Arch. Math., Vol. 65, 244–250 (1995)

0003-889X/95/6503-0244 $ 2.90/0

Lower $\mathscr{H}$-cohomology of higher-dimensional quadrics

By

Bruno Kahn

Let F be a field of characteristic $\neq 2$ and X a smooth variety over F. For all $i, j \geqq 0$, we denote by $H^i(X, \mathscr{H}^j)$ the i-th Zariski cohomology group of X with coefficients in the Zariski sheaf associated to the presheaf $U \mapsto H^j U$, where for any scheme S we denote by $H^j S$ the group $H^j(S_{et}, \mathbb{Z}/2)$. By the Bloch-Ogus theorem [2], this is the i-th cohomology group of the complex:

$$0 \to H^j F(X) \to \bigoplus_{x \in X^{(1)}} H^{j-1} F(x) \to \ldots \to \bigoplus_{x \in X^{(i)}} H^{j-i} F(x) \to \ldots$$

where $X^{(i)}$ denotes the set of points of codimension i in X. In particular, we have $H^i(X, \mathscr{H}^j) = 0$ for $i > j$; moreover, the description of the differentials in the above complex yields isomorphisms $H^i(X, \mathscr{H}^i) \cong CH^i X/2$, where $CH^i X$ is the i-th Chow group of X. The Bloch-Ogus spectral sequence [2]

$$H^i(X, \mathscr{H}^j) \Rightarrow H^{i+j} X$$

yields *cycle class maps* $\mathrm{cl}_X^i \colon CH^i X/2 \to H^{2i} X$. For $i \leqq j$, define maps:

$$\mu^{i,j} \colon H^{j-i} F \otimes CH^i X/2 \to H^i(X, \mathscr{H}^j)$$

by means of cup-product from the cases $j = i$ (explained above) and $i = 0$ (induced by the map $H^j F \to H^j F(X)$).

The case of interest for us is when X is the projective hypersurface with equation $q(x) = 0$, where q is a nondegenerate quadratic form over F. We have $\dim X = \dim q - 2$, where $\dim q$ is the number of variables occurring in q.

We transpose to quadrics some definitions on quadratic forms which only depend on the similarity class: for example, the discriminant $d(X)$ of an even-dimensional quadric X makes sense, as does the Clifford invariant $c(X)$ if $\dim X$ is even and $d(X) = 1$. Similarly, we say that a quadric is a *Pfister quadric* (resp. an *Albert quadric*, a *neighbour*) if a quadratic form defining it is a Pfister form (resp. an Albert form, a Pfister neighbour). (Recall that an Albert form is a 6-dimensional quadratic form with trivial signed discriminant.)

We want to investigate the kernel and cokernel of $\mu^{i,j}$. In fact, we are essentially interested in the case $i = 0$ (unramified cohomology), but the proofs give information on other cases as well. We quote this information when easily available without looking for complete results when $i \neq 0$. By definition, $\mu^{i,i}$ is an isomorphism for any i and it is easy to see that $\mu^{0,1}$ is always an isomorphism. The aim of this paper is to prove Theorems 1, 2 and 3 below. For $a_1, \ldots, a_n \in F^*$, we write $\langle\langle a_1, \ldots, a_n \rangle\rangle$ for the n-fold Pfister form $\langle 1, -a_1 \rangle \otimes \cdots \otimes \langle 1, -a_n \rangle$.

Theorem 1. *If* $\dim X > 2$ *or* $\dim X = 2$ *and* $d(X) \neq 1$, *then* $\mu^{0,2}$ *is an isomorphism.*

Theorem 2. a) *If* $\dim X > 2$, *then* $\mu^{1,2}$ *is injective and there are isomorphisms*:

$$\operatorname{Coker} \mu^{1,2} \cong \operatorname{Ker} \mu^{0,3}$$
$$\operatorname{Coker} \mu^{0,3} \cong \operatorname{Ker} \mathrm{cl}_X^2.$$

b) *If* X *is a neighbour of the anisotropic* 3*-fold Pfister form* $\langle\langle a,b,c \rangle\rangle$, *then* $\operatorname{Ker} \mu^{0,3}$ *is generated by* (a,b,c), $\operatorname{Coker} \mu^{1,2} \cong \mathbf{Z}/2$ *and* $\operatorname{Coker} \mu^{0,3} \cong \mathbf{Z}/2$ *or* 0 *according as* $(-1,a,b,c)$ *is or is not* 0 *in* $H^4 F$.
c) *If* X *is a* 3*-dimensional non-neighbour, then* $\mu^{1,2}$ *and* $\mu^{0,3}$ *are isomorphisms.*
d) *If* X *is an anisotropic Albert quadric, then* $\mu^{1,2}$ *is an isomorphism,* $\mu^{0,3}$ *is injective and* $\operatorname{Coker} \mu^{0,3} \cong \mathbf{Z}/2$.
e) *If* X *is not a* 3*-fold neighbour and either* $\dim X > 4$ *or* $\dim X = 4$ *and* $d(X) \neq 1$ (*e.g.* $\dim X > 6$), *then* $\mu^{1,2}$ *and* $\mu^{0,3}$ *are isomorphisms.*

Theorem 3. *For a quadric* X, *the cycle class map* $\mathrm{cl}_X^2 : CH^2 X/2 \to H^4 X$ *is injective, except in the following cases:*
i) X *is a neighbour of an anisotropic* 3*-fold Pfister form* $\langle\langle a,b,c \rangle\rangle$ *and* $(-1,a,b,c) = 0 \in H^4 F$;
ii) X *is an anisotropic Albert quadric.*
In these two cases, $\operatorname{Ker} \mathrm{cl}_X^2 \cong \mathbf{Z}/2$.

Let $W_{nr}(F(X)/F) = W_{nr}(F(X))$ be the unramified part of the Witt ring of $F(X)$ (relatively to F) [12]. For any $n > 0$, let $I_{nr}^n F(X) = W_{nr}(F(X)) \cap I^n F(X)$. As an application of Theorems 1 and 2, we have:

Corollary. a) *Under the assumptions of Theorem* 1, *the map*

$$W(F)/I^3 F \to W_{nr}(F(X))/I_{nr}^3 F(X)$$

is bijective.
b) *In cases* c) *and* e) *of Theorem* 2, *the map*

$$W(F)/I^4 F \to W_{nr}(F(X))/I_{nr}^4 F(X)$$

is bijective.

Proof. For $n \leq 3$, the isomorphism

$$e^n : I^n F(X)/I^{n+1} F(X) \xrightarrow{\sim} H^n F(X)$$

of Arason, Merkurjev and Rost/Merkurjev-Suslin restricts to an injection

$$e^n : I_{nr}^n F(X)/I_{nr}^{n+1} F(X) \hookrightarrow H_{nr}^n F(X)$$

(compare [12, § 1]). So we have a commutative diagram ($n \leqq 3$):

$$\begin{array}{ccc} I^n F/I^{n+1} F & \xrightarrow{\sim} & H^n F \\ \downarrow & & \downarrow \\ I^n_{nr} F(X)/I^{n+1}_{nr} F(X) & \hookrightarrow & H^n_{nr} F(X). \end{array}$$

Under the assumptions of Theorem 1 (resp. in cases c) and e) of Theorem 2), the right vertical map is an isomorphism, so all maps are isomorphisms, in particular the left vertical one. The corollary follows.

Remarks. 1. By [3, Prop. 1.2], unramified cohomology is rationally invariant. In particular, if X is isotropic, then $F(X)/F$ is a rational extension, so $\operatorname{Ker} \mu^{0,i} = \operatorname{Coker} \mu^{0,i} = 0$ for all i.

2. Theorems 1, 2 and 3 generalise and amplify earlier results of J.-L. Colliot-Thélène and R. Sujatha on unramified H^3 of real anisotropic quadrics [4] and anisotropic 3-fold Pfister quadrics [14]. In particular, the statement of Theorem 2b) on $\operatorname{Coker} \mu^{0,3}$ is due to Sujatha [14]. The proof we give in this paper is slightly different from hers.

3. Similar results on unramified H^4 can be obtained by more sophisticated methods [10].

4. The remaining cases for Theorem 1 are, respectively, $\dim X = 1$ and X is a quaternion surface. Since unramified cohomology is rationally invariant and any 3-dimensional quadratic form is a Pfister neighbour, these two cases are equivalent (compare [4, Lemma 1.3]). If X is a conic curve with invariant $[D]$ ($= c(\varphi)$, where φ is the quaternion form of which a representing form for X is a neighbour), it is known that $\operatorname{Ker} \mu^{0,2}$ is generated by $[D]$ and that $\operatorname{Coker} \mu^{0,2} = \mathbb{Z}/2$ or 0 according as $(-1) \cdot [D]$ is or is not 0 in $H^3 F$ (compare [13, Prop. 2.2]).

5. The remaining cases for Theorem 2 are $\dim X \leqq 2$. In the case of a conic X with invariant $[D]$, it follows from [13, Prop. 2.2] that we have isomorphisms:

$$\begin{aligned} \operatorname{Ker} \mu^{0,3} &\cong F^*/\operatorname{Nrd} D^*, \\ \operatorname{Coker} \mu^{0,3} &\cong \operatorname{Ker}(F^*/\operatorname{Nrd} D^* \xrightarrow{\cdot(-1)\cdot[D]} H^3 F), \\ \operatorname{Ker} \mu^{1,2} &\cong \operatorname{Nrd} D^*/\pm F^{*2}, \\ \operatorname{Coker} \mu^{1,2} &\cong \operatorname{Nrd} D^*/F^{*2}. \end{aligned}$$

The answer is the same for a quaternion surface, at least for $\mu^{0,3}$.

In the case $\dim X = 2$, $d(X) \neq 1$, it is known that $\operatorname{Ker} \mu^{0,3}$ consists of those symbols (a,b,c) such that q is similar to a subform of $\langle\langle a,b,c\rangle\rangle$ (Arason [1]), and it is shown in [10] that $\operatorname{Coker} \mu^{0,3}$ is isomorphic to the subgroup of $\operatorname{Ker} \mu^{0,3}$ formed of those α such that $(-1) \cdot \alpha = 0 \in H^4 F$.

Together with the Bloch-Ogus spectral sequence, we shall use the Hochschild-Serre spectral sequence for the extension to a separable closure of F. We shall use freely the fact that both spectral sequences are compatible with products. For the Hochschild-Serre one, as well as for spectral sequences associated to change of sites in general, this is classical; for the Bloch-Ogus spectral sequence, it follows from Deligne's result that the latter coincides from E_2 on with the change-of-sites spectral sequence associated to the morphism $X_{et} \to X_{Zar}$ [2, footnote p. 195] (we are indebted to Henri Gillet for pointing this out).

1. Preliminaries.

1.1 The Bloch-Ogus spectral sequence. From it we get the following exact sequences:

$$0 \to CH^1X/2 \to H^2X \to H^0(X,\mathscr{H}^2) \to 0 \tag{1}$$

$$0 \to H^1(X,\mathscr{H}^2) \to H^3X \to H^0(X,\mathscr{H}^3) \to CH^2X/2 \to H^4X. \tag{2}$$

(Here again, X may be any smooth variety over F.)

1.2 Chow groups of quadrics.

Lemma 1. *Let $\bar{X} = X \times_F F_s$, where F_s is a separable closure of F.*
a) *If $\dim X > 2$ or $\dim X = 2$ and $d(X) \neq 1$, then $CH^1X/2 \xrightarrow{\sim} H^0(F, CH^1\bar{X}/2)$.*
b) *If X is not a 3-fold neighbour and either $\dim X > 4$ or $\dim X = 4$ and $d(X) \neq 1$ (e.g. $\dim X > 6$), then $CH^2X/2 \xrightarrow{\sim} H^0(F, CH^2\bar{X}/2)$ and the cycle map $CH^2X/2 \to H^4X$ is injective.*

P r o o f. Recall that for any smooth projective quadric X, $CH^nX/torsion$ is generated by h^n for $n < \dim X/2$, where $h \in CH^1X$ is the class of a hyperplane section, and $CH^*\bar{X}$ is torsion-free. Moreover, CH^1X has no torsion and $(CH^2X)_{\text{torsion}}$ is isomorphic to $\mathbb{Z}/2$ if X is a neighbour of an anisotropic 3-fold Pfister form and 0 otherwise [9]. If $\dim X = 2$ and $d(X) \neq 1$, $CH^1X \cong \mathbb{Z}$, $CH^1\bar{X} \cong \mathbb{Z} \oplus \mathbb{Z}$, where the Galois action permutes the two factors, and the natural map $CH^1X \to CH^1\bar{X}$ maps 1 to $(1,1)$. If $\dim X = 4$ and $d(X) \neq 1$ the description is similar for CH^2X (op. cit.). Finally, the cycle maps are isomorphisms over a separable closure of F, which proves the last claim of b).

Lemma 2. *Let X be an anisotropic quaternion surface, with Clifford invariant $c \in H^2F$. Then the image of a generator of CH^2X under the cycle map equals $c \cdot \mathrm{cl}_X(C)$, where C is a hyperplane section of X.*

P r o o f. We follow Szyjewski [15, § 5.3]. First $\mathrm{cl}_C(pt) \in \mathrm{Ker}(H^2C \to H^2\bar{C}) = H^2F$, and then $\mathrm{cl}_C(pt) \in \mathrm{Ker}(H^2F \to H^2F(C)) = \{0, c\}$. Since the cycle map $CH^1C/2 \to H^2C$ is injective, it follows that $\mathrm{cl}_C(pt) = c \in H^2F$. Now the Gysin map $i_*: H^2C \to H^4X$ maps $\mathrm{cl}_C(pt)$ to $\mathrm{cl}_X(pt)$. But

$$i_*(c) = c \cdot i_*(1) = c \cdot \mathrm{cl}_X(C).$$

Proposition 1. a) *For any quadric surface X, the cycle map $\mathrm{cl}_X^2: CH^2X/2 \to H^4X$ is injective.*
b) *If X is a 3-dimensional non-neighbour, the same conclusion holds.*

P r o o f. We first prove a). The case where X is isotropic is clear, since then X has a rational point and the map $CH^2X \to CH^2\bar{X}$ is bijective. Assume now X anisotropic. Extending scalars if necessary to $F(\sqrt{d})$, where $d = d(X)$, we may assume that $d(X) = 1$, i.e. X is a quaternion quadric (observe that $CH^2X \to CH^2X_{F(\sqrt{d})}$ is bijective). In this case, Proposition 1 follows from Lemma 2 *via* the multiplicativity of the Hochschild-Serre spectral sequence by observing that $\mathrm{cl}_X(C) \neq 0$ [15, Lemma 5.3.2 c)]. Finally, b) follows from a) by taking any hyperplane section Z of X and observing that the generator h^2 of CH^2X restricts to the generator of CH^2Z.

1.3 The Hochschild-Serre spectral sequence. This is the spectral sequence

$$H^i(F, H^j \bar{X}) \Rightarrow H^{i+j} X$$

where as in Lemma 1 $\bar{X} = X \times_F F_s$ for a separable closure F_s of F.

If F is separably closed, $H^i X = 0$ for i odd and the cycle maps $CH^i X/2 \to H^{2i} X$ are isomorphisms. In general, define maps:

$$v^{i,j}: H^{j-i} F \otimes CH^i X/2 \to H^{i+j} X$$

by cup-product from the cases $j = i$ (cycle map) and $i = 0$ (functoriality). From Lemma 1 and the Hochschild-Serre spectral sequence, we deduce:

Lemma 3 (compare [15, Lemma 5.2.1]). *If* $\dim X > 2$ *or* $\dim X = 2$ *and* $d(X) \neq 1$, *then the map* $v^{1,1}$ *induces an isomorphism*

$$H^2 F \oplus CH^1 X/2 \xrightarrow{\sim} H^2 X. \tag{3}$$

If $\dim X > 2$, *the map* $v^{1,2}$ *induces an isomorphism*

$$H^3 F \oplus H^1 F \otimes CH^1 X \xrightarrow{\sim} H^3 X. \tag{4}$$

2. Proofs, excluding Albert quadrics.

Proof of Theorem 1. It follows immediately from (1) and (3).

Proof of Theorem 2a). By multiplicativity of the Bloch-Ogus and Hochschild-Serre spectral sequences, the map $v^{1,2}$ factors through $\mu^{1,2}$ and the map $\mu^{0,3}$ factors through $v^{0,3}$. Hence (2) and (4) translate into an exact sequence:

$$\begin{aligned} 0 \to H^1 F \otimes CH^1 X \to H^1(X, \mathscr{H}^2) \to H^3 F \to H^0(X, \mathscr{H}^3) \\ \to CH^2 X/2 \to H^4 X. \end{aligned} \tag{5}$$

The claims of a) follow.

Proof of Theorem 2e). By Lemma 1b), $\mathrm{cl}_{\bar{X}}^2$ is injective; by Arason's theorem [1, Satz 5.6], $\mu^{0,3}$ is injective. The claims of e) follow from these remarks and a).

Proof of Theorem 2b) and c). The statements on $\operatorname{Ker} \mu^{0,3}$ and $\operatorname{Coker} \mu^{1,2}$ follow from Arason's main theorem [1, Satz 5.6] and a). It remains to deal with $\operatorname{Coker} \mu^{0,3}$. In case c), surjectivity of $\mu^{0,3}$ follows from a) and Proposition 1. To prove the last statement of b), we may assume X to be 3-dimensional (compare [4, Lemma 1.3]). By [15, Prop. 5.4.6], the image of the torsion element of $CH^2 X$ in $H^4 X$ by the cycle map is $(-1, a, b, c)$, which proves the statement by a) again.

Proof of Theorem 3, excluding ii). It follows from Lemma 1b), Proposition 1 and (for 3-fold neighbours) Theorem 2a) and b) (it is trivial for conics).

3. Proofs: the case of an Albert quadric. In this section, we prove Theorem 2d) and Theorem 3 in case ii). Let X be an anisotropic Albert quadric.

The injectivity of $\mu^{0,3}$ follows once again from Arason's theorem [1, Satz 5.6], since an anisotropic Albert form is not contained in a 3-fold Pfister form. By [9], $CH^2 X$ and $CH^2 \bar{X}$ are isomorphic to $\mathbb{Z} \oplus \mathbb{Z}$, extension of scalars corresponding to $(x, y) \mapsto (x, 4y)$. It follows that $\operatorname{Ker} \mathrm{cl}_X^2$ is 0 or $\mathbb{Z}/2$. We shall exhibit an explicit element of $H^0(X, \mathscr{H}^3) \backslash H^3 F$, which, with the help of Theorem 2a), will conclude the proof that $\operatorname{Coker} \mu^{0,3} \cong \operatorname{Ker} \mathrm{cl}_X^2 \cong \mathbb{Z}/2$.

Let q be an Albert form defining X and q_1 the anisotropic part of $q_{F(X)}$. Then q_1 is similar to a quaternion form τ, and $\tau \perp -q_1 \in I^3 F(X)$. Let $\tilde{e}^3(q_1) = e^3(\tau \perp -q_1) \in H^3 F(X)$ (compare [7, Prop. 3.2]). As q_1 and τ are unramified, it is clear that $\tilde{e}^3(q_1) \in H^0(X, \mathscr{H}^3)$.

Lemma 4. $\tilde{e}^3(q_1) \neq 0$.

Proof. If $\tilde{e}^3(q_1) = 0$, then $\tau \perp -q_1 \sim 0$ and q_1 represents 1. Consider $q' = q \perp \langle -1 \rangle$. Two cases may occur:

- q' is isotropic. Then $q = q'' \perp \langle 1 \rangle$ with $\dim q'' = 5$. By assumption, one sees that $q''_{F(X)}$ is isotropic. But this is impossible by a result of Hoffmann [6, Main theorem]. Indeed, this result implies that q'' is a neighbour of a 3-fold Pfister form. Then q'' represents its own discriminant -1 and q is isotropic.
- q' is anisotropic. Let $E = F(q')$. By the former case, q_E is isotropic. But this is impossible, this time by a result of Leep [6, Theorem 2], which would imply that q' is similar to a subform of q. □

We now claim that $\tilde{e}^3(q_1)$ is not defined over F. Assume it is. Let $\beta \in H^3 F$ be such that $\beta_{F(X)} = \tilde{e}^3(q_1)$. By [8, Prop. 3], β is a sum of at most two symbols (although this fact is not strictly necessary for the proof). If it is equal to one symbol $e^3(\varphi)$ (φ a 3-fold Pfister form over F), then by the Hauptsatz $\tau \perp -q_1$ is defined over F by φ. Passing to the function field $K = F(\varphi)$, we get that $\tilde{e}^3((q_1)_{K(X)}) = 0$, hence q_K is isotropic by Lemma 4, which is impossible by Merkurjev's index reduction theorem [11]. Assume $\beta = \gamma + \delta$, where γ, δ are symbols. Let φ be the Pfister form with e^3-invariant γ and $K = F(\varphi)$. Over K, q remains anisotropic by [11] again and β_K is one symbol, which is impossible as we have just seen.

Remarks. 1. The exact sequence (5) shows that $\tilde{e}^3(q_1)$ does not map to 0 in $CH^2 X/2$, so that $\operatorname{Ker} \mathrm{cl}_X^2 \cong \mathbb{Z}/2$. In fact, we see from [9] that the image of $\tilde{e}^3(q_1)$ in $CH^2 X/2$ is the class of $4l$, where l is the class of one of the rulings over $\bar{X}$. Also, it follows from (2) that $\tilde{e}^3(q_1)$ does not come from $H^3 X$.

2. Let w_4 be the 4-th Delzant Stiefel-Whitney class [5]. One checks easily that $w_4(q)_{F(X)} = w_4(q_1 \perp \langle 1, -1 \rangle) = (-1) \cdot \tilde{e}^3(q_1)$. So $(-1) \cdot \tilde{e}^3(q_1)$ is defined over F.

3. This proof of Theorem 2d) also implies that τ is an unramified Witt class which does not come from $W(F)$, so that $W(F) \to W_{nr}(F(X)/F)$ is not surjective. To our knowledge, this is the first example of a genuine unramified Witt class over the function field of a quadric to appear in the literature.

Acknowledgements. I wish to thank Hélène Esnault for interesting questions which prompted the former Remarks 1 and 2, and the referee for suggestions which helped improve the presentation of this paper.

Added in proof.*) Colliot-Thélène pointed out that the present proof of Theorem 2b) is very sketchy. Here is a more general agreement. By [9, (2.7)], $CH^p X$/torsion is generated by h^p if the quadratic X is anisotropic and $p \neq \dim X/2$. Applying this to $p = 2$ and our 3-dimensional neighbour X, we get by the same argument as in the proof of Proposition 1b that $\operatorname{Ker} \mathrm{cl}_X^2 \subseteq \operatorname{Im} ((\mathrm{CH}^2 X)_{\text{torsion}} \to CH^2 X/2)$. By [15, Cor. 3.3.2 and Prop. 5.4.6], cl_X^2 maps the nonzero torsion element of $CH^2 X$ to $(-1, a, b, c)_X \in H^4 X$, and the Hochschild-Serre spectral sequence of 1.3 shows that $H^4 F \to H^4 X$ is injective. Theorem 2a) now concludes the proof.

References

[1] J. K. ARASON, Cohomologische Invarianten quadratischer Formen. J. Algebra **36**, 448–491 (1975).
[2] S. BLOCH and A. OGUS, Gersten's conjecture and the homology of schemes. Ann. Sci. École Norm. Sup. (4) **7**, 181–202 (1974).
[3] J.-L. COLLIOT-THÉLÈNE and M. OJANGUREN, Variétés unirationnelles non rationnelles: au-delà de l'exemple d'Artin et Mumford. Invent. Math. **97**, 141–158 (1989).
[4] J.-L. COLLIOT-THÉLÈNE et R. SUJATHA, Unramified Witt groups of real anisotropic quadrics. In: *K*-theory and algebraic geometry: connections with quadratic forms and division algebras, W. Jacob, A. Rosenberg, eds. Proc. Sympos. Pure Math. **58** (2), 127–147 (1995).
[5] A. DELZANT, Définition des classes de Stiefel-Whitney d'un module quadratique sur un corps de caractéristique différente de 2. C. R. Acad. Sci. Paris Sér. I Math. **255**, 1366–1368 (1962).
[6] D. HOFFMANN, Isotropy of 5-dimensional quadratic forms over the function field of a quadric. In: *K*-theory and algebraic geometry: connections with quadratic forms and division algebras, W. Jacob, A. Rosenberg, eds. Proc. Sympos. Pure Math. **58** (2), 217–255 (1995).
[7] B. KAHN, Formes quadratiques de hauteur et de degré 2. To appear in Indag. Math.
[8] B. KAHN, A descent problem for quadratic forms. To appear in Duke Math. J.
[9] N. KARPENKO, Algebro-geometric invariants of quadratic forms (Russian). Algebra i-Analiz. **2**, 141–162 (1990). English translation: Leningrad Math. J. **2**, 119–138 (1991).
[10] B. KAHN, M. ROST and R. SUJATHA, Unramified cohomology of quadrics. In preparation.
[11] A. S. MERKURJEV, Simple algebras and quadratic forms (Russian). Izv. Akad. Nauk SSSR **55**, 218–224 (1991). English translation: Math. USSR Izv. **38**, 215–221 (1992).
[12] R. PARIMALA, Witt groups of affine three-folds. Duke Math. J. **57**, 947–954 (1988).
[13] E. PEYRE, Products of Severi-Brauer varieties and Galois cohomology. In: *K*-theory and algebraic geometry: connections with quadratic forms and division algebras, W. Jacob, A. Rosenberg, eds. Proc. Sympos. Pure Math. **58** (2), 369–401 (1995).
[14] R. SUJATHA, Unramified cohomology and Witt group of anisotropic Pfister quadrics. Preprint, 1995.
[15] M. SZYJEWSKI, The fifth invariant of quadratic forms (Russian). Algebra i-Analiz. **2**, 213–234 (1990). English translation: Leningrad Math. J. **2**, 179–198 (1991).

Eingegangen am 24. 11. 1994**)

Anschrift des Autors:

Bruno Kahn
Institut de Mathématiques
Equipe Théories géométriques
Université de Paris 7
Case 7012
2, Place Jussieu
F-75251 Paris Cedex 05

*) Eingegangen am 14. 6. 1995.
**) Eine revidierte Fassung ging am 3. 3. 1995 ein.