Master-Seminar on Mathematical Physics

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The first meeting will be on the **02.04.2019**, where the topics of the talks will be distributed. All talks will be blackboard talks with an approximate time of 60 **minutes** and a question and discussion session afterwards.

Variational principles (16.04.): Give a mathematical introduction to variational calculus and give physical examples from classical mechanics and field theory. Suggested references: [Kna17, section 8]

Noether Theorem (23.04.): State and prove the Noether Theorem regarding continuous symmetries and conserved quantities (in both the classical and field theoretical version). Explain why this theorem is so important in physics. Suggested references: [Tao91, section 1.4]

Representation of groups (30.04.): Give an introduction to the *representation theory* of finite groups over the field of complex numbers. State and prove the theorem of *Maschke* and explain the classification of the irreducible representations using the *theory of characters*. The talk should be illustrated with small examples (cyclic groups, symmetric groups S_3 etc.). Suggested references: [Ser77] or [FH91].

Young diagrams and representations of Symmetric groups (07.05.): Using the previous talk, explain that the irreducible representations of the symmetric group S_n are in bijection with *partitions* of the integer n. Introduce the notion of *Young diagrams* and *Specht* modules. Explain how the representation theory of the symmetric group is related to the combinatorics of Young diagrams (e.g. *Frobenius character formula*, *Young's rule* or *Littlewood-Richardson rule*). Suggested reference: [Ful97].

Introduction to Lie algebras (14.05.): Give an introduction to the theory of *Lie algebra* over the field of complex numbers. Give a formal definition of an 'abstract' Lie algebra and illustrate it with various examples ($\mathfrak{gl}_n$, $\mathfrak{sl}_n$ etc.). Define the notion of *simple* and *semi-simple* Lie algebras. Explain *Cartan's criterion* for semi-simplicity. Suggested references: [EW06] and [Tao13].

Introduction to Lie groups (21.05.): Give the *formal* definition of a Lie group and prove that every *matrix Lie group* (i.e. a closed subgroup of $GL_n(\mathbb{C})$) is an *embedded submaniflod* of $GL_n(\mathbb{C})$. Thus, it is a Lie group. For this purpose you should define the *Lie algebra* associated to a matrix Lie group and explain the relation with the previous talk. Suggested references: Chapter 1,2 and 3 of [Hal15]. See also [Bak02].

Classification of Cartan-Killing (28.05.): Give an overview of the classification of the simple complex Lie algberas in terms of *root systems* and *Dynkin diagrams*. Suggested references: [EW06] and [Tao13].

Spontaneous symmetry breaking (04.06.): In this talk you explain the concept of spontaneous symmetry breaking and state the Goldstone Theorem. Use examples such as spin systems and Higgs mechanism to illustrate the differences between explicit symmetry breaking, spontaneous symmetry breaking and anomal symmetry breaking. Suggested references: [Gol61], [Nam60], [Hig64], [vD12], [Wei96, section 19 and 21]

Mermin-Wagner-Coleman Theorem (11.06.): State and prove the Mermin-Wagner-Coleman Theorem, which states the non-existence of spontaneous symmetry breaking in one and two dimensions. Suggested references: [Col73], [MW66].

From Newton to Einstein relativity theory (18.06.): Give an overview of the different relativity theories and discuss their connection to group theory. Present Galilei-transformations and Lorentz-Poincare transformations in their respective physical context. Suggested references: [Kna17, section 16], [Wei95, section 2.4].

Introduction to Random Matrix Theory (25.06.): Give an introduction to Random Matrix Theory. Focus on the basic ideas to connect the notion of matrices to probability theory and introduce random matrix models such as GOE. Suggested references: [LNV17], [AGZ09], [EKR15].

Classical McKay correspondence (02.07.): Give an overwiew on the classical McKay correspondence. More precisely, give the classification of the finite subgroups of $SL_2(\mathbb{C})$ up to conjugacy and explain how to relate them with Dynkin diagrams via *McKay graphs*. Suggested reference: [VZ12].

References

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