Homological realization of Nakajima varieties and Weyl group actions

Olivier Schiffmann (ENS Paris)

In this joint work with I. Frenkel and M. Khovanov, we first give a description of Nakajima quiver varieties $\mathcal{M}_0(v, w)$ associated to a quiver Q (with moment map parameter $\lambda = 0$) in terms of (derived category of) representations of the zig-zag algebra $A(Q)$ of Q. This (finite-dimensional) algebra is Koszul dual to the preprojective algebra; for instance, if Q is an affine Dynkin diagram correspondiong to the finite subgroup $\Gamma \subset SL(2, \mathbb{C})$ then $A(Q)$ is Morita equivalent to $\Lambda \mathbb{C}^2 \times \Gamma$. Our result follows from the standard Koszul duality. However, in order to treat the case of arbitrary moment map parameter λ it is necessary to introduce a deformation (of the derived category of representations) of $A(Q)$ (which can be seen as a deformation of $A(Q)$ as an A_∞-algebra), and to use a deformed version of the Koszul duality.

In this realization we show that the action of the Weyl group on the collection of varieties $\mathcal{M}_\lambda(v, w)$ is simply obtained as a tensor product with certain "tilting modules". These titling modules bear a strong ressemblance to the ones used by Khovanov, Seidel and Thomas to define the action of braid groups on certain derived categories.

1