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**Crystal bases of representations and global bases for  $U_q(\mathfrak{n})$**

In the context of exactly solvable models, Date, Jimbo and Miwa considered representations of the quantum group  $U(\mathfrak{gl}_n(\mathbb{C}))$  at  $q = 0$ , which corresponds to the temperature 0. Kashiwara developed the general notion of such a *crystalization of a representation* of an arbitrary symmetrizable Kac-Moody algebra  $\mathfrak{g}$ , and he introduced the notion of a *crystal bases* of a representation.

On the one hand, this led to a completely new tool in algebraic combinatorics, compatible only with the special rôle Young tableaux play for the representations of  $GL_n(\mathbb{C})$ , and, on the other hand, the globalization of the crystal bases led to a different approach to the concept of canonical bases for  $U_q(\mathfrak{n})$ .

We will explain the concept of the crystal basis, explain some of its combinatorial features and realizations, we will speak about Kashiwara's approach to the globalization of the crystal basis, we will talk about the positivity properties of the global/canonical bases proved by Lusztig and about the quantum Frobenius map and its application to algebraic-geometric problems.