Totally disconnected groups (not) acting on three-manifolds

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Abstract : Hilbert's Fifth Problem asks whether every topological group which is a manifold is in fact a (smooth!) Lie group; this was solved in the affirmative by Gleason and Montgomery–Zippin. A stronger conjecture is that a locally compact topological group which acts faithfully on a manifold must be a Lie group. This is the Hilbert–Smith Conjecture, which in full generality is still wide open. It is known, however (as a corollary to the work of Gleason and Montgomery–Zippin) that it suffices to rule out the case of the additive group of p-adic integers acting faithfully on a manifold. I will present a solution in dimension three. The proof uses tools from low-dimensional topology, for example incompressible surfaces, minimal surfaces, and a property of the mapping class group.