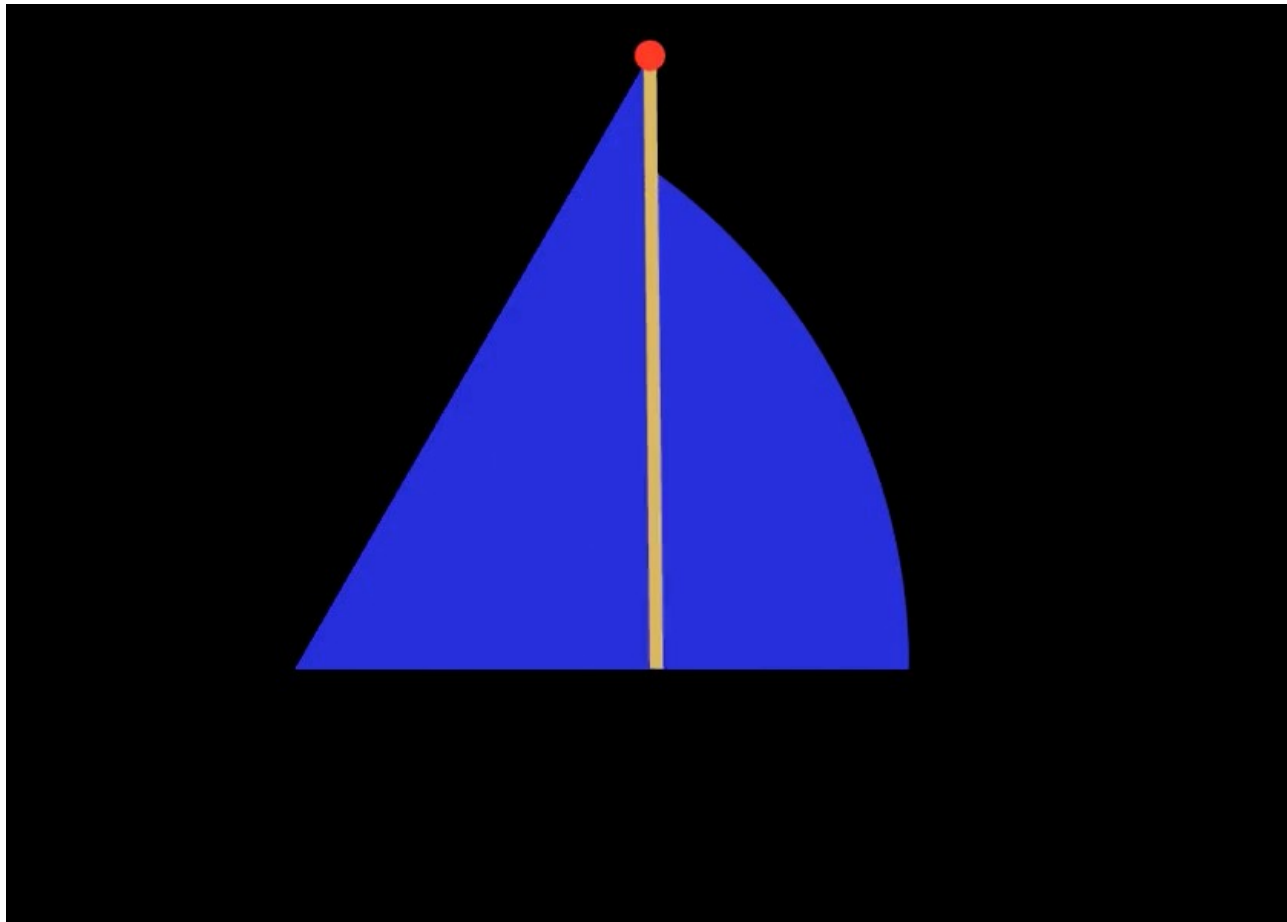
4. Ensembles de Kakeya



plus petit ensemble convexe

$$\frac{1}{\sqrt{3}} \simeq 0,5774$$

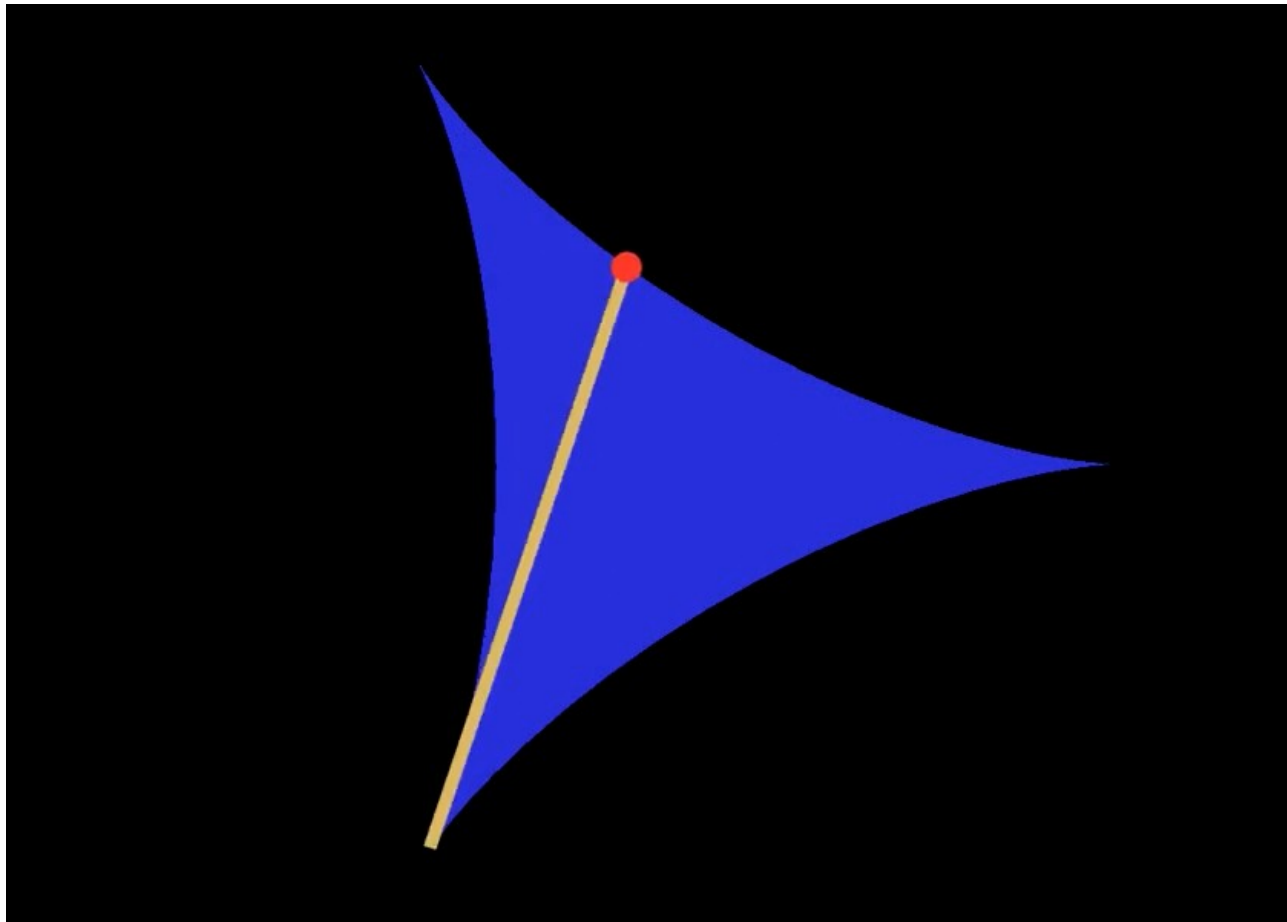
4. Ensembles de Kakeya



plus petit ensemble convexe

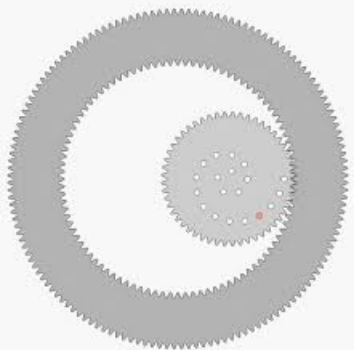
$$\frac{1}{\sqrt{3}} \simeq 0,5774$$

4. Ensembles de Kakeya

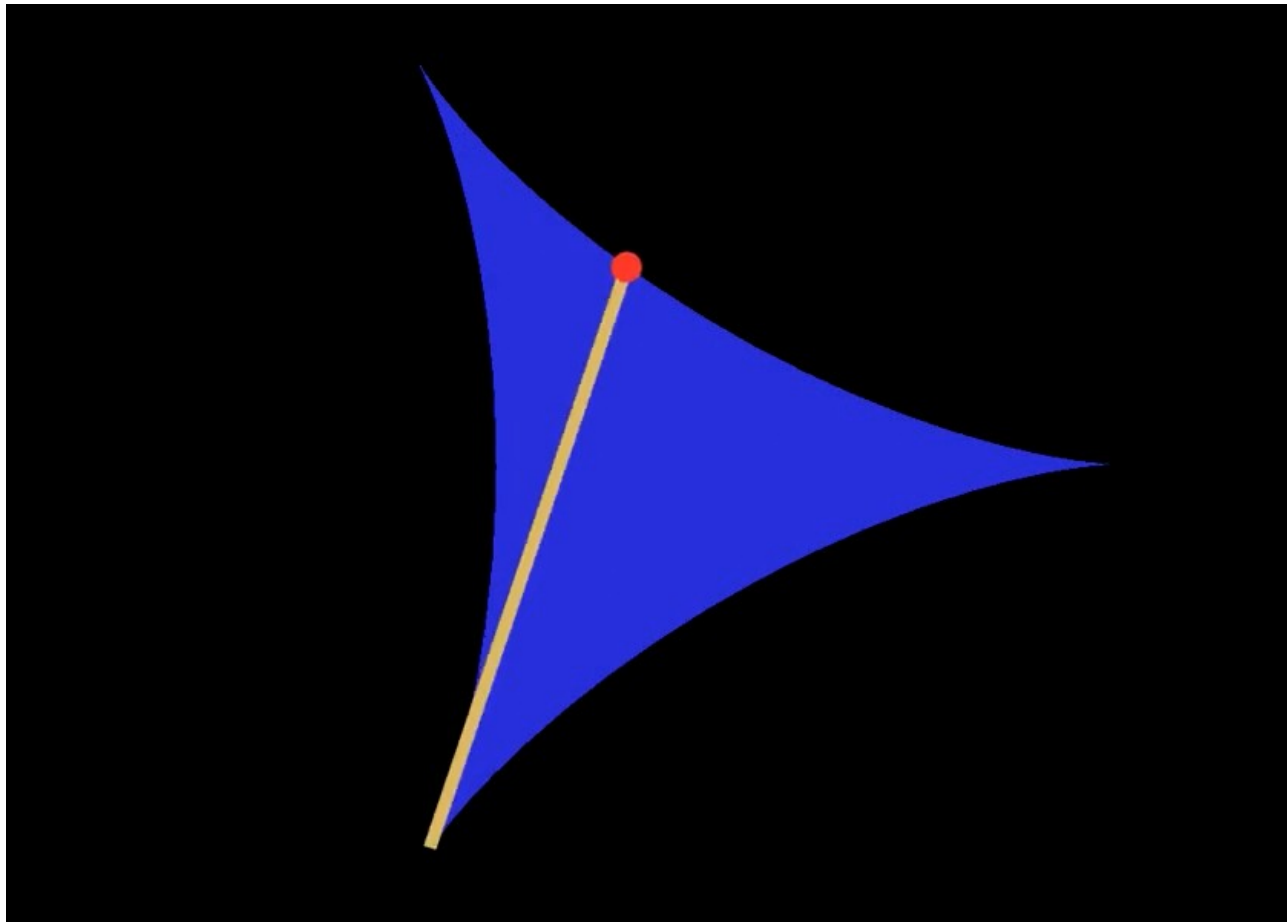


$$\begin{cases} x(t) = 2 \cos(t) + \cos(2t) \\ y(t) = 2 \sin(t) - \sin(2t) \end{cases}$$

$$\frac{\pi}{8} \simeq 0,3927$$

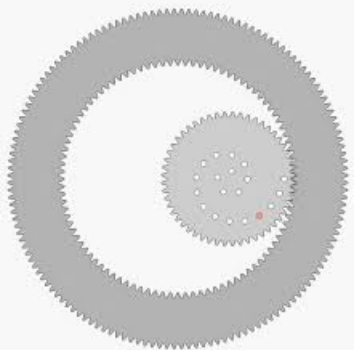


4. Ensembles de Kakeya

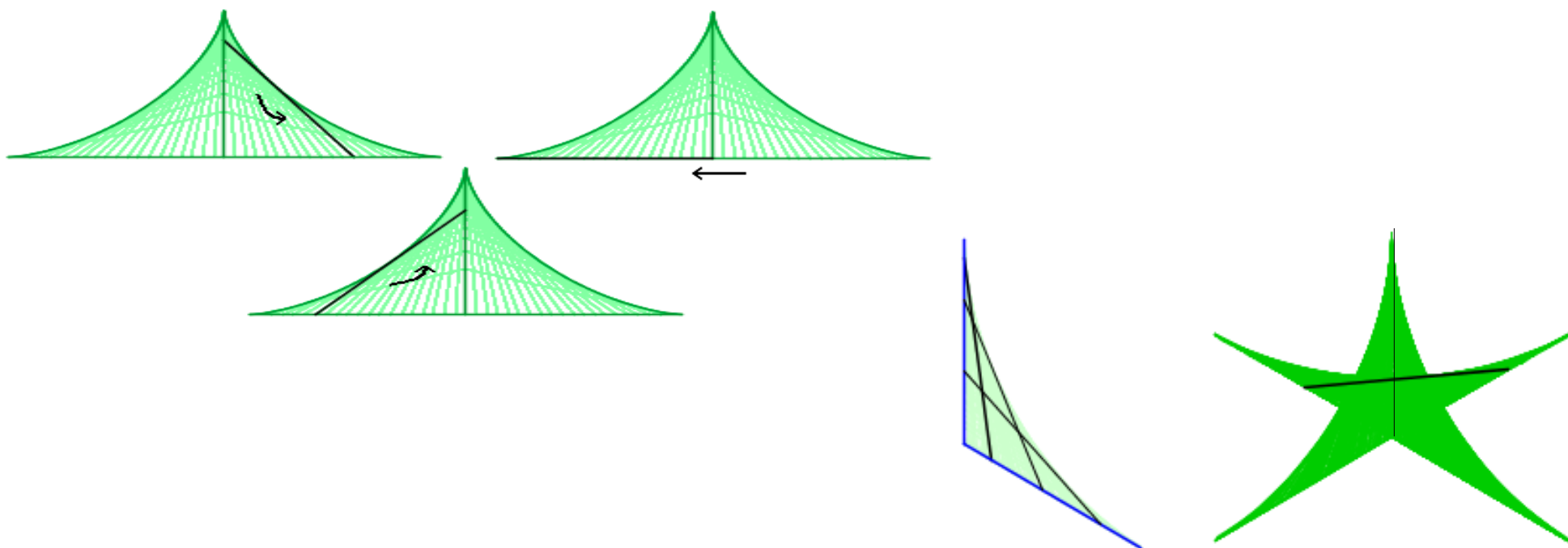


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$$\frac{\pi}{8} \simeq 0,3927$$



4. Ensembles de Kakeya

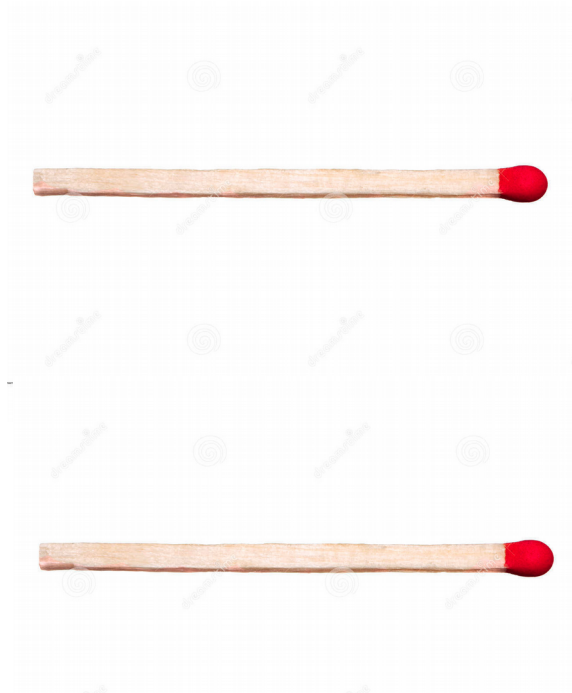


Ensemble étoilé
Cunningham 1971

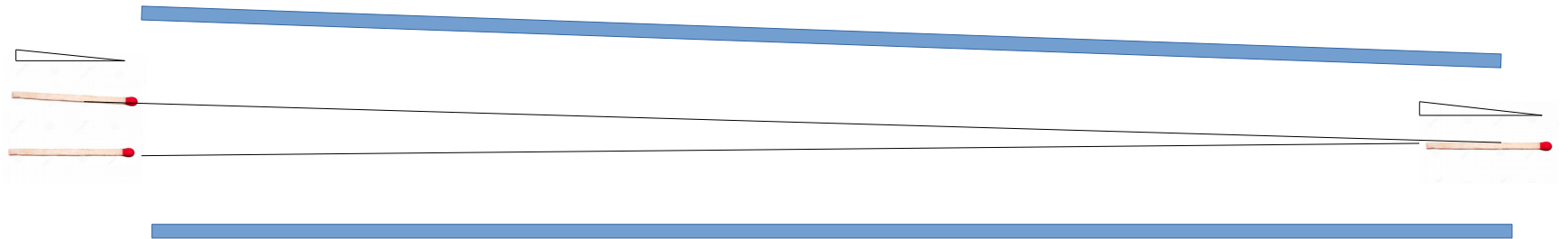
$$> \frac{\pi}{108} \simeq 0,029$$

4. Ensembles de Kakeya

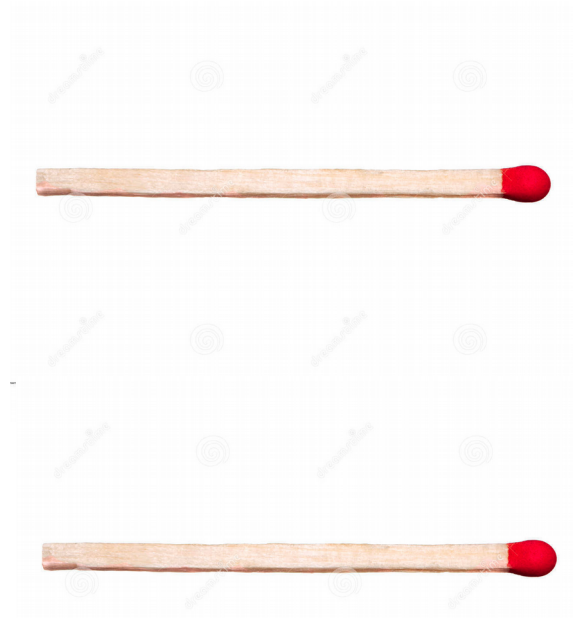
Besicovitch 1928



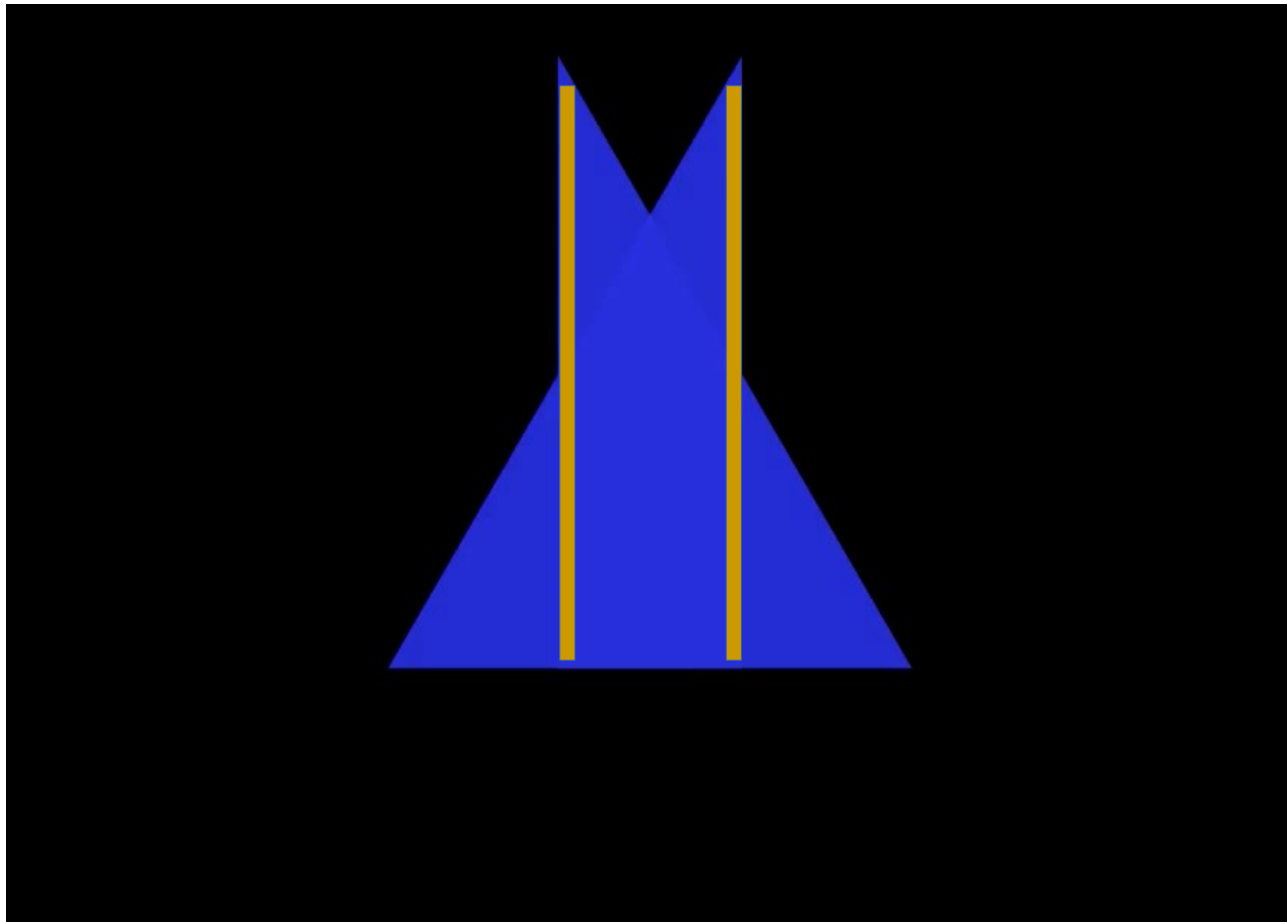
4. Ensembles de Kakeya



Besicovitch 1928

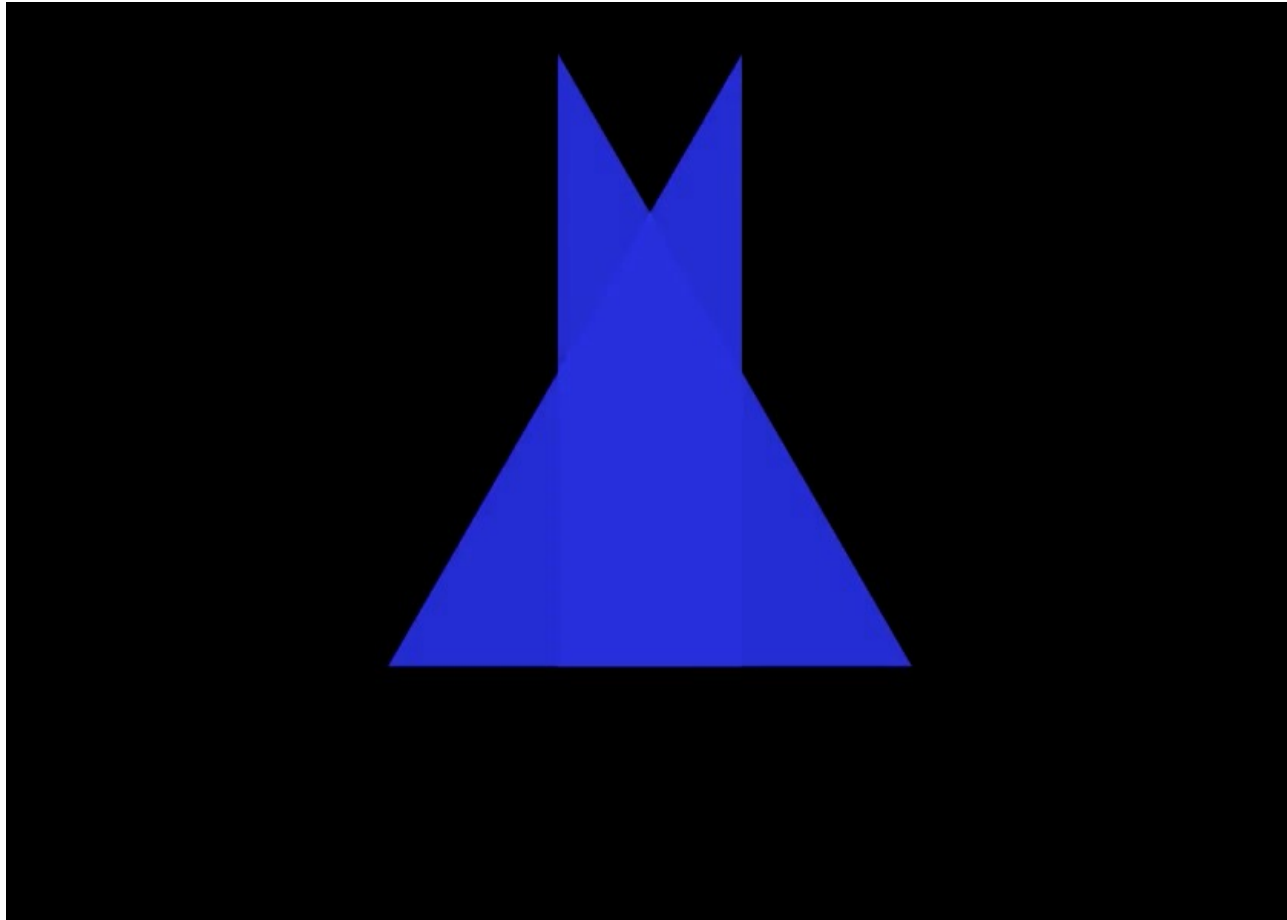


4. Ensembles de Kakeya



Besikovitch

4. Ensembles de Kakeya



Besikovitch

4. Ensembles de Kakeya



Besikovitch

4. Ensembles de Kakeya

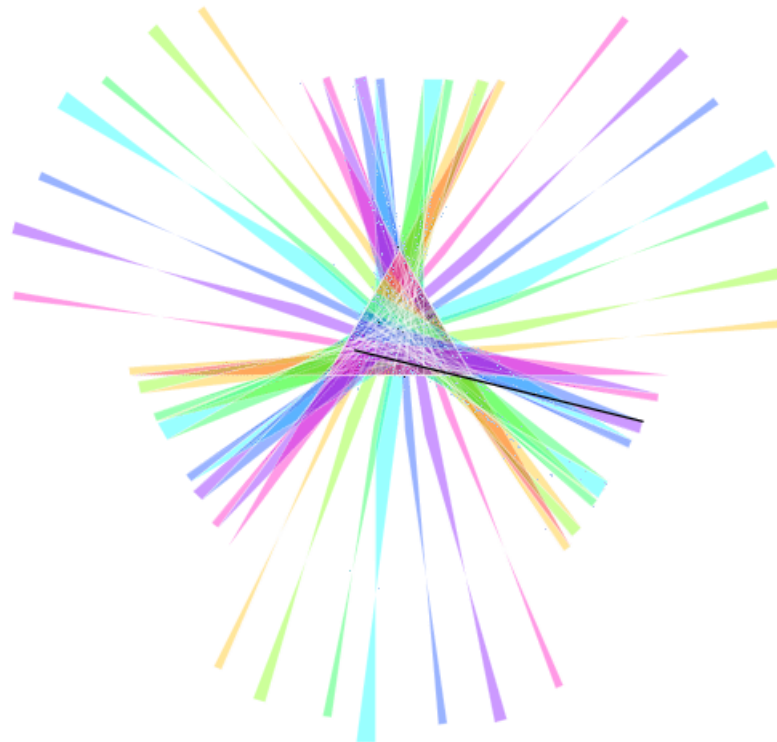


Besikovitch

4. Ensembles de Kakeya

Surface simplement
connexe

$< \varepsilon$



Tore plat de Nash



Vincent Borelli, Said Jabrane,
Francis Lazarus, Damien Rohmer