Module theoretic interpretation of quantum minors

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Let Λ be a the preprojective algebra of type $\mathbb{A}_n$, and let $\mathcal{B}^*$ be the dual canonical basis of the associated quantized algebra U_v^-. The elements in $\mathcal{B}^*$ are indexed by multisegments $\mathbf{m}$.

To each quantum minor $b^*_{\mathbf{m}} \in \mathcal{B}^*$ we associate a Λ-module $L_{\mathbf{m}}$ (this is a laminated module in the sense of Ringel [3]). Our main result is the following:

Theorem. *Let $b^*_{\mathbf{m}}$ and $b^*_{\mathbf{n}}$ be quantum flag minors. Then the following are equivalent:*

(1) *$b^*_{\mathbf{m}}$ and $b^*_{\mathbf{n}}$ are multiplicative, i.e. $b^*_{\mathbf{m}} b^*_{\mathbf{n}} \in v^{\mathbb{Z}}\mathcal{B}^*$;*
(2) $\mathrm{Ext}^1_\Lambda(L_{\mathbf{m}}, L_{\mathbf{n}}) = 0$.

The proof of this theorem uses a combinatorial criterion due to Leclerc, Nazarov and Thibon [2] for two quantum flag minors to be multiplicative. For all missing definitions we refer to [1], [2] and [3].

References

[1] *A. Berenstein, A. Zelevinsky*, String bases for quantum groups of type A_r. I.M. Gelfand Seminar, 51–89, Adv. Soviet Math. **16**, Part 1, Amer. Math. Soc., Providence, RI (1993).

[2] *B. Leclerc, M. Nazarov, J.-Y. Thibon*, Induced representations of affine Hecke algebras and canonical bases of quantum groups. Preprint arXiv:math.QA/0011074 (2000), 1–33.

[3] *C.M. Ringel*, The multisegment duality and the preprojective algebras of type $\mathbb{A}$. AMA Algebra Montp. Announc. 1999, Paper 2, 6pp.