TRANSLATION OF PARTS OF "DIE SÄTZE VON BERTINI FÜR LOKALE RINGE" BY H. FLENNER

[1]

1. Folgerungen

In the following let A be a local noetherian ring with maximal ideal $\mathfrak{m}_A$. A prime ideal $\mathfrak{p} \in \operatorname{Spec} A$ shall be called for short regular (resp. normal, resp. reduced) if $A_\mathfrak{p}$ is a regular (resp. normal, resp. reduced) local ring. The set of regular (resp. normal, resp. reduced) prime ideals we shall denote by $Reg(A)$ (resp. $Nor(A)$, resp. $Red(A)$).
If $x \in A$ and $\mathfrak{p} \subseteq A$ is a regular prime ideal that contains x, so $(A/xA)_\mathfrak{p}$ is regular iff $x \notin \mathfrak{p}^{(2)}$. Therefore follows from (2.1) immediately

Theorem ((3.1))**.** *Let A be a local noetherian ring and $\mathfrak{I} \subseteq \mathfrak{m}_A$ be an ideal. Then there exists an $x \in \mathfrak{I}$ such that*

$$Reg(A/xA) \cap D(\mathfrak{I}) \supseteq Reg(A) \cap V(x) \cap D(\mathfrak{I}).$$

One can always choose the element x so that it avoids a prescribed set of prime ideals of $D(\mathfrak{I})$.

This theorem solves a conjecture by Grothendieck (SGA2 ...). At the same time conjecture (2.5) can be answered positively, as is remarked there. Therefore we show:

Lemma 1. *(3.2)*

Proof. proof of lemma (3.2) □

We remark that the requirements of (3.2) are satisfied if A is an excellent ring or the quotient of a regular local ring.

Theorem. *theorem (3.3)*

Proof. proof of theorem (3.3) □

(3.3) allows a series of corollaries (!? sounds a bit strange in English, a better translation may be "some corollaries"). With the help of Serre's Normalitycriterion and Reducednesscriterion one gets the next two corollaries.

Corollary ((3.4))**.** *Let A be an excellent local ring, $\mathfrak{I} \subseteq \mathfrak{m}_A$ be an ideal and $U := D(\mathfrak{I})$. Then there exists an $x \in \mathfrak{I}$ such that*

$$U \cap Nor(A) \cap V(x) \subseteq U \cap Nor(A/xA).$$

In particular: If A is normal and $hcod(A) \geq 3$ then there exists an $x \in A$ such that A/xA is again normal.

Corollary ((3.5))**.** *Let $A, \mathfrak{I}$ and U like in (3.4). Then there exists an $x \in \mathfrak{I}$ such that*

$$U \cap Red(A) \cap V(x) \subseteq Red(A/xA).$$

In particular: if A is reduced and $hcod(A) \geq 2$ then there exists an $x \in A$ such that A/xA is again reduced.

(3.5) is a generalization of theorem (6.6) in [10].
Now we want to consider the question, under which conditions there exist prime elements in local rings. In (3.5) we have shown that this is always possible if A is excellent, normal and $hcod(A) \geq 3$. More generally holds:

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Corollary ((3.6))**.** *Let A be a local, excellent, analytically irreducible integral domain, that satisfies condition (R_1), and let $\mathfrak{I} \subseteq \mathfrak{m}_A$ be an ideal such that all irreducible components of $V(\mathfrak{I})$ have codimension ≥ 3. Then there exists an $x \in \mathfrak{I}$ such that $\operatorname{Spec}(A/xA) \cap D(\mathfrak{I})$ is an integral scheme. In particular holds: If in addition $\dim A \geq 3$ and hcod$(A) \geq 2$, so A contains prime elements.*

Proof. proof of Cor. (3.6). □