Philippe Caldero (Lyon I)

Title "Rational smoothness of varieties of representations for quivers of type A-D-E"

Abstract : The talk is about a recent paper by R. Bedard and R. Schiffler. Let F be an algebraic closure of the finite field F_q, $\mathbf{d} = (d_1, \dots, d_n) \in \mathbb{Z}^n_{\geq 0}$ and $G_{\mathbf{d}} \prod_{i=1}^n GL_{d_i}(F)$. Let Q be a fixed quiver whose underlying orientation is the Dynkin graph of type A-D-E. The group $G_{\mathbf{d}}$ acts on $E_{\mathbf{d}} = \bigoplus_{i \to j \in Q} \mathrm{Hom}(F^{d_i}, F^{d_j})$. Fix a $G_{\mathbf{d}}$-orbit $\mathcal{O}$, the problem is to characterize which orbit closures $\overline{\mathcal{O}}$ are rationnaly smooth.

Rational smoothness is a topological property of varieties defined using the local intersection cohomology groups of $\overline{\mathcal{O}}$. Let U^+ be the positive part of the quantized enveloping algebra over $\mathbb{Q}(v)$ associated by Drinfeld and Jimbo to the root system of type A-D-E. For each reduced decomposition $\tilde{w_0}$ of the longest element of the Weyl group, there is a PBW-basis $B_{\tilde{w_0}}$. Suppose that $\tilde{w_0}$ is the reduced decomposition associated to the quiver Q, then it is well known that the transition matrix between the canonical basis $\mathbf{B}$ of U^+ and $B_{\tilde{w_0}}$ can be given in terms of the local intersection cohomology groups. This provides a method to study the rational smoothness of the varieties $\overline{\mathcal{O}}$.