Hall algebras for incidence algebras of posets

Justyna Kosakowska

Let $I = (I, \preceq)$ be a finite poset (partially ordered set) and let K be a finite field. We denote by KI the incidence K-algebra of the poset I and by $\mathrm{mod}(KI)$ the category of finite dimensional right KI-modules. Consider the full subcategory $\mathrm{prin}(KI)$ of the category $\mathrm{mod}(KI)$ consisting of prinjective KI-modules in the sense of the following definition. A KI-module X is said to be *prinjective* if there exists a short exact sequence

$$0 \to P_1 \to P_0 \to X \to 0,$$

where P_0, P_1 are projective KI-modules and P_1 is, in addition, semisimple. We call the poset I of *finite prinjective type* if there exist only finitely many isomorphism classes of indecomposable prinjective KI-modules.

The main aim of our talk is to present results concerning the existence of Hall polynomial for prinjective KI-modules in the case I is of finite prinjective type.

Moreover, in this case, we describe the associated Hall algebra by generators and relations.