Mazur's conjecture over totally real fields.

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We shall sketch the proof a conjecture of B. Mazur pertaining to the non-triviality of the p-adic anticyclotomic Euler systems which are attached to CM points on Shimura curves over totally real number fields.

This is a joint work with V. Vatsal (UBC-Vancouver) which generalises to totally real fields the results that we previously obtained for modular curves over **Q** (Cornut, *Mazur's conjecture on Higher Heegner points*, Inventiones **148**; Vatsal, *Uniform distribution of Heegner points*, Inventiones **148** and *Special values of anticyclotomic L-functions*, Duke Math. Journal **116**). They have the same consequences:

1. Non-triviality of special values of (derivatives of) twisted L-functions attached to Hilbert modular forms of weight $(2, \cdots, 2)$ (using the Gross-Zagier type formulas proven by Zhang).
2. Rank one statements for certain Selmer groups, and half main conjectures for p-adic anticyclotomic L-fonctions (using the ad-hoc generalisation of Kolyvagin's method, as in Howard, *Iwasawa theory of Heegner points on abelian varieties of PEL-type*, to appear in Duke).
3. New cases of BSD (combining (1) and (2)).
4. Parity in the BSD conjecture for the Galois representations attached to Hilbert modular forms of weight $(2k, \cdots 2k)$ (using (2) and the theory of *Selmer complexes* developped by Jan Nekovar).

We will first state the result and discuss shortly its consequences. The proof has two main steps, which are best understood in the framework of the Andre-Oort conjecture. The first step is the *geometrical part*: it consists in identifying (and dealing with) the relations that exists among the special points that we consider. The second part is the *chaotical part*, for which we now have two proofs: one that uses a theorem of M. Ratner (Ergodic theory of unipotent actions of p-adic Lie group on homogeneous spaces), one that uses a proven case of the Andre-Oort conjecture.