

# A classification of torsion torsionfree triples in module categories

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In 1965 Jans ([2]) introduced the concept of **torsion torsionfree (TTF) triples** in an abelian category. They are triples  $(\mathcal{C}, \mathcal{T}, \mathcal{F})$  of full subcategories such that  $(\mathcal{C}, \mathcal{T})$  and  $(\mathcal{T}, \mathcal{F})$  are both torsion theories. In case the ambient category is the module category  $ModA$  over an associative ring  $A$  with unit, he gave a bijection between the set of those triples and the set of (two-sided) idempotent ideals of  $A$ . A TTF triple as above is called **centrally split** when both constituent torsion theories are split. Jans also proved that the above bijection restricted to another one between centrally split TTF triples of  $ModA$  and (ideals generated by) central idempotents of  $A$ . On the other hand, the existence of **one-sided split** TTF-triples (i.e. such that only one of  $(\mathcal{C}, \mathcal{T})$  and  $(\mathcal{T}, \mathcal{F})$  is a split torsion theory) has been known for a long time (cf. [4]). However, the idempotent ideals of  $A$  associated to them by Jans' correspondence had not been identified.

In this joint work with Pedro Nicolás (see [3]), we identify those idempotent ideals, thus providing a full classification of one-sided split TTF- triples in module categories. In the particular case when  $A$  is an Artin algebra the bijections obtained (for left and right split TTF-triples, respectively) can be obtained one from each other by duality and were considered in an earlier work with Assem ([1]).

## References

- [1] ASSEM, I.; SAORIN, M.: Abelian exact subcategories closed under predecessors. *Comm. Algebra* **33** (2005), 1205-1216
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- [3] NICOLAS, P.; SAORIN, M.: Classification of split torsion torsionfree triples in module categories. Preprint [www.arxiv.org/math.RA/0511159](http://www.arxiv.org/math.RA/0511159)
- [4] TEPLY, M.L.: Homological dimension and splitting torsion theories. *Pacific J. Math.* **34** (1970), 193-205