Alexander Premet

NILPOTENT COMMUTING VARIETIES AND PUNCTUAL HILBERT SCHEMES

Let G be a reductive algebraic group over an algebraically closed field k and $\mathfrak{g} = \operatorname{Lie} G$. Let $\mathcal{N}$ denote the nilpotent variety of $\mathfrak{g}$ and $\mathfrak{C}^{\rm nil}(\mathfrak{g})$ the variety of all commuting pairs $(x, y) \in \mathcal{N} \times \mathcal{N}$). The aim of my talk is to announce that under some standard (and mild) assumptions on the characteristic of k the variety $\mathfrak{C}^{\rm nil}(\mathfrak{g})$ is equidimensional and its components are parametrised by the distinguished nilpotent orbits in $\mathfrak{g}$. In characteristic 0, this confirms a conjecture of V. Baranovsky. When applied to $\mathfrak{gl}(n)$ our result implies that the punctual Hilbert scheme $\mathcal{H}_r$, which parametrises the ideals of colength r in the formal power series ring $k[[x, y]]$, is irreducible. This result is new when $\operatorname{char} k < r/2$.