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## Groupoids, and Faà di Bruno formulae for Green functions in bialgebras of trees

In his work on gauge field theories, van Suijlekom discovered that in the Connes-Kreimer Hopf algebra of Feynman graphs, a certain infinite series Y satisfies the Faà di Bruno comultiplication formula, dual to the formula for composition of formal power series. Except for some denominators, Y is the combinatorial Green function (sum of all graphs weighted by inverses of symmetry factors). In this talk I will explain a version of this result in the setting of operadic trees, and give a very conceptual proof in which the Faà di Bruno formula drops out as the homotopy cardinality of an equivalence of groupoids. The use of operadic trees rather than the usual combinatorial trees is essential for the construction, and also allows the transfer of the formula back to the realm of graphs.

The talk will start with the classical Faà di Bruno bialgebra, and move on to the bialgebra of operadic trees; then I shall explain some facts about groupoids, homotopy quotients and homotopy cardinality, and finally put the things together in an easy proof of the Faà di Bruno formula for operadic trees.

This is joint work with Imma Galvez and Andy Tonks.