NEARBY CYCLES OF AUTOMORPHIC ÉTALE SHEAVES, II — ERRATUM

KAI-WEN LAN AND BENOÎT STROH

(1) In Sec. 3, the reference [Mil90, Ch. III, Sec. 6, Rem. 6.1], which claims that the Galois finite étale cover $\mathsf{X}_{\mathcal{H}(\ell^r)} \underset{\mathbb{Z}}{\otimes} \mathbb{Q} \to \mathsf{X}_{\mathcal{H}} \underset{\mathbb{Z}}{\otimes} \mathbb{Q}$ in (3.2) has Galois group exactly $\mathcal{H}_\ell^c/\mathcal{U}_\ell(\ell^r)^c$, is incorrect. (We thank Yihang Zhu for asking us about this reference and discussing with us about its validity.)

To see this, let $\mathrm{T} := \ker(\mathrm{G} \to \mathrm{G}^c)$, which is a torus by the definition of G^c. (We will not even need to know that T is $\mathbb{Q}$-anisotropic but $\mathbb{R}$-split.) The claim in [Mil90, Ch. III, Sec. 6, Rem. 6.1] would be valid only if, for nontrivial T, the cardinalities of $\mathrm{T}(\mathbb{Q})\backslash\mathrm{T}(\mathbb{A}^\infty)/\mathcal{H}_\mathrm{T}$ (which is a finite set by [Bor63, Thm. 5.1]) remain unchanged for all sufficiently small open compact subgroups $\mathcal{H}_\mathrm{T}$ of $\mathrm{T}(\mathbb{A}^\infty)$. This implies that the closure $\overline{\mathrm{T}(\mathbb{Q})}$ of $\mathrm{T}(\mathbb{Q})$ has finite index in $\mathrm{T}(\mathbb{A}^\infty)$, but contradicts the fact that $\overline{\mathrm{T}(\mathbb{Q})}$ has infinite index in $\mathrm{T}(\mathbb{A}^\infty)$ for *every* nontrivial torus T over $\mathbb{Q}$. (See [PR94, Prop. 7.13(2)], which explains that the same failure occurs, more generally, for algebraic groups over number fields that are connected but not simply-connected.)

This does not affect the construction of automorphic étale sheaves for representations of G^c, since all we need is that the Galois group is a quotient of $\mathcal{H}_\ell/\mathcal{U}_\ell(\ell^{r(m)})$ by construction, and admits $\mathcal{H}_\ell^c/\mathcal{U}_\ell(\ell^{r(m)})^c$ as a quotient. In particular, in (3.3), the contraction product can be formed using the action of $\mathcal{H}_\ell/\mathcal{U}_\ell(\ell^{r(m)})$ instead, whose pullback to $\mathsf{X}_{\mathcal{H}(\ell^{r(m)})} \underset{\mathbb{Z}}{\otimes} \mathbb{Q}$ is still isomorphic to $\underline{V}_{0,\ell^m}$ by construction. The remainder of Sec. 3 is unaffected.

References

[Bor63] A. Borel, *Some finiteness properties of adele groups over number fields*, Publ. Math. Inst. Hautes Étud. Sci. **16** (1963), 5–30.

[Mil90] J. S. Milne, *Canonical models of (mixed) Shimura varieties and automorphic vector bundles*, Automorphic Forms, Shimura Varieties, and L-Functions. Volume I (L. Clozel and J. S. Milne, eds.), Perspectives in Mathematics, vol. 10, Academic Press Inc., Boston, 1990, pp. 283–414.

[PR94] V. Platonov and A. Rapinchuk, *Algebraic groups and number theory*, Pure and Applied Mathematics, vol. 139, Academic Press Inc., New York and London, 1994, translated by Rachel Rowen.

University of Minnesota, Minneapolis, MN 55455, USA
Email address: kwlan@math.umn.edu

C.N.R.S. and Institut de Mathématiques de Jussieu–Paris Rive Gauche, 75252 Paris Cedex 05, France
Email address: benoit.stroh@imj-prg.fr

Published in *Cohomology of Arithmetic Groups: On the Occasion of Joachim Schwermer's 66th birthday, Bonn, Germany, June 2016*, Springer Proceedings in Mathematics & Statistics 245 (2018), ch. 4, 24 pp., doi:10.1007/978-3-319-95549-0_4.