The irreducible Components of Lusztig's Nilpotent Variety and Crystal Graphs

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Let Q be a quiver without loops. We denote the quantized version of the enveloping algebra of the negative part of the corresponding Kac-Moody-Lie algebra by U^-. Lusztig has defined varieties $\mathcal{R}(\Pi(Q); d)_0$, also called Lusztig's nilpotent varieties, consisting of nilpotent representations of the preprojective algebra $\Pi(Q)$ of Q of dimension vector d. It was shown by Kashiwara and Saito ([1]) that the irreducible components of the various $\mathcal{R}(\Pi(Q); d)_0$, where d runs through all possible dimension vectors of Q, form the crystal of U^-.
The principal aim of this talk is to describe the irreducible components of $\mathcal{R}(\Pi(Q); d)_0$ using so-called nilpotent class representations (nc-representation) of Q with dimensions vector d. Informally a nc-representation assigns to Q and d certain nilpotent classes, so that the generic nc-representations are in natural bijection with the irreducible components of $\mathcal{R}(\Pi(Q); d)_0$.
This construction leeds naturally to certain similar varieties $\mathcal{R}(F_n; \lambda^0, \ldots, \lambda^n)_0$ associated to the free associative algebra F_n and $n+1$ nilpotent classes λ^i. Then we can define for any n an associated local crystal graph $\mathcal{C}_n$. We show, how one can reconstruct the global crystal graph from the local crystal graphs in a purely combinatorial way.

[1] M. Kashiwara, Y. Saito: Geometric construction of crystal bases. Duke Mat. J. 89 (1997), no. 1, 9 - 36