ENVELOPING ALGEBRAS OF SLODOWY SLICES THROUGH THE MINIMAL NILPOTENT OEBIT

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ABSTRACT. Let G be a simple algebraic group over an algebraically closed field of characteristic $p \geq 0$ and $\mathfrak{g} = \operatorname{Lie} G$. In finite characteristic we assume that $p > 3$ and $\mathfrak{g}$ admits a G-invariant nondegenerate trace form Ψ. Let (e, h, f) be an $\mathfrak{sl}_2$-triple in $\mathfrak{g}$ with e and f being long root vectors, and let $\chi \in \mathfrak{g}^*$ be such that $\chi(x) = \Psi(e, x)$ for all $x \in \mathfrak{g}$. Let $\mathcal{S}$ be the Slodowy slice through $\Omega = (\operatorname{Ad} G) \cdot e$ and let $H = H_\chi$ be the universal enveloping algebra of $\mathcal{S}$.

In my talk an explicit presentation of H will be given. In characteristic 0, a close relationship between H and the localisation of $U(\mathfrak{g})$ at f will be established, and a homeomorphism between $\operatorname{Prim} H$ and the spectrum of all primitive ideals of infinite codimension in $U(\mathfrak{g})$ will be presented. This homeomorphism respects Goldie rank and Gelfand-Kirillov dimension. Some general properties of the enveloping algebras of Slodowy slices will be discussed, if time permits, and the associated varieties of related primitive ideals of $U(\mathfrak{g})$ will be determined. An explicit Whittaker model for the Joseph ideal of $U(\mathfrak{g})$ will be presented and dimension formulae for finite dimensional H-modules will be given in some cases.

We shall also mention a finite dimensional modular analogue $H^{[p]}$ of H. As it turned out, if G is not of type A then the reduced enveloping algebra $U_\chi(\mathfrak{g})$ has a unique simple module of dimension $p^{(\dim \Omega)/2}$. For $\mathfrak{g} = \mathfrak{sp}_{2n}$ this module is just a restricted version of the Weil representation, but for simple Lie algebras of other types the modules are new (except in types A_n and D_4). Highest weights of these 'minimal' modules are found in all cases.

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