

# TQFT, groupes quantiques et invariants non commutatifs

IMJ-PRG, Université Paris Diderot, Université Sorbonne-Paris-Cité\*

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## **Non-degenerate ribbon categories from unrolled quantum groups**

Azat Gainutdinov

Hambourg/Tours

There exist several versions of non-semisimple analogues of the modular tensor categories: (i) transparent objects in  $C$  are trivial; (ii) the Drinfeld centre of  $C$  is equivalent to the Deligne tensor product of  $C$  with  $C^{op}$ ; (iii) the Hopf pairing on the Lyubashenko's coend is non-degenerate. In the semi-simple case, all these conditions lead to the non-degeneracy of the Hopf link or the  $S$ -matrix. It was proven recently (by Shimizu) that all three conditions (i)-(iii) are equivalent and we can thus talk about “the non-degenerate ribbon categories” as the replacement for “the modular tensor categories”. I will present a large class of new non-degenerate (finite) ribbon categories that come from representation theory of the unrolled quantum group and will discuss mapping class group representations on their Hom spaces. The motivation for our new construction comes from conformal extensions of vertex-operator algebras that appear in Logarithmic CFT.

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## **Modified trace on quantum $sl(2)$ and logarithmic invariants**

Nathan Geer

Utah State University, Logan

In this talk I will discuss a link invariant arising from the restricted quantum of  $sl(2)$  at a  $2p$ -th root of unity. In particular, I will show how we can use a modified trace on the quantum group itself to define a Logarithmic invariant of colored links. Using the integer on the quantum group, this invariant can be extended to an invariant of colored links in a 3-manifold. We expect this 3-manifold invariant to lead to a TQFT. This is joint work with Anna Beliakova and Christian Blanchet.

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## **Invariants of mapping tori in Quantum Teichmüller theory**

Rinat Kashaev

Genève

Quantum Teichmüller theory allows to construct unitary projective representations in infinite dimensional Hilbert spaces of the mapping class groups of punctured surfaces. A direct application of that theory in 3-manifold topology is the construction of invariants of mapping tori of surface homeomorphisms by taking the traces of the associated quantum operators. A priori, the trace of a unitary operator is either well defined (finite) or ill defined (infinite). The volume conjecture for the Teichmüller TQFT implies that those traces should be finite at least in the case of hyperbolic mapping tori. I will discuss the cases of finite cyclic coverings of the trefoil and figure-eight knot complements.

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## **Géométrie différentielle sur la variété des caractères**

Julien Marché

IMJ-PRG, Paris 6

La variété des caractères d'une variété de dimension 3 à bord admet des structures naturelles: formes symplectiques ou formes volumes. Ces structures sont reliées des objets de nature "quantique", les modules de Kauffman, avec "au premier ordre", une équation différentielle satisfaite conjecturalement par la torsion de Reidemeister.

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## **Invariants of Q-links from representations of singular biracks**

Bertrand Patureau

Université de Bretagne-Sud, Vannes

This is a joint work in progress with Christian Blanchet, Nathan Geer and Nicolai Reshetikhin. A birack is a set with a map  $B \times B \rightarrow B \times B$  which satisfies some properties including the set theoretical Yang-Baxter equation. It is singular if the above map is partially defined. I will explain how a representation of a generically defined singular birack in a pivotal category leads to invariants of quandle-colored links. Such representations appear in the category of cyclic representation of  $U_q \mathfrak{sl}_2$  at root of unity using the "holonomy R-matrices" introduced by Rinat Kashaev and Nicolai Reshetikhin.

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# On solutions to the Yang-Baxter equation related to quantum groups at roots of unity

Reshetikhin Nicolai  
Berkeley

In this talk an explicit formula will be given for solutions to the holonomy R-matrix for quantum  $sl_2$ .

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## Homotopy quantum field theories

Virelizier Alexis  
Lille

Homotopy quantum field theory (HQFT) is a branch of quantum topology concerned with maps from manifolds to a fixed target space. The aim is to define and to study homotopy invariants of such maps using methods of quantum topology. I will focus on low dimensional HQFTs with aspherical target. (When the target space is a point, one recovers the more familiar notion of TQFT.) In particular, these HQFTs provide numerical invariants of principal bundles over closed 3-manifolds. To construct such HQFTs, the relevant algebraic ingredients are monoidal categories which are graded by a discrete group. If time permits, I will explore the case of topological groups.

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