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An overview of the manuscript collection. Results are grouped by subject; entry numbers follow the catalog order and do not indicate a ranking. PDF links follow each summary.

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Number theory

001. **Milne's rationality conjecture.** Proves Milne's rationality conjecture for abelian varieties over $\overline{\mathbb{Q}}$ with good reduction: specialized Hodge classes pair rationally with complementary divisor products, independently of cohomology theory. Together with result 032, every specialized Hodge class is represented by a single rational algebraic cycle simultaneously in all prime-to-p and crystalline realizations, for every residue characteristic p.
Milne's rationality conjecture for abelian varieties

002. **The full Birch–Swinnerton-Dyer formula in Selmer coranks zero and one.** Proves the full Birch–Swinnerton-Dyer leading-term formula for every elliptic curve over $\mathbb{Q}$ whose full q-power Selmer group has corank zero or one for some prime q, including finiteness of the Tate–Shafarevich group. With result 006, this gives full BSD for a density-one set of quadratic twists of every elliptic curve over $\mathbb{Q}$.
Exact Birch–Swinnerton-Dyer Formula from Low Selmer Corank · The Selmer converse for elliptic curves at every prime · The two-primary Birch–Swinnerton-Dyer formula in Selmer corank at most one

003. **The quasi-Riemann hypothesis.** Proves that every Dirichlet L-function, including $\zeta(s)$, is zero-free in $\Re s > 7/8$, resolving the quasi-Riemann hypothesis. The same half-plane is zero-free for every finite-order Hecke L-function over $\mathbb{Q}(\sqrt{-3})$. A companion gives a different proof of the zero-free half-plane $\Re s > 11/12$.
The Quasi-Riemann Hypothesis: A Zero-Free Half-Plane $\Re s > 7/8$ · The Quasi-Riemann Hypothesis (alternate 11/12 proof) · Uniform exclusion of Landau–Siegel zeros

004. **Hilbert's tenth problem over $\mathbb{Q}$.** Proves that no algorithm decides whether an integer-coefficient polynomial in an arbitrary number of variables has a rational zero, resolving Hilbert's tenth problem over $\mathbb{Q}$ negatively.
Hilbert's tenth over $\mathbb{Q}$ · Pointwise 2-converse with rational two-torsion

005. **Irrationality of Catalan's constant.** Proves that Catalan's constant $G = \sum_{j\geq 0}(-1)^j/(2j+1)^2$ is irrational.
Catalan's constant is irrational

006. **Goldfeld's conjecture.** Proves Goldfeld's conjecture for quadratic twists of every elliptic curve over $\mathbb{Q}$: analytic ranks zero and one each have density $1/2$, and the mean analytic rank tends to $1/2$. Both statements order signed squarefree twist parameters by absolute value.
Analytic density and 2-converse · Mean analytic rank

007. **Two-point Chowla and the corrected binary Elliott conjecture.** Proves the ordinary two-point Chowla conjecture, with a bound $O(X/(\log X)^c)$ for Liouville correlation sums along fixed nonproportional affine forms, where $c > 0$ is absolute. More generally, proves the binary corrected Elliott conjecture for complex multiplicative functions bounded by one when one factor is uniformly nonpretentious against each fixed Dirichlet character times n^{it} for $|t| \leq X$.
Ordinary two-point correlations of multiplicative functions

008. **The Deligne–Drinfeld conjecture.** Proves that the rational Grothendieck–Teichmüller Lie algebra, with the Ihara bracket, is freely generated by one element in each odd weight $3, 5, 7, \ldots$, resolving the Deligne–Drinfeld conjecture.
The Deligne-Drinfeld conjecture

009. **Bogomolov–Pop and Milnor K-theoretic reconstruction.** Reconstructs function fields of transcendence degree at least two over algebraically closed constants from K_1^{M}/ℓ, K_2^{M}/ℓ, and their product. These data recover the perfect closure and constants when ℓ differs from the characteristic, and the original field and its named base in equal characteristic. Also proves Bogomolov–Pop reconstruction from abelian-by-central pro-ℓ Galois data away from the characteristic.
Reconstruction of Function Fields from Mod-ℓ Milnor K-Theory · Reconstruction from Milnor K-theory modulo the characteristic · The Bogomolov-Pop reconstruction theorem

010. **Fontaine–Mazur modularity at the prime 2 and 2-adic pro-modularity.** Every continuous odd absolutely irreducible two-dimensional 2-adic representation of $G_{\mathbb{Q}}$ unramified outside finitely many primes occurs in a completed Hecke algebra at some odd tame level. Also proves classical modularity up to Tate twist for irreducible odd representations with these finiteness conditions that are de Rham at 2 with distinct Hodge–Tate weights, resolving the dyadic Fontaine–Mazur case without residual restrictions.
Unrestricted pro-modularity · Two-adic Hecke dimension · Fontaine–Mazur modularity at the prime 2

011. **The Ford–Konyagin–Luca conjecture on prime predecessors.** Proves that the normalized ordered logarithms of the prime factors of $p-1$, counted with multiplicity, converge jointly to the Poisson–Dirichlet law PD(1) as p ranges uniformly over primes up to x and $x \to \infty$. This resolves the Ford–Konyagin–Luca conjecture. It also proves that infinitely many integers n have more than $n^{1-\varepsilon}$ totient preimages, for every $\varepsilon > 0$.
Weighted dilation graphs, smooth shifted primes and totient fibers · The Poisson-Dirichlet law for prime predecessors · Prime Predecessors with an Even Number of Prime Factors

012. **Independent largest prime factors of consecutive integers.** Resolves the Erdős–Pomerance joint Dickman conjecture: the logarithmic sizes of the largest prime factors of n and $n+1$ are asymptotically independent in ordinary natural density. In particular, the integers satisfying $P^+(n) < P^+(n+1)$ have density $1/2$.
The joint Dickman law for consecutive integers

013. **Ostmann's inverse Goldbach conjecture.** Proves that no finite modification of the primes can be written as $A+B$ with $A, B \subseteq \mathbb{Z}_{\geq 0}$ each containing at least two elements. This resolves Ostmann's inverse Goldbach conjecture on additive indecomposability.
The additive indecomposability of the primes

014. **Restricted geometric Langlands, generic Ramanujan and Arthur parameters.** Proves the restricted geometric Langlands equivalence for connected reductive groups on smooth projective connected curves over $\overline{\mathbb{F}}_q$ under the four stated Lie-theoretic characteristic hypotheses. Over arbitrary algebraically closed fields of characteristic $p > 0$, the same conclusion holds assuming additionally that p is very good for the group and $p \nmid |W_G|$. Over global function fields, proves Ramanujan at every place for globally generic cuspidal representations of split adjoint absolutely simple exceptional groups, without characteristic or ramification-depth restrictions, and at every unramified place for cuspidal representations of split adjoint absolutely simple groups with a generic unramified component. Assuming the finite-level Ramanujan–Arthur decomposition, constructs global Arthur enhancements of occurring cuspidal excursion parameters for split connected semisimple groups at full finite level, recovering the given parameters by diagonal specialization on the entire Weil group, including inertia.
Global Arthur Enhancements of Cuspidal Excursion Parameters · Unramified Arthur filtration · Arthur decompositions at full finite level · Temperedness at ramified places for globally generic exceptional groups · Restricted geometric Langlands · Constructible tame Hecke eigensheaves in positive characteristic · Tame Hecke Eigensheaves with Several Marked Points · Frobenius Structures on Tame Hecke Eigensheaves

015. **Torus-packet equidistribution in prime, quartic, and sextic degrees.** Proves Haar equidistribution without escape of mass for complete volume-weighted torus packets from totally real fields: arbitrary lattices and prescribed local types in fixed prime degree at least five, arbitrary-order Picard packets in primitive quartic fields, and maximal-order ideal-class packets in primitive sextic fields. Here primitive means having no proper intermediate field; the relevant order or field discriminant tends to infinity.
Equidistribution of Prime-Degree Torus Packets with Arbitrary Local Type · Equidistribution of primitive quartic torus packets for arbitrary orders · Equidistribution of Primitive Sextic Torus Packets

016. **Zilber–Pink in abelian varieties and the Siegel threefold.** Proves the abelian Zilber–Pink conjecture over $\overline{\mathbb{Q}}$: every irreducible subvariety has finitely many maximal atypical subvarieties relative to its smallest containing torsion coset. It also proves the full curve case in the Siegel threefold $\mathcal{A}_2$ for Hodge-generic curves defined over $\overline{\mathbb{Q}}$, without boundary or reduction assumptions.
Abelian Zilber–Pink · $E \times$ CM component · Quaternionic division points · Elliptic squares

017. **The irrationality exponent of π is 2.** Proves that the irrationality exponent of π is exactly 2: for every $\varepsilon > 0$ and all sufficiently large denominators q, every rational p/q satisfies $|\pi - p/q| \geq q^{-2-\varepsilon}$. This also proves convergence of the Flint–Hills series $\sum_{n\geq 1} 1/(n^3 \sin^2 n)$, with angles in radians.
The irrationality exponent of pi is 2

018. **The Margulis–Platonov conjecture over global fields.** Proves the Margulis–Platonov conjecture over every global field, including function fields of characteristic two. For an absolutely almost simple simply connected algebraic group G over k, every noncentral abstract normal subgroup of $G(k)$ is the inverse image of an open normal subgroup in the finite product of its anisotropic nonarchimedean local groups.
The Margulis–Platonov conjecture over global function fields · The Margulis–Platonov conjecture over number fields

019. **The p-adic section conjecture.** Proves that rational points on every smooth proper geometrically connected curve of genus at least two over a finite extension of $\mathbb{Q}_p$ correspond bijectively to conjugacy classes of sections of its full arithmetic étale fundamental group. It also proves Grothendieck's section conjecture over $\mathbb{Q}$ for the modular curves $X_0(N)$ and $X_1(N)$ of genus at least two.
Prescribed exterior sheet · p-adic section conjecture

020. **Squarefree quartics and power-free polynomial values.** Proves the squarefree-values conjecture for irreducible integer quartics with no fixed prime-square divisor: squarefree values on positive integers have the predicted positive Euler-product density. More generally, establishes the $(d-2)$-power-free density for irreducible integer polynomials of degrees four through eight under the necessary local condition; together with Browning's higher-degree theorem, this covers every $d \geq 4$.
Squarefree values of quartics and power-free values of polynomials

021. **A quadratic bound for Jacobsthal's function.** Answers Jacobsthal's quadratic-bound question: every interval of Ck^2 consecutive integers contains an integer coprime to any prescribed positive integer with at most k distinct prime divisors, for an absolute constant C. The bound is uniform over prime sets and interval positions and removes the classical logarithmic loss.
A quadratic bound for Jacobsthal's function

022. **The weak inhomogeneous Duffin–Schaeffer conjecture.** Proves that for every real shift γ and finite-valued $\psi : \mathbb{N} \to [0, \infty)$, divergence of $\sum_q \phi(q)\psi(q)/q$ implies $\|qx - \gamma\| < \psi(q)$ for infinitely many q, for almost every x. Here ϕ is Euler's totient and the norm is distance to the nearest integer. Numerators are unrestricted; no monotonicity or Diophantine condition on γ is needed.
The Weak Inhomogeneous Duffin–Schaeffer Conjecture

023. **Patterson's first moment for cubic Gauss sums.** Proves unconditionally the all-primary-prime form of Patterson's first-moment asymptotic: normalized cubic Gauss sums over primary Eisenstein primes of norm at most X, including both conjugates, have an explicit positive main term of order $X^{5/6}/\log X$. Every fixed nonzero prime-angle Fourier mode has smaller order.
An unconditional first moment for cubic Gauss sums

024. **An asymptotic formula for the number of totients.** Gives an asymptotic equivalent for the number $V(x)$ of distinct totient values up to x, with a positive bounded phase-dependent factor determined by convergent arithmetic approximations. In particular, $V(cx)/V(x) \to c$ for every fixed $c > 0$, answering Erdős and Hall's scaling question.
An asymptotic formula for the number of totients

025. **Erdős's short Egyptian-fraction conjecture.** Every rational a/b with $1 \leq a < b$ is a sum of $O(\log\log b)$ distinct positive unit fractions. The worst-case minimum number of terms has the same order, resolving Erdős's conjecture on short Egyptian fractions.
Short Egyptian fractions

026. **Positive lower density of large prime gaps.** For every fixed $C > 0$, a positive proportion of consecutive prime gaps exceed $C \log p_n$, throughout every sufficiently large initial segment of the primes. The proportion may depend on C. Consequently, the indices where p_n/n increases have positive lower density, answering Erdős and Prachar.
Positive lower density of large prime gaps

027. **Integral density on curve character varieties.** Resolves the determinant-one curve case of Litt's integral-density question. For every smooth connected complex algebraic curve and every rank, integral points become Zariski dense in every component of its SL_r character variety over the full ring of integers of one number field. Prescribed quasi-unipotent boundary conjugacy classes are allowed, including nonsemisimple classes.
Integral points on character varieties of curves

028. **The Gaussian moat conjecture.** Proves the Gaussian moat conjecture: no infinite walk through distinct Gaussian primes can have uniformly bounded steps. More strongly, for every distance bound D, the graph joining Gaussian primes at distance at most D has uniformly bounded finite component sizes, depending only on D, including primes on the coordinate axes.
Bounded-Step Walks on Gaussian Primes

029. **Artin's primitive root conjecture: infinitude for every base.** Proves the infinitude assertion in Artin's primitive root conjecture for every integer a that is neither -1 nor a square. For each such base, at least $c_a x/(\log x)^2$ primes in every sufficiently large interval $(x, 2x)$ have primitive root a, with $c_a > 0$.
Primitive roots for every admissible integer base · Simultaneous primitive roots: a conditional lower bound for prime bases

030. **Modularity over imaginary quadratic fields.** Proves the modularity conjecture for elliptic curves over imaginary quadratic fields: every elliptic curve over every imaginary quadratic field is modular, with matching local parameters at every place.
Modularity of elliptic curves over imaginary quadratic fields

031. Uchida's conjecture for open Galois homomorphisms. Proves Uchida's conjecture: every continuous open homomorphism between Galois groups of possibly infinite solvably closed Galois extensions of number fields comes from a unique equivariant field embedding in the opposite direction. No restriction on the kernel or separate cyclotomic-compatibility assumption is needed.
Open Homomorphisms of Global Solvably Closed Galois Groups

Algebraic and complex geometry

032. The rational Hodge conjecture for CM abelian varieties and products of K3 surfaces. Proves the rational Hodge conjecture for every complex CM abelian variety, in every dimension and codimension. Through Milne's theorems, this also gives the Tate conjecture for all abelian varieties over finite fields and the Hodge standard conjecture for abelian varieties in every characteristic. Companion results prove rational Hodge for arbitrary products of projective complex K3 surfaces and algebraicity of the Kuga–Satake correspondence for every such surface.
The rational Hodge conjecture for CM abelian varieties · The rational Hodge conjecture for products of K3 surfaces · Algebraicity of Kuga–Satake Correspondences for K3 Surfaces · Algebraic Kuga–Satake correspondences and Hodge conjectures on a K3 quadratic locus · Weil classes and Hodge classes on abelian powers · Abelian covers, Gale correspondences, and the Hodge conjecture for powers · Algebraicity of Weil classes on split abelian eightfolds · A Conditional Reduction for Algebraic Kuga–Satake Correspondences

033. Campana's orbifold Iitaka conjecture and logarithmic subadditivity. Proves Campana's orbifold Iitaka subadditivity conjecture for smooth Fujiki-class-$\mathcal{C}$ manifolds with rational simple-normal-crossing boundaries. For projective fibrations $f : U \to V$ of smooth complex quasi-projective varieties with connected fibers, general fiber F, and $\bar{\kappa}(V) \geq 0$, proves Popa's inequality $\bar{\kappa}(U) \geq \kappa(F) + \max\{\bar{\kappa}(V), \mathrm{Var}(f)\}$, where variation measures the whole geometric generic fiber.
Orbifold and logarithmic subadditivity · Whole-fiber variation · Reverse inequality and additivity · Projective Hodge lines · Kähler b-semiampleness

034. Log abundance and effective Iitaka fibrations. Using logarithmic Iitaka subadditivity, proves log abundance in every dimension for normal compact Kähler log canonical pairs with effective rational boundary: an analytically nef $\mathbb{Q}$-Cartier adjoint is semiample. It also proves projective log abundance over every algebraically closed field of characteristic zero and the effective Iitaka fibration conjecture for smooth projective varieties of nonnegative Kodaira dimension in that setting. A further result resolves the finite-rational-coefficient index conjecture for connected projective semi-log-canonical log Calabi–Yau pairs over such fields in each fixed dimension and for each fixed finite set of rational boundary coefficients, with a uniform index independent of the number of components.
Log abundance for compact Kähler spaces · Uniform indices for semi-log-canonical log Calabi–Yau pairs · Conditional Kähler good minimal models · Log abundance in characteristic zero · Minimal metrics and interior injectivity · Fourfold nonvanishing · Lifting sections from reduced support · Schnell fiber spaces · Uniform log Iitaka fibrations · Uniform pluricanonical Iitaka fibrations · Log Calabi–Yau fibrations · Arithmetic Stein-degree bounds · Log Iitaka fibrations for fourfolds · Compact Kähler fourfolds after nonvanishing

035. Threefold log abundance in numerical dimension one in characteristic $p > 3$. Proves log abundance for projective log canonical threefold pairs over algebraically closed fields of characteristic $p > 3$ when the effective boundary is rational and the $\mathbb{Q}$-Cartier adjoint is nef of numerical dimension one. The adjoint is semiample, without requiring the original variety to be terminal or $\mathbb{Q}$-factorial.
Log abundance in numerical dimension one for threefolds in positive characteristic · Abundance in numerical dimension one for terminal threefolds in positive characteristic

036. Numerical semiampleness and generalized minimal models. Proves numerical semiampleness for nef adjoints $K_X + B + M$ with $K_X + B$ pseudo-effective and M nef rational, for projective klt rational pairs over algebraically closed characteristic-zero fields and smooth compact Kähler rational klt simple-normal-crossing pairs, using Bott–Chern cohomology in the latter case. Separately, projective generalized log canonical rational pairs over such fields admit minimal models for pseudo-effective adjoints and Mori fiber spaces otherwise, with nef b-data fixed.
Compact Kähler numerical semiampleness · Projective numerical semiampleness · Generalized log canonical Q-pairs · Numerical dimension one

037. The sharp ordinary-double-point volume gap. Proves the ordinary-double-point volume-gap conjecture: every singular complex algebraic klt germ of dimension $n \geq 2$, with zero boundary, has normalized volume at most $2(n-1)^n$. Equality holds precisely for an analytic ordinary double point.
Gap in every dimension · Dimension-four normalized-volume gap

038. Fujita's freeness conjecture. Proves Fujita's freeness conjecture at its sharp bound in every dimension: for a smooth projective complex variety X of dimension n and an ample line bundle L, the adjoint $K_X + mL$ is globally generated for every integer $m \geq n + 1$.
Fujita's freeness conjecture

039. **Nagata's conjecture and maximal Seshadri constants.** Proves Nagata's strict inequality $\sum_i m_i < d\sqrt{r}$ for every nonzero effective plane curve of degree d through $r \geq 10$ very general complex points, with arbitrary multiplicities m_i. It also proves maximal multipoint Seshadri constants $(L^n/r)^{1/n}$ for every smooth polarized projective variety of dimension $n \geq 2$ and all sufficiently large r: at very general points over $\mathbb{C}$, and at the geometric generic tuple over any algebraically closed field of positive characteristic.
Plane-curve Nagata conjecture · Maximal Seshadri constants · Maximal Multipoint Seshadri Constants in Higher Dimensions · Maximal multipoint Seshadri constants in positive characteristic

040. **Bloch's conjecture for complex surfaces.** Proves Bloch's conjecture: for every smooth connected projective complex surface S with $p_g(S) = 0$, the Albanese map $\mathrm{CH}_0(S)^0 \to \mathrm{Alb}(S)(\mathbb{C})$ on integral degree-zero zero-cycles is an isomorphism. This combines the new $p_g = q = 0$ theorem with the classical theorem of Bloch, Kas, and Lieberman.
Bloch's conjecture for surfaces with $p_g = 0$

041. **Hyperkähler SYZ and projective-space bases.** Proves the strong hyperkähler SYZ conjecture: every holomorphic line bundle with nonzero nef isotropic first Chern class on a compact irreducible holomorphic symplectic Kähler manifold is semiample. It also proves that every projective Lagrangian fibration with normal projective base has projective space as its base, in every dimension and deformation type.
Projective-space bases · Strong hyperkähler SYZ

042. **Oka classification for K3 surfaces and other compact complex surfaces.** Proves that every complex K3 surface is Oka, including nonprojective surfaces. More generally, every connected minimal compact complex surface of Kodaira dimension zero is Oka; a connected minimal compact complex surface of class VII is Oka exactly when it is a Hopf or Enoki surface.
Every complex K3 surface is Oka

043. $P = W$ **for fixed-determinant moduli spaces.** Proves $P_k = W_{2k} = W_{2k+1}$ on the full rational cohomology of smooth coprime fixed-determinant, trace-free Higgs moduli spaces and their character varieties for composite ranks over smooth projective complex curves of genus at least two. This includes variant cohomology and, together with the known prime-rank theorems, establishes the fixed-determinant $P = W$ conjecture in every coprime rank.
P=W in composite rank for fixed determinant

044. **The equivariant cohomological Hikita conjecture for quivers.** Proves the equivariant cohomological Hikita correspondence for every finite quiver, including loops and multiple arrows, with arbitrary dimension and framing vectors and commuting flavor torus. When every semistable point is stable and the gauge action is free, the equivariant cohomology of the Nakajima variety is canonically the coordinate ring of the scheme-theoretic cocharacter-fixed locus of its flavor-deformed Coulomb branch.
The equivariant cohomological Hikita conjecture for arbitrary quivers

046. **Counterexamples to Shafarevich holomorphic convexity.** Constructs a smooth projective complex fourfold with large fundamental group whose universal cover contains no positive-dimensional compact analytic subvariety but is neither Stein nor holomorphically convex. A separate smooth projective complex surface already disproves unrestricted Shafarevich holomorphic convexity in dimension two.
A projective fourfold with large fundamental group and non-Stein universal cover · A surface counterexample to Shafarevich holomorphic convexity

047. **Complex counterexamples to cancellation and affine fibrations.** Constructs an integral complex affine fourfold $X \not\cong \mathbb{A}^4$ with $X \times \mathbb{A}^1 \cong \mathbb{A}^5$, disproving affine-space cancellation over $\mathbb{C}$ in dimension four. It also disproves the Dolgachev–Weisfeiler affine-fibration conjecture: smooth surjections $X \to \mathbb{A}^1$ and $\mathbb{A}^5 \to \mathbb{A}^2$ have every residue-field fiber isomorphic to affine three-space but are not Zariski-locally trivial.
An explicit failure of complex affine-space cancellation

048. **A characteristic-zero counterexample to Lipman–Zariski.** Constructs a singular normal affine complex surface with free rank-two tangent sheaf, disproving the characteristic-zero Lipman–Zariski conjecture.
A singular normal affine surface with free tangent sheaf

049. **A stable-coordinate counterexample in four variables.** Constructs a polynomial in four complex variables that is not a coordinate but becomes one after adjoining a single variable, disproving the Stable Coordinate conjecture in four variables. Every fiber is affine three-space, yet none of its embeddings is rectifiable, also disproving the Abhyankar–Sathaye conjecture even when all fibers are affine spaces.
A stable coordinate that is not a coordinate in four variables · An explicit noncoordinate polynomial with affine three-space zero fibre

050. **A counterexample to Griffiths' positivity conjecture.** Constructs ample rank-two bundles on $\mathbb{P}^1 \times \mathbb{P}^1$ with no smooth Hermitian metric of strictly Griffiths-positive curvature, disproving Griffiths' positivity conjecture already on the quadric surface.
Ample rank-two bundles on the quadric surface without Griffiths-positive metrics

051. **Kobayashi's canonical-ampleness conjecture.** Every compact connected Kähler manifold of positive complex dimension with no nonconstant entire curve has ample canonical bundle and is therefore projective. This proves Kobayashi's canonical-ampleness conjecture in the smooth compact Kähler setting.
Canonical ampleness of compact hyperbolic Kähler manifolds

052. **Tangent-bundle splittings and universal covers.** A splitting of the tangent bundle of a compact Kähler manifold into two integrable holomorphic subbundles induces a compatible product decomposition of its universal cover, proving the two-summand form of Beauville's splitting conjecture. On smooth rationally connected projective manifolds, both summands are automatically integrable, establishing Höring's conjecture and the corresponding product decomposition.
Kähler universal-cover splitting · Integrability on rationally connected manifolds

053. **A counterexample to Pixton's original completeness conjecture.** Constructs a tautological relation on a moduli space of stable pointed curves that vanishes in rational Chow, hence in rational cohomology, but lies outside Pixton's original relation span. This disproves the Chow and rational-cohomological forms of his original completeness conjecture.
A high-arity counterexample to Pixton completeness in Chow

054. **Counterexamples to Kuznetsov's rationality conjecture.** For every sufficiently large admissible Hassett discriminant, a very general smooth complex cubic fourfold is irrational despite having both an untwisted geometric K3 category and an integral Hodge-theoretic K3 association. This disproves Kuznetsov's rationality conjecture and the sufficiency of the associated-K3 criterion for rationality; the discriminant threshold is ineffective.
Irrational cubic fourfolds with geometric K3 categories

055. **Toda's Gepner conjecture and large-volume stability.** Proves Toda's Gepner conjecture for every smooth complex quintic threefold, constructing a numerical Bridgeland stability condition with the prescribed phase shift 2/5. Also constructs numerical Bridgeland stability conditions at every sufficiently large volume on all smooth projective complex threefolds with trivial canonical bundle, with the exact ordinary and square-root-Todd central charges.
Gepner stability on quintic threefolds · Prescribed large-volume charges

056. **Termination of fourfold minimal model programs.** Proves termination of every existing generalized log canonical flip sequence on globally Weil $\mathbb{Q}$-factorial compact Kähler fourfolds, with rational boundary, fixed rational analytically nef b-data, and projective small flip diagrams with the prescribed ample signs. Also proves termination of arbitrary permitted minimal model programs for projective log canonical fourfolds with rational boundary in characteristic zero.
Termination of generalized log canonical flips on compact Kähler fourfolds · Termination of generalized-canonical flips on compact Kähler fourfolds · Log canonical fourfold termination · Ordinary Kähler fourfold termination · Finite Kähler fourfold programs

057. **Campana's abelianity conjecture and special varieties.** Proves Campana's abelianity conjecture: special compact Kähler manifolds have virtually abelian fundamental groups. Using this theorem, establishes the same conclusion for order-two root orbifolds of smooth projective complex fourfolds along one nonempty smooth connected divisor, when special in the stated differential-line sense. For smooth special complex quasi-projective varieties, proves that every finite-dimensional complex linear representation of the fundamental group has virtually nilpotent image of class at most two.
Special compact Kähler abelianity · Two-step monodromy · A conditional abelianity theorem for special fourfold pairs with a half-weight divisor

058. **The Kollár–Pardon universal-cover conjecture.** Proves the Kollár–Pardon conjecture: the semialgebraic universal covers of connected normal projective complex varieties are exactly products $D \times \mathbb{C}^m \times F$, with D bounded symmetric and F simply connected, normal, and projective. A universal cover is quasi-projective exactly when the bounded symmetric factor is absent. In particular, a smooth projective variety covered by $\mathbb{C}^n$ has a finite étale cover by an abelian variety.
Semialgebraic universal covers · Bounded-domain symmetry

059. **Counterexamples to Zariski's multiplicity conjecture.** Disproves Zariski's multiplicity conjecture by constructing reduced holomorphic hypersurface germs that are ambiently homeomorphic but have different multiplicities. The examples include hypersurfaces in $\mathbb{C}^4$ with isolated critical points and multiplicities four and five.
Multiplicities two and three · Multiplicities four and five

060. **The Global Spherical Shell conjecture.** Every connected minimal compact complex surface of class VII with $b_2 > 0$ contains a global spherical shell, proving the positive-b_2 Global Spherical Shell conjecture. Such a shell is a holomorphically embedded neighborhood of the standard three-sphere in $\mathbb{C}^2 \setminus \{0\}$ whose complement is connected.
Global Spherical Shells on Minimal Surfaces of Class VII

062. **The LeBrun–Salamon conjecture and projective contact classification.** Proves the LeBrun–Salamon conjecture: every closed connected positive quaternionic-Kähler manifold of real dimension at least eight is homothetic to a compact symmetric Wolf space. It also proves contact-Fano homogeneity and classifies smooth connected complex projective contact manifolds of complex dimension at least three: those with $b_2 = 1$ are adjoint varieties with their canonical contact structures, while those with $b_2 \geq 2$ have underlying manifold $\mathbb{P}(T^*Z)$ for a smooth projective variety Z.
Contact Fano manifolds and the LeBrun–Salamon conjecture

063. **The generalized Mukai conjecture.** Proves the generalized Mukai conjecture: every positive-dimensional smooth complex projective Fano manifold of dimension n, Picard number ρ and pseudoindex ι satisfies $\rho(\iota - 1) \leq n$, with equality exactly for $(\mathbb{P}^{\iota-1})^\rho$. Here the pseudoindex is the least anticanonical degree of a rational curve.
The generalized Mukai conjecture

064. **The μ-constant problem for surface singularities.** Proves topological right-triviality for every holomorphic one-parameter family of isolated hypersurface singularities in $\mathbb{C}^3$ with constant Milnor number, resolving the surface case of the μ-constant problem. After shrinking the parameter disk and representatives, ambient homeomorphisms vary jointly continuously, fix the origin section and preserve the defining functions.
Topological triviality of mu-constant families of surface singularities

065. **Virasoro constraints for complete intersections and projective bundles.** Proves the full ordinary unreduced descendant Virasoro conjecture for smooth complete intersections in complex projective space, in every genus and curve class with arbitrary cohomology insertions. The constraints also pass from any smooth projective complex base satisfying them to the projectivization of every algebraic vector bundle of rank at least two, and hence to projective-bundle towers.
Virasoro Constraints under Projectivization · Virasoro Constraints for Projective Complete Intersections

066. **Bounded klt complements for Fano contractions.** Proves the finite-rational-coefficient form of Shokurov's bounded-klt-complement conjecture for ϵ-lc complex Fano-type pairs with nef anti-log-canonical divisor. For ϵ-lc Fano contractions over any algebraically closed characteristic-zero field, it gives klt complements near every base point, with index bounded only by dimension and positive rational ϵ.
Bounded klt complements · Uniform Cartier sections

067. **The Campana–Peternell conjecture in dimension six.** Proves the Campana–Peternell conjecture in complex dimension six: every smooth connected complex projective Fano sixfold with nef tangent bundle is rational homogeneous.
The Campana–Peternell conjecture in dimension six

068. **Anticanonical nonvanishing under smooth semipositivity.** If X is a smooth connected complex projective variety and $-K_X$ admits a smooth Hermitian metric with nonnegative curvature, then $H^0(X, -mK_X) \neq 0$ for some $m > 0$. Thus smooth semipositivity forces a nonzero section of a positive tensor power of the anticanonical bundle in every dimension.
Smooth semipositivity · Invariant indices and twisted differentials · Bounded metrics on klt pairs · Cohomological transfer · Rank-preserving contractions · Effectivity with controlled boundary · Exact orders and invariant linear systems

069. Quantum geometric Langlands at irrational level. Proves the unramified de Rham quantum geometric Langlands equivalence for every connected simple complex algebraic group on every smooth projective connected complex curve, at every shifted level $c \in \mathbb{C} \setminus \mathbb{Q}$. It identifies the full derived categories of twisted D-modules for the group and its Langlands dual, retaining all global forms and connected components.
Global quantum geometric Langlands at irrational level

Real and complex analysis

071. Koebe's circle-domain conjecture and circle-domain rigidity. Resolves the existence part of Koebe's circle-domain conjecture: every domain in the Riemann sphere is conformally equivalent to a domain whose complementary components are round disks or points. It also proves that circle domains with conformally removable boundary are rigid, meaning every conformal equivalence to another circle domain is Möbius, establishing this direction of the He–Schramm conjecture.
Removable boundaries and rigidity · Koebe's circle-domain conjecture

072. Brennan's conjecture and a counterexample to Kraetzer's prediction. Proves Brennan's conjecture: for every conformal bijection ϕ from a simply connected plane domain onto the disk, $|\phi'|^s$ is area-integrable for $4/3 < s < 4$. The sharp universal integral-means identity is $B_\delta(t) = |t| - 1$ for $t \le -2$. A strict bound $B_b(-1) < 1/4$ for bounded univalent functions disproves Kraetzer's prediction at that parameter.
Brennan's conjecture · Strict inverse-first-power bound

073. The Falconer distance conjecture. Resolves the Falconer distance conjecture in every dimension $d \ge 2$: every compact set $E \subset \mathbb{R}^d$ with Hausdorff dimension greater than $d/2$ determines a set of Euclidean distances of positive Lebesgue measure.
The Falconer distance conjecture in all dimensions

074. Kakeya in three and four dimensions. Resolves the Kakeya maximal conjecture in three dimensions and the Hausdorff-dimension conjecture in four. In three dimensions, the radius-δ tube maximal operator maps $L^3(\mathbb{R}^3)$ to $L^3(S^2)$ with norm $O_\varepsilon(\delta^{-\varepsilon})$ for every $\varepsilon > 0$. In four dimensions, every set containing a unit segment in every direction has Hausdorff dimension four.
3D Kakeya maximal conjecture · 4D Kakeya Hausdorff dimension

075. The $L \log L$ Fourier-convergence conjecture. Proves that the ordinary symmetric Fourier partial sums of every complex-valued function in $L \log L(\mathbb{T})$ converge almost everywhere along the full sequence. This resolves the classical sufficiency conjecture at the $L \log L$ scale.
Almost-everywhere Fourier convergence in L log L

076. Ultraflat real Littlewood polynomials. Constructs polynomials with N consecutive coefficients in $\{-1, 1\}$ whose modulus is $(1 + o(1))\sqrt{N}$ uniformly on the entire unit circle, for every sufficiently large integer length N. Thus real Littlewood polynomials are ultraflat, including at the real endpoints. Their binary merit factors tend to infinity, disproving Turyn's bounded-merit-factor conjecture.
Ultraflat real Littlewood polynomials · Nearly minimal maxima and positive minima of Littlewood polynomials · Asymptotically minimal maxima of real Littlewood polynomials

077. Fourier restriction for positively curved surfaces. Proves the diagonal Fourier extension conjecture for positively curved surfaces in three dimensions. For every compact smooth positively curved surface $\Sigma \subset \mathbb{R}^3$, including surfaces with boundary, the extension operator is bounded from $L^p(\Sigma)$ to $L^p(\mathbb{R}^3)$ for every $p > 3$.
Sphere restriction · Diagonal Fourier extension

078. The three-dimensional Bochner–Riesz conjecture. Resolves the three-dimensional Bochner–Riesz conjecture in its strict range: the Bochner–Riesz multipliers of order δ are bounded on $L^p(\mathbb{R}^3)$ for every $1 \le p \le \infty$ whenever $\delta > \max\{3|1/p - 1/2| - 1/2, 0\}$.
Bochner–Riesz multipliers in three dimensions

079. Sogge's local smoothing conjecture in dimension three. Resolves Sogge's local smoothing conjecture for the Euclidean wave equation in three spatial dimensions. The estimate holds throughout the full strict range $2 < p < \infty$, with Sobolev regularity above $\max\{0, 1 - 3/p\}$. In particular, the critical L^3 estimate holds with every positive Sobolev loss.
Critical local smoothing for the three-dimensional wave equation

080. The Sobolev endpoint in Carleson's Schrödinger convergence problem. Proves almost-everywhere convergence $e^{it\Delta}f \to f$ as $t \downarrow 0$ for every $f \in H^{n/(2(n+1))}(\mathbb{R}^n)$ and every dimension $n \geq 2$. This attains the sharp Sobolev equality case of Carleson's Schrödinger convergence problem, including the planar endpoint $H^{1/3}$.
Planar endpoint convergence · Higher-dimensional endpoint convergence

081. The David–Semmes Riesz-transform problem in higher codimension. Resolves the remaining higher-codimension Riesz-transform rectifiability problem: for $d \geq 4$ and $2 \leq n \leq d-2$, an n-Ahlfors–David regular Radon measure on $\mathbb{R}^d$ is uniformly n-rectifiable whenever its n-dimensional Riesz transform is uniformly L^2-bounded over all positive hard truncations. The rectifiability bounds depend only on dimension, regularity and operator bounds.
Riesz transforms and uniform rectifiability in higher codimension

082. Maximal and variational bounds for the triangular Hilbert transform. Proves maximal and annular r-variation bounds, for every $r > 2$, from complex $L^3(\mathbb{R}^2) \times L^3(\mathbb{R}^2)$ to $L^{3/2}(\mathbb{R}^2)$. The maximal estimate controls both hard truncation endpoints and gives almost-everywhere and norm convergence. Pairing with a third input settles the triangular Hilbert transform estimate at the symmetric $L^3 \times L^3 \times L^3$ point.
Annular variation of the triangular Hilbert transform at the symmetric point · An L^3 bound for the dyadic triangular Hilbert form · The maximal triangular Hilbert transform at the symmetric point

083. Stein's conjecture for Hilbert transforms along Lipschitz directions. Proves a uniform strong L^2 bound for the planar Hilbert transform along any Lipschitz unit vector field, at integration lengths bounded by an absolute multiple of its reciprocal Lipschitz constant. The estimate is uniform over inner truncations and yields an L^2-bounded principal-value operator, establishing Stein's weak-type conjecture at this short scale.
A uniform Hilbert transform estimate for Lipschitz directions

084. The geometric case of the Erdős similarity conjecture. For every fixed $q \in (0,1)$, constructs compact subsets of $[0,1]$ with measure arbitrarily close to one containing no translated and nontrivially dilated copy of $\{q^n : n \geq 1\}$, with dilations of either sign. This resolves the geometric-progression case of the Erdős similarity conjecture for every ratio.
The geometric case of the Erdős similarity conjecture · The dyadic case of the Erdős similarity conjecture

085. Endpoint regularity of the planar centered maximal function. Resolves the planar centered-disk case of the Hajłasz–Onninen maximal-function regularity problem. For every real $f \in W^{1,1}(\mathbb{R}^2)$, the centered disk maximal function satisfies $\|\nabla Mf\|_1 \leq C\|\nabla f\|_1$ with an absolute constant. It belongs locally to $W^{1,1}$ and has a globally integrable weak gradient.
An Endpoint Gradient Bound for the Centered Disk Maximal Operator

086. An L^3 bound for the trilinear Hilbert transform. Proves that the principal-value trilinear Hilbert transform with shifts $x-t$, $x-2t$, $x-3t$ is bounded from $L^3(\mathbb{R})^3$ to $L^1(\mathbb{R})$. This resolves the L^3 exponent case of the standard conjecture for slopes $1, 2, 3$.
An L^3 bound for the trilinear Hilbert transform

Convex and metric geometry

087. The Mahler conjectures and symplectic width. Resolves the symmetric and nonsymmetric geometric Mahler conjectures in every dimension, with Hanner polytopes and simplices as the respective volume-product minimizers and all equality cases classified. The corresponding sharp functional Mahler inequalities also hold. For $n \geq 2$, every symmetric polar product $K \times K^\circ$ in dimension $2n$ has Gromov width 4.
Symmetric Mahler equality cases · General Mahler conjecture · Symplectic balls in polar products

088. Petty's projection-volume conjecture and simplex counterexamples. Proves Petty's projection-volume conjecture in the remaining dimensions $n \geq 4$: ellipsoids uniquely minimize projection-body volume at fixed body volume. Also establishes the full Lutwak–Petty projection inequalities. In contrast, products of simplices exceed Brannen's proposed simplex maximum for normalized projection-body volume by an exponential factor in every sufficiently large dimension.
Petty's projection-volume conjecture · Product counterexample to simplex maximum

089. **Bounded-distortion L_1 embeddings of planar and bounded-treewidth graphs.** Resolves the planar and bounded-treewidth cases of the Gupta–Newman–Rabinovich–Sinclair conjecture. Shortest-path metrics of finite connected graphs with arbitrary positive edge lengths embed into real L_1 with universal distortion for planar graphs, and distortion depending only on treewidth for bounded-treewidth graphs. The corresponding multicommodity flow–cut gaps are uniformly bounded.
Planar graph embeddings · Bounded-treewidth embeddings

090. **Universal optimality of the triangular lattice.** Proves that the triangular lattice minimizes the lower limit of energy per particle for every nonnegative completely monotone potential of squared distance among locally finite planar configurations of centered-disk density one. It also minimizes unit-background renormalized Riesz energies for $0 < s < 2$ and Coulomb energy, resolving Sandier–Serfaty and the two-dimensional Brauchart–Hardin–Saff conjecture on the linear term of optimal spherical logarithmic energy.
Atomic certificate · Universal optimality · Fourier circle-packing certificate · Coulomb renormalized energy

091. **The logarithmic Brunn–Minkowski conjecture.** Proves the logarithmic Brunn–Minkowski inequality for origin-symmetric convex bodies in every dimension, and the scalar-dilation B-conjecture for all even log-concave Radon measures. For Lebesgue volume it also proves the additive L_p Brunn–Minkowski inequality for full-dimensional origin-symmetric convex bodies throughout $0 < p < 1$.
The logarithmic Brunn–Minkowski conjecture

092. **The optimal order of convex-body covering density.** Determines the optimal worst-case covering density as $\Theta(n \log n)$, for both lattice and unrestricted translative coverings. Every convex body in $\mathbb{R}^n$, $n \geq 2$, admits a lattice covering of density at most $Cn \log n$; centrally symmetric examples in every sufficiently large dimension require at least $cn \log n$ even without the lattice restriction, for absolute $c, C > 0$.
Single-lattice covering bound · Translative covering densities

093. **Dimension-free logarithmic Sobolev inequality for subgaussian log-concave measures.** Proves a dimension-free logarithmic Sobolev inequality for centered log-concave densities with uniformly subgaussian linear marginals, with constant bounded by a universal multiple of the squared linear subgaussian parameter.
Subgaussian logarithmic Sobolev inequality

094. **Subpolynomial dimension reduction in L_p.** For every fixed $1 < p < \infty$ and distortion $D > 1$, every n-point subset of real L_p embeds into ℓ_p^d with distortion at most D and dimension $d = n^{o(1)}$, answering Naor's sublinear-dimension question for $p \neq 2$. In contrast, exact embeddings require worst-case dimension $\Theta(n^2)$ when $p \neq 2$.
Subpolynomial dimension reduction in Lp

095. **Hyperbolicity cones without semidefinite lifts.** Disproves the Projected Lax conjecture: some hyperbolicity cones are not spectrahedral shadows. The examples admit no exact finite affine semidefinite lift, regardless of the number of auxiliary variables or the real coefficients used. This also disproves the generalized Lax conjecture that every hyperbolicity cone is spectrahedral.
Hyperbolicity Cones Without Semidefinite Lifts · A nonspectrahedral hyperbolicity cone · An Exact Semidefinite Lift of a Nonspectrahedral Hyperbolicity Cone

096. **The Gaussian propeller conjecture.** Proves that the sum of squared Gaussian first moments of any finite measurable partition is at most $9/(8\pi)$. In dimension at least two, three planar sectors of angle $2\pi/3$, extended orthogonally, attain the bound. Combined with the separate Unique Games theorem, this proves NP-hardness of improving the loss factor $(8\pi/9)(1 - 1/k)$ for identity-target kernel clustering with fixed $k \geq 3$ on rational centered positive semidefinite inputs.
The Gaussian propeller bound in every dimension

097. **The Euclidean Steinitz–Bergström conjecture.** Proves that any finite sequence in the Euclidean unit ball of $\mathbb{R}^d$ admits signs keeping every partial sum within $C\sqrt{d}$, independently of length. Consequently every zero-sum family can be reordered with the same bound on unsigned partial sums. A matching lower bound gives the optimal order $S_2(d) = \Theta(\sqrt{d})$.
The Euclidean Steinitz–Bergström theorem

098. **A negative answer to the Lang–Plaut problem.** Every infinite-dimensional real Banach space contains a compact doubling set that admits no bi-Lipschitz embedding into any finite-dimensional normed space. The doubling constant is universal. This answers the Lang–Plaut problem negatively, even for compact subsets of Hilbert space.
A doubling Hilbert subset with no finite-dimensional bi-Lipschitz embedding

099. The sharp distortion of edit distance into ℓ_1. Determines the least distortion of embedding edit distance on words of length at most d into real ℓ_1: it is $\exp(\Theta(\sqrt{\log d \log\log d}))$. Insertions, deletions and substitutions have unit cost. The constants are uniform over all finite alphabets with at least two symbols, even when the alphabet grows with d; binary words already force the lower bound.
Matching bounds · Finite circles · Tree constructions

100. A counterexample to Bang's cylinder-covering bound. Covers the entire closed regular tetrahedron by finitely many cylinders with compact triangular perpendicular bases whose total area is less than half its smallest orthogonal projection area. This disproves Bang's half-area cylinder-covering bound and the stronger directionwise normalized conjecture in dimension three.
Angular cylinder covers · Ruled-set approximation · Slope-field perturbations · Radial-sweep approximation

101. The sharp simplex conjecture for isotropic constants. Proves that simplices uniquely maximize the isotropic constant among convex bodies in every dimension, resolving the strong isotropic constant conjecture. Also establishes the sharp entropy lower bound for log-concave probability densities, with equality precisely for invertible affine images of products of one-sided exponential laws.
A sharp entropy bound and the simplex inequality for isotropic constants

Theoretical computer science

102. The Unique Games Conjecture and optimal approximation thresholds. Proves Khot's Unique Games Conjecture. Independent direct reductions also establish NP-hardness, on unweighted graphs, of approximation beyond the Goemans–Williamson ratio for Max-Cut, below factor two for Vertex Cover, and within any fixed constant factor for Min-UnCut and directed feedback vertex set. These direct proofs use established PCP and Label Cover hardness results.
Unique Games theorem · Optimal Max-Cut hardness · Factor-two Vertex Cover hardness · Constant-factor Min-UnCut hardness · Directed feedback vertex set hardness

103. Derandomization of logarithmic space: $\mathsf{L} = \mathsf{RL} = \mathsf{BPL}$. Proves $\mathsf{L} = \mathsf{RL} = \mathsf{BPL}$, resolving derandomization for bounded-error logarithmic-space computation. An effective compiler converts each randomized polynomial-time logarithmic-space machine deciding a language with one-sided or two-sided error into a deterministic logarithmic-space decider with explicit polynomial running-time bounds.
Exact derandomization of logarithmic space: L = RL = BPL

104. Quasipolynomial algorithms for mean-payoff, stochastic and parity games. Gives deterministic algorithms using $2^{O((\log(L+2))^2)}$ bit operations, for complete binary input length L, for ordinary mean-payoff games and two separate extensions. They compute exact values and optimal positional strategies in ordinary games, the nonnegative expectation-of-liminf value set in turn-based stochastic games, and the winning set for nonnegative liminf mean payoff conjoined with parity. Signed rewards, rational chance probabilities, and parity priorities are unrestricted and binary-encoded.
Turn-Based Stochastic Mean-Payoff Games in Deterministic Quasipolynomial Time · Mean-payoff parity games in quasipolynomial time · Deterministic algorithm · Randomized algorithm

105. The 2-to-1 Games Conjecture with perfect completeness. Proves Khot's 2-to-1 Games Conjecture with perfect completeness: for every fixed rational $\delta \in (0,1)$, it is NP-hard to distinguish satisfiable games from games whose optimum is at most δ, on explicit unweighted instances. The alphabet depends only on δ, and every right-hand label has exactly two preimages under each constraint map.
Perfect completeness for 2-to-1 games

106. Hardness of coloring three-colorable graphs. It is NP-hard to color a three-colorable graph using any fixed number $c \geq 3$ of colors. More strongly, for every fixed $0 < \delta < 1/3$, a deterministic polynomial-time reduction from 3SAT produces simple unweighted graphs that are three-colorable in the satisfiable case and have no independent set of size δn otherwise, where n is the number of vertices.
Hardness of finding large independent sets in three-colorable graphs

107. Matrix multiplication with exponent at most $9/4$. Proves $\omega \leq 9/4$ over $\mathbb{C}$, giving $O_\varepsilon(n^{9/4+\varepsilon})$ arithmetic operations for square matrix multiplication. In characteristic zero, some inner dimension n^a with $a > 0.465$ permits $n^{2+o(1)}$ rectangular multiplication. Further square bounds give $\omega < 2.258$ outside finitely many positive characteristics and $\omega < 2.371054886006746$ over every fixed field.
Exponent at most 9/4 over $\mathbb{C}$ · Earlier complex and rectangular bounds · Staggered extraction over every field

108. A cubic permanent–determinant lower bound. Proves an $\Omega(n^3)$ lower bound for the border determinantal complexity of the $n \times n$ permanent over $\mathbb{C}$. Even coefficientwise limits of determinants of affine-linear matrices require matrix size at least cn^3, for an absolute $c > 0$ and all sufficiently large n; the same bound therefore holds for exact representations.
A cubic lower bound for border determinantal complexity of the permanent

109. Integer multiplication below $n \log n$. Multiplies two n-bit integers exactly at every input length in deterministic worst-case time $O(n(\log n)^{1-\kappa})$, with $\kappa = 2^{-182}$, on one fixed finite-alphabet Turing machine with finitely many one-dimensional tapes. This disproves the Schönhage–Strassen $n \log n$ optimality conjecture in the ordinary multitape bit model.
Integer multiplication below n log n

110. Optimal-order randomized k-server on arbitrary metrics. Establishes a randomized competitive ratio $O(\log^2(k+1))$ for k-server on every metric space, matching the worst-case lower-bound order. One policy serves every finite oblivious request sequence, including on infinite unbounded metrics. On finite rational metrics, a uniform implementation has polynomial preprocessing and per-request bit cost in the input length and $\log(t+1)$ at request t, with a finite instance-dependent additive movement constant.
Squared-logarithmic randomized k-server · Uniform computation

111. One-sample matroid prophet inequalities against an almighty adversary. For every finite matroid known in advance, gives a distribution-independent online rule using one independent sample per element and earning a universal constant fraction of the expected offline optimum. Values are independent and nonnegative, with finite expected optimum. The guarantee holds even when the arrival-order adversary sees all samples, values, and the rule's entire random seed; no polynomial-time implementation is asserted.
One Sample Suffices for Matroid Prophet Inequalities against an Almighty Adversary

112. Beyond the square-root exponent for depth-three circuits. Constructs a single language in deterministic polynomial time whose n-bit membership function requires $2^{\omega(\sqrt{n})}$ total gates in unbounded-fan-in OR–AND–OR circuits, at every sufficiently large input length. This crosses the square-root-exponent threshold for explicit depth-three Boolean circuit lower bounds.
Beyond the Square-Root Exponent for Depth-Three Boolean Circuits

113. Approximate counting and the perfect-matching entropy conjecture. Gives a fully polynomial randomized approximation scheme for counting perfect matchings in arbitrary finite simple graphs, with exact detection of zero counts. Also proves the perfect-matching entropy conjecture of Anari, Oveis Gharan, and Vinzant, bounding the maximum entropy of a matching law at every feasible edge-marginal vector in a loopless labelled multigraph, including boundary points.
Perfect-matching FPRAS · Entropy and face dimension

114. Approximate counting of common integer polymatroid bases. Gives a fully polynomial randomized approximation scheme for counting common integer bases of two integral polymatroids of equal total rank, supplied by exact rank-value oracles. Capacities are binary-encoded, each integer vector counts once, and oracle calls and bit operations outside the oracles are polynomial on every execution. For matroids presented by independence oracles, the results also cover common independent sets of prescribed, unrestricted, or maximum cardinality, even when the ranks differ.
An FPRAS for Common Integer Polymatroid Bases with Binary Capacities · Approximate counting of common bases of two matroids

115. Sampling and counting contingency tables with arbitrary margins. For nonnegative integer matrices with prescribed row and column sums, gives exact uniform sampling in expected polynomial bit time and almost-uniform sampling in worst-case polynomial bit time. The dimensions and binary-encoded margins are unrestricted. Also gives a fully polynomial randomized approximation scheme for counting such tables with arbitrary individual cell bounds, including structural zeros, with polynomial cost on every execution.
Exact uniform sampling · Cell-bounded counting FPRAS

116. Uniform identity testing for noncommutative formulas. For each characteristic, constructs in deterministic polynomial bit time a polynomial-dimensional matrix tuple detecting every nonzero division-free noncommutative formula of bounded size over any field of that characteristic. Rational formulas over $\mathbb{Q}$ also admit polynomial-size hitting lists whenever they have a defined rational-matrix evaluation.
Uniform Matrix Hitting Points in Every Positive Characteristic · One rational matrix hitting point · Rational-formula hitting lists

117. **Uniform sparsest cut: hardness and semidefinite gaps.** Proves that approximating Uniform Sparsest Cut within any fixed constant factor is NP-hard, even with nonnegative rational capacities and unit demands. The Goemans–Linial semidefinite relaxation also has integrality gaps of order at least $\sqrt{\log n}/(\log\log n)^3$, approaching the square-root-logarithmic upper bound.
Constant-factor hardness · Near-square-root logarithmic integrality gaps

118. **Unbounded bin-packing gaps and the modified integer round-up conjecture.** Disproves the modified integer round-up conjecture of Scheithauer and Terno: the integral bin-packing optimum can exceed its configuration linear-programming value by an arbitrarily large additive constant. Approximating the optimum within any fixed additive constant is also NP-hard, even when every item exceeds $1/6$ and each bin holds at most five items.
Additive hardness and unbounded configuration gaps in bin packing

119. **The Courtade–Kumar and Hellinger conjectures.** Proves the Courtade–Kumar conjecture: among Boolean functions of independent uniform bits, a single coordinate retains the most mutual information after independent bit-flip noise. A stronger theorem treats randomized binary summaries at fixed initial information. The Hellinger conjecture is also proved for every Boolean output bias and noise correlation.
Sharp binary-information contraction · Arbitrary-bias Hellinger contraction

120. **Almost-linear-time exact matching in general graphs.** Gives a randomized algorithm finding an exact maximum-cardinality matching in any simple undirected graph in $(n+m)^{1+o(1)}$ word time, with success probability at least $2/3$. The time bound holds on every computation path. The same guarantees apply to finding a spanning subgraph with prescribed admissible vertex degrees, or deciding that none exists.
Almost-Linear-Time Maximum-Cardinality Matching in General Graphs

121. **Almost-linear expected-time approximation of edit distance.** For every fixed rational $\varepsilon \in (0,1)$, gives a randomized $(1+\varepsilon)$ approximation to unit-cost edit distance in worst-case expected time $N^{1+o(1)}$, with success probability at least $2/3$. The strings have total length N and polynomially bounded integer symbols. This is an asymptotic guarantee at fixed accuracy.
An Almost-Linear Approximation Scheme for Edit Distance

122. **Superpolynomial lower bounds and quasipolynomial reconstruction from deletion traces.** At every fixed deletion probability in $(0,1)$, reconstructing an arbitrary length-n binary string requires $n^{\Omega(\log\log n)}$ independent traces, ruling out polynomial-sample reconstruction. A uniform decoder achieves quasipolynomial sample and running-time bounds for known fixed rational retention probabilities. When the deletion probability is at most $n^{-\varepsilon}$ for fixed $\varepsilon > 0$, both bounds become polynomial in the input and parameter encoding.
Uniform quasipolynomial-time trace reconstruction · A latest-anchor induction with spectrally compact masks for worst-case trace reconstruction · Quantitative lower bounds for trace reconstruction

124. **Polynomial-time scheduling on three identical machines.** Resolves the three-processor unit-job scheduling problem of Garey and Johnson: a deterministic polynomial-time algorithm minimizes makespan for nonpreemptive unit-length jobs with arbitrary precedence constraints on three identical parallel machines. For an explicitly given precedence graph, it decides deadline feasibility exactly and constructs a feasible schedule.
A Polynomial-Time Algorithm for Three-Machine Unit-Job Scheduling

125. **The approximation threshold for metric k-median.** Gives a deterministic polynomial-time $(1 + 2/e + \varepsilon)$-approximation for finite rational metric k-median with specified candidate facilities, for every fixed $\varepsilon > 0$. Assuming $P \neq NP$, the optimal infimum approximation factor is $1 + 2/e$.
Single-exponential recovery · Approximation threshold

126. **Exponential semidefinite complexity of perfect matching.** Proves that every exact semidefinite lift of the perfect matching polytope has exponential size, answering Rothvoss's polynomial-size lift question negatively. The bound holds even for the positive semidefinite rank of its odd-cut slack matrix after any fixed shift $0 < \rho < 1$, allowing arbitrary real positive semidefinite factors.
Exponential PSD rank of positively shifted matching matrices

127. **The asymptotic Gotsman–Linial conjecture.** Proves that a degree-at-most-d polynomial threshold function on the uniform n-dimensional Boolean cube has average sensitivity at most $8d\sqrt{n}$, uniformly for $1 \le d \le n$. Average sensitivity counts expected output changes under single-bit flips. This establishes the asymptotic Gotsman–Linial conjecture, allowing polynomial zeros with $\operatorname{sign}(0) = 1$.
Average sensitivity of polynomial threshold functions

128. **A factor-two approximation for shortest common superstring.** Gives a deterministic polynomial-time algorithm constructing a common superstring of length at most twice the optimum for every finite family of explicitly represented strings. The running time is polynomial in the full encoded input length, including symbol labels.
A Polynomial-Time 2-Approximation for Shortest Common Superstring

129. **Exponential state costs for two-way automata.** Proves exponential lower bounds both for complementing two-way nondeterministic finite automata and for simulating one-way nondeterministic automata by two-way deterministic ones. The latter resolves the Sakoda–Sipser state-succinctness conjecture over growing finite alphabets; both results rule out polynomial state bounds independent of alphabet size.
Nondeterministic complementation lower bound · Two-way deterministic state lower bound

130. **Fourier transforms below $n \log n$.** Gives a deterministic length-n discrete Fourier transform algorithm using $O(n(\log n)^{1-\delta})$ operations for every n, with explicit $\delta = 10^{-13}$. The model uses exact complex arithmetic, unrestricted coefficients and a supplied root of unity, and counts scalar preparation and logarithmic-word indexing.
Explicit DFT power saving · Tensor savings and Fourier circuits

131. **Polynomial mixing of graph switches with prescribed degrees.** Resolves the simple-undirected Kannan–Tetali–Vempala conjecture: the lazy edge-switch chain mixes in $O(n^8)$ time for every graphical labeled degree sequence. The same degree-constrained graphs can also be sampled exactly uniformly by an almost-surely terminating algorithm with expected polynomial bit running time.
Polynomial mixing of the switch chain for every graphical degree sequence

132. **A counterexample to the quadratic sensitivity conjecture.** Constructs total Boolean functions with block sensitivity $\mathrm{bs}(f) \geq s(f)^{\alpha}$ for a fixed $\alpha > 2$, disproving the quadratic strengthening of the Sensitivity Conjecture. Here $s(f)$ counts influential individual-bit flips, while block sensitivity allows disjoint groups of bits to change together.
A superquadratic separation between sensitivity and block sensitivity

133. **The complexity of Weisfeiler–Leman refinement.** Proves unconditional $n^{\Omega(k)}$ deterministic time lower bounds for joint and separate k-dimensional Weisfeiler–Leman equivalence, for sufficiently large fixed k in the specified sequential adjacency-matrix models. With dimension as input, joint equivalence is EXPTIME-complete even on subcubic graphs; deciding whether refinement identifies a graph is also EXPTIME-complete.
Parity lifts and bounded-treewidth witnesses · Graph-identification complexity · Unconditional time lower bounds · Variable-dimension equivalence

134. **Generalized star height at most three.** Every regular language over a finite alphabet has a generalized regular expression with at most three nested Kleene stars, allowing union, concatenation and complement over the same alphabet. This establishes an absolute bound independent of automaton size, resolving the uniform-boundedness version of the generalized star-height problem.
Finite-monoid computations · Star height at most four · Star height at most three

135. **Sharp homogeneous depth-five complexity of matrix products.** Over every characteristic-zero field, the $(1,1)$ entry of a product of n independent $n \times n$ variable matrices requires $n^{\Theta(\sqrt{n})}$ gates in homogeneous depth-five sum–product circuits. This sharp bound allows shared gates and bottom linear forms involving all variables.
Homogeneous depth-five lower bounds for iterated matrix multiplication

136. **The quasilinear PCP-for-PPAD conjecture.** Resolves the quasilinear PCP-for-PPAD conjecture. An End-of-Line instance of length N reduces to numerical circuit constraints of total length $N(\log N)^{O(1)}$ such that any polynomially encoded rational assignment satisfying all but a fixed fraction to fixed accuracy yields an endpoint solution. Such assignments always exist, giving robust local verification with only quasilinear size overhead.
The PCP-for-PPAD conjecture: a quasilinear reduction

137. **One-tape time simulation in two-fifths-power space.** Determines the halting and finite-control outcome of a fixed deterministic one-writable-tape machine up to time T using $O(T^{2/5}\log^{C}(T+2))$ space, improving the square-root exponent. Heads move at most one cell per step; finitely many read-only input heads are allowed. Initial contents are independent of T, and contents and input symbols have polylogarithmic-space access. Simulation time is unrestricted.
Simulating One-Tape Time in Two-Fifths-Power Space

138. Subset Sum in $O(2^{0.49n})$ time. Gives a uniform randomized classical algorithm for worst-case Subset Sum in ordinary $O(2^{0.49n})$ word-RAM time on polynomial-bit inputs, where n counts the integers. The time bound holds on every execution and success probability is at least 2/3 on every input. Inputs may repeat positive integers; words have $O(n + b)$ bits for maximum input bit length b.
Subset Sum in Time $O(2^{0.49n})$ · A Low-Space Algorithm for Worst-Case Subset Sum

139. Subpolynomial queries for log-concave sampling. For C^2 potentials with a supplied minimizer and $I \preceq \nabla^2 V \preceq 2I$, proves that sampling within total variation 1/10 requires only $C_\varepsilon d^\varepsilon$ exact value-and-gradient queries for every fixed $\varepsilon > 0$. The bound holds on every run, with unrestricted computation between queries. A logarithmic lower bound also holds, so the optimal power-law exponent in this oracle model is zero.
Subpolynomial query complexity for well-conditioned log-concave sampling

140. Memory–sample lower bounds for noiseless Gaussian regression. For fixed $A > 0$, a one-pass learner with Ad^2 persistent bits needs $\Omega_A(d \log(1/\epsilon))$ noiseless Gaussian samples to recover a unit vector to angular error $0 < \epsilon \le 1/10$ with probability 2/3, uniformly in accuracy for large d. Computation and randomized updates are unrestricted, but output uses only the terminal state, stopping index and fresh randomness.
Memory and precision · Posterior replicas · Localization costs · Projection moments and Riesz estimates · Observation replacement · Subsphere methods

141. The existential theory of the reals and existential–universal sentences in the counting hierarchy. Proves that the existential theory of the reals lies in the counting hierarchy. More generally, truth of existential–universal real sentences can be decided at one fixed level of that hierarchy, even when their integer polynomials are specified by arithmetic circuits.
Existential–universal real sentences in the counting hierarchy

142. Deterministic polynomial factorization over prime fields. Gives a uniform deterministic algorithm that completely factors every nonzero dense degree-n polynomial over a prime field $\mathbb{F}_p$, including multiplicities, in bit complexity polynomial in $(n + 1) \log p$. The prime is supplied in binary. No randomness, integer-factorization or primitive-root oracle, or GRH assumption is required.
Deterministic Polynomial Factorization over Prime Fields

Dynamical systems and ergodic theory

143. Uniform limit-cycle bounds in Hilbert's sixteenth problem. Resolves the uniform boundedness assertion in Hilbert's sixteenth problem: the number of isolated periodic orbits of a real planar polynomial vector field is bounded by a finite constant depending only on its degree. For classical quintic Liénard systems, the exact maximum is two limit cycles.
Uniform limit-cycle bounds · Quintic Liénard limit cycles

144. Banach's simple Lebesgue-spectrum problem. Resolves the probability-preserving form of Banach's simple Lebesgue-spectrum problem within smooth dynamics. A smooth volume-preserving diffeomorphism of the standard-volume three-torus has simple Lebesgue spectrum on its entire complex mean-zero L^2 space: the bilateral iterates of one real observable form an orthonormal basis of that space.
A smooth three-torus diffeomorphism with simple Lebesgue spectrum

145. Rokhlin's multiple-mixing problem. Proves that every invertible mixing probability-preserving transformation is mixing of all finite orders, resolving Rokhlin's multiple-mixing problem for a single transformation. Correlations among any finite collection of measurable sets converge to the product of their measures whenever all pairwise time separations diverge.
Rokhlin's multiple-mixing problem for one transformation

146. Sinai's positive-entropy conjecture for the standard map. Proves that the standard sine map on the two-dimensional torus has positive metric entropy with respect to area for every sufficiently large positive parameter. This establishes Sinai's positive-parameter-measure conjecture for the original family, with the stronger conclusion of a full parameter tail.
Positive Metric Entropy for the Standard Map at Large Parameters

147. The near-boundary Birkhoff conjecture. Resolves the near-boundary Birkhoff conjecture for smooth strictly convex planar billiards of positive curvature. Such a billiard is an ellipse whenever a full grazing annulus is continuously foliated by individually invariant essential curves. A continuous physical collar of smooth closed convex caustics also suffices.
Analytic caustic collars · Billiard rigidity

148. **The dimension formula for self-similar measures on the line.** For every self-similar measure on the line generated by finitely many contracting similarities, proves $\dim_H \mu = \min\{1, h_{RW}/\chi\}$, where h_{RW} is the entropy rate of random composed maps and χ the average logarithmic contraction. This resolves the entropy-rate dimension conjecture without a separation assumption, allowing exact overlaps and unequal contraction ratios.
The entropy-rate dimension formula for self-similar measures on the line

149. **Permanence for weakly reversible reaction networks.** Proves the permanence conjecture for every finite weakly reversible mass-action system with fixed positive rate constants. Every positive stoichiometric compatibility class, even an unbounded one, has a common compact convex forward-invariant absorbing set. All positive trajectories in that class therefore eventually share positive lower and finite upper concentration bounds.
Uniform Permanence in Weakly Reversible Mass-Action Systems · Boundedness and persistence of weakly reversible mass-action systems

150. **Weak mixing of irrational triangular billiards.** Proves that the billiard flow in every nondegenerate Euclidean triangle with at least one angle irrational relative to π is weakly mixing for normalized area times uniform direction. This strengthens ergodicity on the entire irrational-angle class, with no genericity or Diophantine restrictions.
Weak mixing of triangular billiards with an irrational angle · Ergodicity of triangular billiards with an irrational angle

151. **A C^1 counterexample to Shub's entropy conjecture.** Constructs a noninvertible C^1 self-map of a compact smooth manifold with zero topological entropy but eigenvalue 2 on second homology. This disproves the homological entropy lower bound for general C^1 self-maps: homological growth need not force positive orbit complexity.
A C^1 Counterexample to the Entropy Conjecture

152. **Zero entropy does not guarantee a smooth positive-volume model.** Constructs a zero-entropy ergodic invertible transformation of a standard nonatomic probability space that is not measurably conjugate to any C^∞ diffeomorphism preserving a strictly positive smooth probability density on a compact finite-dimensional manifold. One example rules out every finite dimension.
A zero-entropy system without a smooth positive-volume model

153. **Arithmetic classification of Bernoulli convolutions.** Classifies singular and absolutely continuous unbiased Bernoulli convolutions for every $\lambda \in (0,1)$ by an infinite, one-sided approximation condition using explicit finite sets of algebraic units. It also proves singularity at reciprocals of every quartic Salem number in $(1,2)$, giving examples beyond reciprocal Pisot parameters.
Arithmetic classification and non-Pisot singularity for Bernoulli convolutions

154. **Pointwise multiple ergodic averages for mixing transformations.** Proves almost-everywhere convergence of consecutive multiple ergodic averages of every finite length for invertible mixing probability-preserving transformations. For each fixed tuple of bounded functions, the limit is the product of their integrals, along all positive averaging lengths. No mixing rate or standardness assumption on the probability space is required.
Pointwise Multiple Ergodic Averages for Mixing Transformations · Pointwise convergence of fourfold ergodic averages for mixing transformations · Triple ergodic averages with distinct integer slopes · Pointwise convergence of triple ergodic averages for mixing transformations

Combinatorics

155. **A counterexample to periodic tiling in dimension three.** Constructs a finite translational tile in $\mathbb{Z}^3$ that tiles space but admits no fully periodic tiling, disproving the periodic tiling conjecture in the smallest possible lattice dimension. Its unit-cube thickening gives the same counterexample in $\mathbb{R}^3$, even with arbitrary real translation vectors.
A translational tile with no fully periodic tiling in dimension three

156. **Borsuk's conjecture fails in dimension nine.** Constructs a compact subset of $\mathbb{R}^9$ that cannot be covered by ten sets of strictly smaller diameter, disproving Borsuk's covering assertion already in dimension nine. The example consists of rank-one orthogonal projectors onto lines in $\mathbb{R}^4$, with the Frobenius metric.
A nine-dimensional counterexample to Borsuk's covering assertion

157. Counterexamples to the Hadwiger and Colin de Verdière conjectures. Disproves Hadwiger's conjecture even for fractional coloring: arbitrarily large finite simple graphs with independence number at most two satisfy $\chi_f(G) > h(G)$, where $h(G)$ is the largest clique-minor order. Also disproves the fractional Colin de Verdière chromatic bound $\chi_f(G) \le \mu(G) + 1$. In the positive direction, every finite nonempty graph satisfies $\chi_{\text{list}}(G) \le Ch(G)$ for a universal constant C.
Hadwiger counterexample · Colin de Verdière counterexample · Linear list-coloring bound

158. The Euclidean plane cannot be colored with five colors. Proves that every five-coloring of the Euclidean plane has a monochromatic pair at distance one, with no restriction on the color classes. This advances the Hadwiger–Nelson problem: together with the classical seven-coloring, only six and seven remain possible chromatic numbers of the plane.
The Euclidean plane is not five-colorable

159. Erdős's reciprocal-sum conjecture and quasipolynomial Szemerédi bounds. Proves Erdős's conjecture that every set of positive integers with divergent reciprocal sum contains arithmetic progressions of every finite length. Quantitatively, for each fixed $k \ge 3$, every subset of $\{1, \dots, N\}$ with no nonconstant k-term progression has size at most $C_k N \exp[-c_k(\log N)^{\epsilon_k}]$, with positive constants depending only on k.
Quasipolynomial Bounds for Arithmetic Progressions

160. Superexponential van der Waerden numbers. Resolves Erdős's superexponential-growth question for van der Waerden numbers. If $W_r(k)$ is the least interval length forcing a monochromatic k-term progression in every r-coloring, then $W_r(k) > k^{ck\lfloor \log_2 r \rfloor}$ for an absolute $c > 0$, all $r \ge 2$ and sufficiently large k, uniformly in r. In particular, $W_r(k)^{1/k} \to \infty$ for each fixed r.
Quantitative Superexponential Bounds for van der Waerden Numbers

161. Counterexamples to Sidorenko's conjecture and the forcing conjecture. Disproves Sidorenko's conjecture with a connected bipartite pattern on 35 vertices and 66 edges that occurs less frequently than in a random graph of the same edge density. The same pattern disproves the forcing conjecture of Skokan and Thoma: matching its density and the edge density of a constant graphon need not force quasirandomness.
A counterexample to Sidorenko's conjecture

162. Counterexamples to Ryser's covering and Gyárfás's tree-cover conjectures. Disproves Ryser's covering conjecture by constructing intersecting $(q+1)$-partite, $(q+1)$-uniform hypergraphs with covering number $q+1$, rather than the predicted bound q, for every sufficiently large prime q. A separate construction over extension fields also disproves Gyárfás's monochromatic tree-cover conjecture.
Balanced prime-order counterexamples · Original counterexample

164. Hindman's finite sums and products conjecture. Proves Hindman's finite sums and products conjecture: every finite coloring of the positive integers contains sets of any prescribed finite size whose nonempty subset sums and nonempty subset products all have one common color.
Monochromatic finite sums and products in the positive integers

165. Exact crossing numbers of complete and complete bipartite graphs. Resolves the Harary–Hill conjecture and Turán's brickyard problem in the Zarankiewicz formulation, determining the crossing numbers of every complete and complete bipartite graph. The result proves the optimality of the classical drawings among all plane drawings with continuous edge arcs.
Complete graphs · Complete bipartite graphs

166. The higher-dimensional Erdős distinct-distances conjecture. For every fixed $d \ge 3$, any $n \ge 2$ distinct points in $\mathbb{R}^d$ determine at least $c_d n^{2/d}$ distinct distances, with $c_d > 0$ depending only on dimension. This matches the integer-grid order and resolves the higher-dimensional Erdős distinct-distances conjecture with a constant-factor bound.
The higher-dimensional Erdős distinct-distances conjecture

167. Pinned distances and a power saving for planar unit distances. Proves the weak pinned Erdős distance conjecture: for every fixed $\varepsilon > 0$, all but $o(n)$ points of any n-point planar set determine at least $n^{1-\varepsilon}$ distinct nonzero distances. A complementary theorem bounds the number of unit-distance pairs by $O(n^{4/3-\delta})$ for an absolute $\delta > 0$.
Weak pinned planar distances · Planar unit-distance power saving

168. Combinatorial invariance of Kazhdan–Lusztig polynomials. Resolves the full combinatorial invariance conjecture: isomorphic Bruhat intervals in arbitrary Coxeter systems have identical equal-parameter Kazhdan–Lusztig polynomials. Thus the abstract order of the interval determines the polynomial, even across different Coxeter systems.
Combinatorial invariance of Kazhdan–Lusztig polynomials

169. The Shareshian–Wachs e-positivity conjecture. Resolves the elementary-positivity part of the Shareshian–Wachs conjecture: the chromatic quasisymmetric function of every natural unit interval graph has elementary-basis coefficients in $\mathbb{N}[q]$. The coefficients count explicitly described permutations, giving a combinatorial explanation of positivity.
Elementary positivity of chromatic quasisymmetric functions

170. Sharp logarithmic exponents for off-diagonal Ramsey numbers. For every fixed integer $s \geq 5$, proves $r(s,t) = t^{s-1}/(\log t)^{s-2+o(1)}$ as $t \to \infty$, determining the logarithmic exponent and matching the classical upper bound at that scale. Here $r(s,t)$ is the least number of vertices forcing an s-clique or a t-vertex independent set.
$r(5,t)$ logarithmic exponent · Fixed off-diagonal Ramsey numbers

171. The hypercube Ramsey conjecture. Resolves the Burr–Erdős hypercube Ramsey conjecture: the two-color Ramsey number of the n-dimensional cube is $\Theta(2^n)$. Thus every red-blue coloring of a complete graph on a universal constant times the cube's number of vertices contains a monochromatic copy of the cube.
The hypercube Ramsey number has linear order

172. Classification of finite Euclidean Ramsey configurations. Classifies finite point configurations that occur monochromatically, at their original scale, in every finite coloring of sufficiently high-dimensional Euclidean space. The characterization is an algebraic condition over the coordinate field. It also disproves the Leader–Russell–Walters conjecture that every such configuration is a subset of a finite transitive set.
A classification of finite Euclidean Ramsey configurations

173. Seymour's second-neighborhood conjecture. Proves Seymour's second-neighborhood conjecture: every nonempty finite oriented graph has a vertex with at least as many vertices at directed distance exactly two as at directed distance one. Oriented graphs may be arbitrary apart from the exclusion of loops and oppositely directed edge pairs.
A proof of Seymour's second-neighborhood conjecture

174. Deterministic construction of strong thin spanning trees. Resolves the strong thin-tree conjecture constructively. Every finite loopless k-edge-connected multigraph on at least two vertices has a spanning tree containing at most a universal C/k fraction of the edges of every cut. Such a tree can be found deterministically in polynomial time, even with binary-encoded parallel-edge multiplicities.
Strong thin tree conjecture · Polynomial-time construction

175. Talagrand's conjectures and graph decompositions at expectation thresholds. Proves that integral and fractional expectation thresholds differ by at most a universal factor, and resolves Talagrand's discrete-convexity conjecture. An application proves the Ascoli–He–Park–Talagrand graph-decomposition conjecture: every graph's edges split into a universally bounded number of fixed pieces, each with containment threshold at most a universal constant times the original graph's integral expectation threshold. The pieces' embeddings need not agree on shared vertices.
Graph Decompositions at the Integral Expectation Threshold · Expectation-threshold equivalence · Discrete convexity

176. The second Kahn–Kalai conjecture. Proves the second Kahn–Kalai conjecture: for every finite simple graph H with $h \geq 1$ edges and at most n vertices, its appearance threshold in $G(n,p)$ is at most $Cp_{\mathrm{E}}(n,H)(1+\log_2 h)$, with universal C. Here p_{E} is the least density at which every subgraph of H has expected copy count at least $1/2$.
The second Kahn–Kalai conjecture

177. Bounded-degree coboundary expanders in every dimension. Constructs arbitrarily large finite d-dimensional simplicial complexes, for every $d \geq 3$, with uniformly bounded vertex degrees and uniform $\mathbb{F}_2$ coboundary expansion in every degree below d. Together with the known graph and two-dimensional cases, this establishes the existence of such expanders in every positive dimension.
Bounded-degree coboundary expanders in every dimension

178. Deterministic nonbipartite Ramanujan graphs in every fixed degree. For every fixed $d \geq 3$, constructs a simple d-regular nonbipartite Ramanujan graph on every sufficiently large even number n of vertices, with every nonconstant adjacency eigenvalue strictly between $-2\sqrt{d-1}$ and $2\sqrt{d-1}$. A deterministic algorithm outputs the full adjacency list in polynomial bit time, with exponent depending on d.
Deterministic nonbipartite Ramanujan graphs in every fixed degree

179. The circulant Hadamard conjecture. Proves that real circulant Hadamard matrices exist exactly in orders 1 and 4, resolving the circulant Hadamard conjecture. Together with classical Barker-sequence results, this shows that binary sequences whose nontrivial aperiodic autocorrelations have magnitude at most 1 exist at lengths $n > 1$ exactly when $n \in \{2, 3, 4, 5, 7, 11, 13\}$.
The circulant Hadamard conjecture

180. Barnette's Hamiltonian-cycle conjecture. Proves that every finite simple cubic bipartite planar 3-vertex-connected graph has a Hamiltonian cycle, resolving Barnette's conjecture. Equivalently, every three-edge path in a finite simple cubic 3-vertex-connected bipartite Pfaffian graph lies in a Hamiltonian cycle.
Paired states and Hamiltonian cycles in cubic bipartite planar graphs

181. The Erdős–Gallai cycle-decomposition conjecture. Proves that the edges of every finite simple undirected graph on n vertices can be partitioned into at most Cn simple cycles and single edges, for an absolute constant C. This resolves the Erdős–Gallai cycle-decomposition conjecture, bounding the number of pieces linearly even for dense graphs.
A linear cycle-and-edge decomposition of every graph

182. Power savings for polynomial-difference-free sets. For every fixed intersective integer polynomial h of degree $k \geq 2$ with positive leading coefficient, proves that a subset of $\{1, \dots, N\}$ avoiding nonzero values $h(1), h(2), \dots$ as differences has size $O_h(N^{1-c_k})$, with $c_k > 0$ depending only on degree. Here intersective means having a root modulo every modulus. For prime arguments, a power saving also holds when h has a unit root modulo every modulus, with exponent allowed to depend on h.
A power saving for intersective polynomial differences with an exponent depending only on the degree · A Power Saving for Polynomial Differences at Prime Arguments · A power saving for square-difference-free sets

183. Power savings for planar halving lines and k-sets. Improves the planar halving-line bound to $O(n^{4/3-\varepsilon})$ for sets with no three collinear and an absolute $\varepsilon > 0$. More generally, an n-point set with no three collinear has $O(n(k+1)^{1/3-\varepsilon_0})$ strictly separable k-subsets for $1 \leq k \leq n/2$, with an absolute $\varepsilon_0 > 0$. The constants and positive exponents are nonquantitative.
A power saving for planar halving lines

184. Coloring and independence in graphs with forbidden subgraphs. Proves the Alon–Krivelevich–Sudakov coloring conjecture in correspondence-coloring form: graphs avoiding any fixed subgraph F need $O_F(\Delta/\log\Delta)$ colors when their maximum degree Δ is sufficiently large. Also proves the Ajtai–Erdős–Komlós–Szemerédi independence conjecture: for fixed $r \geq 4$, every n-vertex K_r-free graph of average degree $d \geq 2$ has an independent set of size $\Omega_r(n \log d/d)$.
Correspondence coloring graphs with a forbidden clique · A logarithmic independence bound for clique-free graphs

185. Counterexamples to infinite matroid intersection and packing/covering. Disproves the unrestricted infinite matroid intersection and packing/covering conjectures in ZFC, using two self-dual partitional matroids on a countably infinite ground set. The same examples answer Joó's partitional-matroid question negatively. They are neither finitary nor cofinitary, so Nash-Williams' original finitary conjecture remains outside the result.
A Counterexample to the Infinite Matroid Packing/Covering Conjecture

186. The Friedgut–Kalai graph and hypergraph threshold conjectures. Proves the Friedgut–Kalai threshold-width conjectures for graphs and fixed-uniformity hypergraphs. For fixed $0 < \varepsilon < 1/2$, every nontrivial increasing relabeling-invariant property crosses from probability ε to $1 - \varepsilon$ within width $O((\log n)^{-2})$ for graphs and $O_r((\log n)^{-r/(r-1)})$ for r-uniform hypergraphs, $r \geq 3$. The hypergraph influence bound also applies to nonmonotone properties.
A uniform influence bound for hypergraph properties · A Sharp Threshold Bound for Monotone Graph Properties

187. Snaky in 21 Maker moves. Settles the Snaky achievement problem: Maker can force the six-cell Snaky shape within 21 of its own moves on the initially empty infinite square board. Maker moves first, each player claims one free cell per turn, and translations, rotations and reflections count as wins.
Snaky in 21 Maker moves

188. The sharp constant in random triangle removal. Starting from the complete graph on n vertices, repeatedly delete a uniformly chosen remaining triangle. The terminal edge count is asymptotic to $n^{3/2}/(2\sqrt{2})$, with mean-square convergence after normalization by $n^{3/2}$. This proves the triangle case of the Joos–Kühn sharp-constant conjecture.
The sharp terminal leave in random triangle removal

189. Exact cycle–clique Ramsey numbers. Proves the Erdős–Faudree–Rousseau–Schelp conjecture: $R(C_m, K_n) = (m-1)(n-1)+1$ for every $m \geq n \geq 3$, except $R(C_3, K_3) = 6$. This is the exact threshold forcing a red m-cycle or a blue n-clique in every red–blue coloring of a complete graph.
Cycle–clique Ramsey numbers

190. Polynomial removal fails for ordered binary matrices. Disproves polynomial ordered binary matrix removal with one fixed 66×66 zero–one pattern. Matrices can require many binary-entry changes to become pattern-free while their copy density is smaller than every proposed polynomial bound in that distance. Copies preserve row and column orders and match both zeros and ones.
Polynomial removal fails for ordered binary matrices

191. A power improvement in the Heilbronn triangle problem. For every sufficiently large n, constructs n points in the unit square such that every triangle has area at least n^{-2+c} for one absolute $c > 0$. This disproves the conjectured almost-n^{-2} upper bound in Heilbronn's triangle problem, which asks how large the smallest determined triangle can be.
A power improvement in the Heilbronn triangle lower bound

192. A counterexample to the Gopalan–Servedio conjecture. Disproves the proposed square-root bound relating a Boolean function's linear Fourier coefficients to its polynomial degree. For every $C > 0$, there is a sign-valued Boolean function f with $\sum_i \hat{f}(\{i\}) > C\sqrt{\deg(f)}$. Thus its total signed correlation with individual input bits can exceed the proposed bound by an arbitrary factor.
Unbounded Violations of the Square-Root Degree Bound

Algebra

193. Serre's intersection-multiplicity conjecture. Proves strict positivity of Serre's intersection multiplicity $\chi^R(M,N)$ for nonzero finitely generated modules over any regular local ring, provided $M \otimes_R N$ has finite length and $\dim M + \dim N = \dim R$. This resolves the positivity conjecture, including ramified mixed characteristic.
Positivity of Serre's Intersection Multiplicity

194. Lech's multiplicity conjecture. Proves $e(R) \leq e(S)$ for every flat local homomorphism of nonzero Noetherian local rings, where e is Hilbert–Samuel multiplicity. This resolves Lech's conjecture in every dimension and characteristic.
Lech's multiplicity conjecture

195. A counterexample to the small Cohen–Macaulay module conjecture. Constructs a three-dimensional complete Noetherian normal local domain over $\mathbb{C}$ with no nonzero finitely generated maximal Cohen–Macaulay module. A three-dimensional local domain essentially of finite type over $\mathbb{C}$ has the same property, disproving the domain form of the small Cohen–Macaulay module conjecture.
A Complete Local Domain without a Small Cohen–Macaulay Module

196. A counterexample to Kaplansky's zero-divisor conjecture. Constructs a finitely presented torsion-free group G whose group algebra $\mathbb{F}_2[G]$ has nonzero zero divisors, disproving Kaplansky's zero-divisor conjecture. The group has a finite two-dimensional classifying space.
A Torsion-Free Group Algebra with Zero Divisors

197. Nonsofic groups and group-ring counterexamples. Constructs a finitely presented torsion-free nonsofic group whose group algebra over $\mathbb{F}_2$ is not directly finite, disproving Kaplansky's conjecture even without torsion. Companion examples give injective nonsurjective cellular automata on all configurations, refuting Gottschalk's surjunctivity conjecture. Another counterexample is an integral group-ring matrix, invertible over the rational group ring, with Fuglede–Kadison determinant strictly between zero and one, disproving the unrestricted Determinant Conjecture.
Torsion-free direct-finiteness counterexample · Characteristic two · Determinant · Odd characteristic

198. A counterexample to the little finitistic-dimension conjecture. Constructs a finite-dimensional complex algebra whose finite-dimensional modules have unbounded finite projective dimensions. This disproves the little finitistic-dimension conjecture.
An algebra of infinite little finitistic dimension

199. **Counterexamples to conjectures of Auslander–Reiten, Tachikawa, and Nakayama.** Constructs finite-dimensional algebras over a characteristic-two rational-function field that disprove the Auslander–Reiten and Gorenstein-projective conjectures, and Tachikawa's second conjecture. An associated endomorphism algebra also disproves the classical, generalized and strong Nakayama conjectures, the Auslander–Gorenstein conjecture, and the Wakamatsu tilting conjecture. The counterexamples persist under every extension of the base field.
Auslander–Reiten counterexample · Tachikawa's second conjecture

200. **Eisenbud–Green–Harris and lex-plus-powers in characteristic zero.** Proves the Eisenbud–Green–Harris and lex-plus-powers conjectures over every characteristic-zero field. Any homogeneous ideal containing a regular sequence, of arbitrary length and degrees at least two, admits a lex-plus-powers ideal with the same Hilbert function and no smaller graded Betti numbers.
Artinian lex-plus-powers Betti theorem · Commuting division-coefficient forms

201. **A counterexample to Kurosh's division-ring problem.** Constructs a countable characteristic-zero division ring that is algebraic over its center and generated by two elements over that center, but has infinite dimension over it. This answers Kurosh's division-ring problem on local finiteness negatively.
A Counterexample to Kurosh's Division-Ring Problem

202. **The blockwise Alperin weight conjecture.** Proves the numerical blockwise Alperin weight conjecture for every prime and every finite group: the number of irreducible Brauer characters in a block equals the number of conjugacy classes of its weights.
The Blockwise Alperin Weight Conjecture

203. **Donovan's conjecture over fields and discrete valuation rings.** Proves Donovan's conjecture: over each fixed algebraically closed field of characteristic p, blocks of finite groups with bounded defect-group order have only finitely many Morita-equivalence classes, for every prime p. Also proves the integral form over each fixed complete mixed-characteristic discrete valuation ring with algebraically closed residue field.
Donovan over algebraically closed fields · Integral Donovan over Witt vectors

204. **Tensor saturation for even spin groups.** Proves saturation factor one for $\mathrm{Spin}(2n)$, $n \geq 2$: for three dominant integral weights whose sum lies in the root lattice, an invariant at any common positive integral dilation already gives an invariant at the original weights. This resolves the type-D part of the simply-laced saturation conjecture.
Tensor saturation for even spin groups

205. **Saxl's conjecture and universal tensor squares.** Proves Saxl's conjecture: the tensor square of every staircase representation contains every irreducible complex representation of the corresponding symmetric group. More generally, every S_n with $n \notin \{2, 4, 9\}$ has an irreducible representation whose tensor square contains all irreducibles.
Universal tensor squares · Cyclic polytabloid proof

206. **Finite lattice representation: counterexamples and undecidability.** Some finite lattices are not congruence lattices of any finite algebra, answering the finite lattice representation problem negatively. Moreover, no algorithm decides whether a finite lattice has such a representation, or whether it is a full subgroup interval of a finite group.
Characterization and undecidability · Negative solution

207. **The ℓ^1-Bass and complex Bass trace conjectures.** Proves the ℓ^1-Bass conjecture for every discrete group: Hattori–Stallings traces of idempotent matrices over $\ell^1(G)$ are supported on finitely many finite-order conjugacy classes. The algebraic companion proves the integral Bass trace conjecture and Kaplansky's idempotent conjecture for torsion-free groups over every commutative unital characteristic-zero domain.
The ℓ^1-Bass Conjecture for Discrete Groups · The Bass trace conjecture and the characteristic-zero Kaplansky idempotent conjecture

208. **The finite Benson–Etingof–Ostrik conjecture.** Proves that every finite symmetric tensor category over an algebraically closed field k of characteristic $p > 0$ admits a k-linear exact faithful strong symmetric monoidal fiber functor to a higher Verlinde category Ver_{p^n}. The level may depend on the category, and the theorem includes characteristic two, resolving the finite case of the Benson–Etingof–Ostrik conjecture.
Fiber functors for finite symmetric tensor categories in positive characteristic

209. **Integral counterexamples to Gersten's conjecture.** Disproves unrestricted integral Gersten injectivity in degrees 3 and 5. Two explicit two-dimensional ramified regular local rings of mixed characteristic $(0,5)$ have nonzero integral K-theory classes that vanish over their fraction fields.
Integral Gersten counterexample · Degree-three Gersten counterexample

210. **Foulkes' conjecture for sixth powers and quadratic stabilization.** Proves the sixth case of Foulkes' conjecture: $\mathrm{Sym}^6(\mathrm{Sym}^b V)$ embeds equivariantly in $\mathrm{Sym}^b(\mathrm{Sym}^6 V)$ for every $b \geq 6$ and finite-dimensional complex V. More generally, the canonical multiplication map $\mathrm{Sym}^b(\mathrm{Sym}^a V) \to \mathrm{Sym}^a(\mathrm{Sym}^b V)$ is surjective for $a \geq 2$ and $b \geq a(a-1)$, giving dimension-independent quadratic stabilization.
Sixth symmetric power · Quadratic stabilization

Probability and statistical mechanics

211. **Geometry, diffusion, and spectra of random planar maps.** Critical Fortuin–Kasteleyn planar maps converge to Liouville quantum gravity spheres for $0 < q \leq 4$ and to the Brownian continuum random tree for $q > 4$, establishing the surface-to-tree geometric transition. For FK–Ising and spanning-tree-weighted maps, stationary random walks converge to Liouville Brownian motion on the limiting sphere. The FK–Ising spectral result also gives convergence of eigenvalues and heat traces, using the stated Brownian/LQG inputs.
Random Walks on Critical FK–Ising Maps and Liouville Brownian Motion · Spectral convergence for critical FK–Ising planar maps · A Linear Clock for Random Walk on Tree-Weighted Planar Maps · Subcritical conformal limits · Critical sphere at $q = 4$ · FK and spanning-tree metric-measure limits · Brownian CRT limits for $q > 4$

212. **No bigeodesics and smooth limit shapes in planar first-passage percolation.** Proves that planar first-passage percolation has no doubly infinite geodesic for iid nonnegative nonatomic edge weights when the minimum of four weights has finite second moment. For exponential weights, the limit shape is strictly convex with C^1 boundary. Differentiability also holds for every Gamma law with positive shape and rate.
No bigeodesics · Strict convexity and differentiability

213. **No infinite critical clusters on quasi-transitive graphs.** Resolves the Benjamini–Schramm criticality conjecture for bond percolation on every infinite connected locally finite quasi-transitive graph with $p_c < 1$: at the critical probability, there is almost surely no infinite cluster. The family also establishes this conclusion for both nearest-neighbor bond and site percolation on $\mathbb{Z}^3$.
Cubic-lattice bond and site percolation · Quasi-transitive graphs at criticality

214. **The Benjamini–Schramm nonuniqueness conjecture.** Proves $p_c < p_u$ for Bernoulli bond percolation on every infinite connected locally finite nonamenable quasi-transitive graph, resolving the Benjamini–Schramm nonuniqueness conjecture. Thus there is a nonempty range of probabilities with infinitely many infinite clusters. A stronger operator bound also establishes the critical triangle condition.
Nonuniqueness of percolation on nonamenable quasi-transitive graphs

215. **Massive continuum limits and exact mass asymptotics for planar $O(n)$ models.** Constructs the canonical continuum limit of the two-dimensional nearest-neighbor $O(3)$ model: a non-Gaussian local relativistic theory with a unique vacuum and a positive mass gap. For the square-lattice $O(4)$ model, determines the exact leading asymptotic of the full transfer gap, $m_{\mathrm{lat}}(\beta) \sim 32 e^{\pi/4-1/2}\sqrt{\beta}\, e^{-\pi\beta}$. The family also proves exponential spin-correlation decay for two-dimensional nearest-neighbor $O(n)$ models with $n \geq 3$ at every positive temperature.
O(3) massive continuum limit · O(3) isolated particle pole · Exact mass asymptotics for the two-dimensional $O(4)$ lattice model · O(4) mass bounds · O(n) exponential decay

216. **Critical logarithmic corrections and BKT scaling for the planar XY model.** For the square-lattice nearest-neighbor cosine XY model, proves critical axis correlations $C_{\beta_c}(r) \sim A r^{-1/4}(\log r)^{1/8}$ and the Berezinskii–Kosterlitz–Thouless essential singularity $\sqrt{\beta_c - \beta}\log \xi(\beta) \to B$, with $A, B > 0$ after the free-box thermodynamic limit. For finite square-symmetric interactions containing nearest neighbors, discrete Gaussian heights converge to Gaussian fields throughout the rough phase, including its threshold, along geometric torus sizes. Critical center-magnetization and spin-field conclusions retain their stated height, renormalization, and field-input assumptions.
The critical logarithmic correction for the planar XY model · Critical Center Magnetization in the Planar XY Model · The Critical Spin Field of the Planar XY Model · Essential Singularity of the Correlation Length in the Planar XY Model · Critical XY correlation exponent · Height and spin-field universality

217. **The low-temperature Sherrington–Kirkpatrick fluctuation law.** For every fixed inverse temperature $\beta > 1$, determines the fluctuation scale and limiting law of the zero-field Gaussian Sherrington–Kirkpatrick log partition function. Its variance is asymptotic to $c_\beta n^{1/3}$, with $c_\beta > 0$, confirming the predicted $n^{1/6}$ standard-deviation scale. Exact centering and standardization give full-sequence convergence to a uniquely characterized nondegenerate law.
Free-energy limiting law · Fluctuation scale

218. **Conformal universality for weakly interacting and disordered Ising models.** Weak finite-range square-symmetric even multispin perturbations of the square-lattice Ising model preserve critical bulk spin and energy limits; weak square-symmetric contour interactions also yield chordal SLE_3 interface limits. With sufficiently weak iid bond disorder of any fixed bounded nondegenerate mean-zero law, critical spin interfaces converge to the same law in probability over environments.
Quenched SLE_3 limits for general weak random-bond Ising models · Quenched SLE_3 Universality for the Weak Random-Bond Ising Model · Logarithmic Relative Fluctuations in the Weakly Disordered Planar Ising Model · Bulk correlation universality · SLE_3 universality · Stopping-band resolution

219. **GOE universality for random regular graphs with weak disorder.** For every fixed degree $d \geq 3$, the bulk adjacency-eigenvalue point process of a uniform simple random d-regular graph converges to the Gaussian orthogonal ensemble law, including for cubic graphs. The same fixed-energy universality persists under sufficiently weak fixed iid uniform diagonal disorder, throughout compact bands strictly inside the clean spectral edges.
Fixed-energy universality for weak Anderson disorder on random regular graphs · GOE bulk universality for fixed-degree random regular graphs

220. **Directional zero–one laws and ballisticity in random environments.** On $\mathbb{Z}^d$, $d \geq 3$, directional escape has probability zero or one for iid strictly elliptic nearest-neighbor environments, and for stationary ergodic finite-range-dependent environments under uniform ellipticity. In iid uniformly elliptic environments with $d \geq 2$, almost-sure directional transience implies a deterministic limiting velocity with positive projection in that direction, resolving the ballisticity conjecture.
A directional zero–one law for finite-range-dependent random environments · Strict-ellipticity zero–one law · Directional transience implies ballisticity

221. **The Mézard–Parisi formula for diluted spin glasses.** Proves the Mézard–Parisi hierarchical cavity formula for Poisson-diluted even-arity Ising models satisfying the Panchenko–Talagrand factorization and positivity assumptions, with only first-moment integrability. The limiting free energy equals the infimum over finite-depth hierarchical trial laws. This includes the Viana–Bray model, symmetric diluted even-spin models, and weighted soft even-K satisfiability.
The Mézard–Parisi formula for diluted spin glasses

222. **Perceptron free energies and microscopic jamming.** Determines finite-temperature variational free energies for Gaussian Ising perceptrons with bounded Borel log-potentials and Gaussian spherical perceptrons with bounded continuous potentials, at every positive pattern density. A spherical extension treats bi-orthogonally invariant disorder with compact limiting singular-value distributions and no outliers. At margin -1, the quadratic-penalty spherical model has a sharp feasibility threshold and limiting gap and force laws, with system size, zero temperature, and critical density taken in that order.
Ising perceptron free energy · Negative spherical perceptron jamming · Bi-orthogonally invariant disorder · Spherical perceptron free energy

223. **Conformal limits of square-lattice random-cluster interfaces.** Proves chordal SLE_κ limits for critical square-lattice random-cluster interfaces for $0 < q \leq 4$, with $\kappa = 4\pi/\arccos(-\sqrt{q}/2)$: bounded Jordan domains are allowed for $q \geq 1$, and smooth Jordan domains for $q < 1$, under the stated marked-boundary approximations. For $1 \leq q \leq 4$, complete nested plane loops converge to CLE_κ.
FK interfaces and nested loops ($1 \leq q < 4$) · Self-dual interfaces ($q < 1$) · Quenched SLE Universality for Weakly Disordered FK–Ising Interfaces · Thermal FK–Ising interfaces and massive SLE · Natural Occupation Measures for Critical Square-Lattice FK Interfaces · Conformal Limits of Critical Square-Lattice Random-Cluster Interfaces

224. **Critical and near-critical universality for Voronoi percolation.** Proves Cardy's formula for annealed critical Poisson–Voronoi crossing probabilities in every bounded Jordan quadrilateral. With each model normalized by its own expected unit-square pivotal count, the conditional joint near-critical crossing-threshold laws for rational polygonal quads converge in environment probability to the triangular-lattice reference law. This establishes quenched near-critical universality for crossing thresholds.
From critical crossings to quenched near-critical universality in Voronoi percolation · A Pivotal Amplitude for Voronoi Percolation from Cardy's Formula · Cardy's formula for critical Poisson–Voronoi percolation

225. **Gaussian free field limits for the balanced six-vertex model.** The balanced square-lattice six-vertex height field with $a = b = 1$ and $0 < c \leq 2$ converges to a Gaussian free field, including at the endpoint $c = 2$. The plane state is defined by balanced-torus limits. For unit height increments and Green kernel $-(2\pi)^{-1}\log|x-y|$, the exact variance multiplier is $1/\arcsin(c/2)$.
The Gaussian free field limit of the balanced six-vertex model with variance multiplier 1/arcsin(c/2)

226. **The double-dimer CLE_4 conjecture in the half-plane.** Resolves the half-plane Temperleyan form of the double-dimer scaling-limit conjecture: the complete loop ensemble formed by two independent dimer coverings of the Temperleyan square lattice converges to nested CLE_4. Convergence matches every macroscopic loop as an unparametrized curve, upgrading convergence of loop observables to convergence of the loops themselves.
The curve scaling limit of half-plane double dimers

227. **The dynamical phase transition in the Sherrington–Kirkpatrick model.** For zero-field Gaussian SK heat-bath dynamics with rate-one updates per spin, proves worst-start cutoff on the $\log n$ scale for fixed $0 \leq \beta < 1$, mixing time $n^{2/3+o(1)}$ at $\beta = 1$, and stretched-exponential mixing from a Gibbs-sampled fixed starting configuration for $\beta > 1$, in probability over disorder. At criticality, rescaled stationary and quench autocorrelation processes have universal random limits for Gaussian and Rademacher disorder; the quench limit relaxes to the stationary limit.
Universality of critical quench autocorrelations in the Sherrington–Kirkpatrick model · Functional universality of critical SK autocorrelations · High-temperature spectral gap · High-temperature cutoff · Critical slowing down · Typical-start stretched-exponential barriers · Low-temperature typical-start upper bound · Critical mixing

228. **Continuum phase transitions for radial pair potentials.** Constructs stable distance-dependent pair interactions for three-dimensional classical particles with a first-order phase transition: the canonical free energy has a derivative jump at one inverse temperature throughout an open density interval. One potential has a divergent repulsive core; another is bounded and continuous with an integrable power-law tail, realizing the type of transition sought in Simon's continuum problem.
Temperature singularity · Algebraic-decay phase transition

229. **Sharp three- and four-state reconstruction thresholds.** Proves the exact reconstruction threshold $d\lambda^2 > 1$, with nonreconstruction at equality, for three-state symmetric and four-state ferromagnetic broadcasting on regular trees ($d \geq 2$) and observed Poisson trees (mean $d > 1$ and $d > 0$, respectively), with Poisson advantage averaged without conditioning on survival. The three-state theorem allows both signs of λ and gives the exact weak-recovery threshold for the symmetric three-community stochastic block model.
The Reconstruction Threshold for the Ferromagnetic Four-State Potts Model · A Capacity Criterion for Four-State Potts Reconstruction on Trees · The exact reconstruction threshold for the three-state symmetric channel

230. **Exact Hausdorff measure for SLE.** Resolves Schramm's Hausdorff-measure question for chordal SLE_κ, $0 < \kappa < 8$. The explicit gauge $r^d(\log\log(1/r))^{(2-d)/2}$, $d = 1 + \kappa/8$, gives almost surely positive finite measure to every trace segment $\gamma([s,t])$ with $0 < s < t < \infty$, and finite expected measure to the trace in every bounded disk.
Exact Hausdorff gauge · Explicit exact Hausdorff gauge

231. **The free uniform spanning forest is a factor of IID.** On every infinite connected locally finite simple unweighted graph, the free uniform spanning forest is a factor of independent vertex labels, by one isomorphism-equivariant rule using no root. Translation-invariant strongly Rayleigh binary processes on every countable group, including invariant determinantal processes with Hermitian positive-contraction kernels, are also factors of IID.
The free uniform spanning forest is a factor of IID

232. **Gaussian fields and SLE interfaces for Lipschitz heights.** Proves Gaussian free field limits on bounded smooth simply connected domains for triangular-lattice height models: uniform odd heights with increments $0, \pm 2$ and two-arc boundary values ± 1, and zero-boundary integer Lipschitz heights weighted by fixed $x \in [1/\sqrt{2}, 1]$. Uniform real Lipschitz heights also converge to a Gaussian field; at a tuned opposite-boundary amplitude, their interface converges to chordal SLE_4, establishing Schramm's real-field/interface predictions.
Weighted integer heights · Integer heights with two-arc boundary · Uniform real Lipschitz surfaces

233. **The joint critical Ashkin–Teller scaling limit.** Identifies the joint scaling limit of Ashkin–Teller heights and both complete current-cluster collections throughout the critical line, including the four-state Potts endpoint. In bounded Jordan domains with admissible lattice approximations and wired primal/free dual boundaries, the height converges to the predicted Gaussian free field, and the clusters to canonical recursive sets of that same field, retaining every nesting depth.
The joint scaling limit of critical Ashkin-Teller currents

234. All-temperature pressure for orthogonally invariant Ising spin glasses. Gives an exact variational formula for the limiting pressure of orthogonally invariant Ising spin glasses at every fixed temperature, both almost surely and in expectation. The coupling matrix is a Haar-random rotation of a deterministic spectrum converging to a compactly supported law, with extreme eigenvalues converging to its support edges. The zero-field ground-state energy follows as temperature tends to zero.
All-temperature pressure for orthogonally invariant Ising spin glasses

235. Random-SAT thresholds, sharp variance and computability. For random k-SAT with independent uniformly signed proper clauses sampled with replacement, proves finite positive limiting thresholds and hitting-time variance $\Theta_k(n)$ for every fixed $k \geq 3$, and computability of the 3-SAT threshold. We credit Gaia Carenini with priority for resolving the threshold-existence conjecture in her concurrent ECCC TR26-229, made public October 5, 2026; this family supplies another proof and the sharper variance and computability results.
Fixed-clause-size limiting threshold · Linear Variance of the Random 3-SAT Hitting Time · Random k-SAT hitting-time variance · Computing the random 3-SAT threshold

236. The factor-of-IID threshold for free Ising states on trees. Determines when the free zero-field ferromagnetic Ising state on the infinite d-regular tree is a factor of independent vertex labels: exactly when $\tanh\beta \leq (d-1)^{-1/2}$, including equality, for $d \geq 3$ and $\beta \geq 0$. The construction uses no root and is almost surely equivariant for each fixed tree automorphism, resolving the ferromagnetic case of Lyons's question.
The sharp factor-of-IID threshold for the free Ising model on regular trees

237. The three-quarter diameter exponent for honeycomb walks. Proves the diameter form of Nienhuis's predicted three-quarter exponent: a uniformly chosen n-step self-avoiding walk on the honeycomb lattice has diameter $n^{3/4+o(1)}$. Its local mass and covering numbers have exponent 4/3. These estimates hold at every sufficiently large fixed length, simultaneously across scales, with arbitrarily high polynomial probability.
Radial transfer and polygon length laws · Chords with prescribed boundary endpoints · Cylinder loop weights and planar nesting · Mass and covering exponents for fixed-length walks · Signed cylinder propagation · Logarithmic bridge-length windows · Marked polygons and one-arc bounds · Disk transfer and confined bridge mass · Polynomial vacuum representations · Renewal and changes of law · Critical strip-crossing mass · Finite bridge moments · Cap-selected amplitudes and triangle chords

238. Optimal logarithmic mixing of the Thorp shuffle. Proves that the Thorp shuffle randomizes $N = 2^d$ labeled cards in $\Theta(\log N)$ physical shuffles, settling its optimal mixing order for power-of-two deck sizes. Convergence is in total variation from the worst initial ordering and concerns the entire permutation, not just individual card positions.
Optimal-order mixing of the Thorp shuffle · Random coordinate frames · Partial-information Fourier bounds · Deterministic-sweep information · Permutations with revealed switching · Routing and representation contraction · Row–column symmetry and sweep contraction · Random-subspace tests and trace smoothing · Compatibility entropy and spectrum · Signed tensor densities and diagram budgets · Conditional sweeps and analytic transfer · Strict four-row permanent inequality

239. Sharp singularity rates for symmetric sign matrices. Determines the sharp exponential singularity rate of symmetric random sign matrices with independent entries on and above the diagonal. Uniform signs give $\Pr(\det A_n = 0) = (1/2 + o(1))^n$; for fixed bias $p \in (0,1) \setminus \{1/2\}$, the rate is $(p^2 + (1-p)^2 + o(1))^n$. In the biased case, agreeing rows attain this rate.
The sharp exponential rate of singularity for symmetric Bernoulli matrices · The sharp singularity rate for biased symmetric sign matrices

Mathematical logic

240. Shelah's eventual categoricity conjecture. Proves Shelah's eventual categoricity conjecture in ZFC: for each bound on the Löwenheim–Skolem number, a uniform threshold makes categoricity of an abstract elementary class in one cardinal above that threshold imply categoricity throughout the same tail. Categoricity means uniqueness up to isomorphism at a given cardinality. Under the continuum hypothesis, a proposed specific Hanf threshold need not suffice.
CH threshold obstruction · Eventual categoricity

241. Rigidity of the Turing degrees. Every order automorphism of the Turing degrees is the identity, resolving their rigidity problem. Thus no nontrivial relabeling of degrees preserves the ordering by relative computability.
Rigidity of the Turing degrees

242. **Single-fold Diophantine representations.** Every recursively enumerable set of tuples of natural numbers has a Diophantine representation with exactly one auxiliary solution for each member and none for nonmembers. This proves the single-fold conjecture and hence the finite-fold conjecture. Diophantine solvability over the nonnegative integers remains undecidable even with an at-most-one-solution promise.
Single-fold Diophantine representations

243. **Choiceless counting does not capture polynomial time.** Confirms the Blass–Gurevich–Shelah noncapture conjecture: consistency of a linear system over $\mathbb{F}_3$ defines a polynomial-time query on unordered finite structures that choiceless polynomial time with counting cannot express. A separate result shows that adding witnessed symmetric choice strictly increases expressive power. Both separations hold for the full counting formalism, allowing hereditarily finite sets of arbitrary finite rank.
Failure to capture polynomial time · Witnessed symmetric choice

244. **The partition principle does not imply choice.** Assuming ZF is consistent, constructs a model in which every surjective image of a set injects into that set, yet the axiom of choice fails. Choice for ordinal-indexed families still holds. From any countable transitive model of ZFC, a separate construction gives a transitive symmetric extension with these properties and no new countable sequences of ground-model elements.
The Partition Principle does not imply Choice

245. **The β-Barendregt–Geuvers–Klop conjecture.** Proves that weak normalization implies strong normalization for every pure type system: if every legal expression in every valid context has a β-normal form, every β-reduction sequence terminates. This resolves the β-Barendregt–Geuvers–Klop conjecture, including nonfunctional rules and open contexts.
Weak and strong normalization in pure type systems

Group theory

246. **Cannon's conjecture.** Every word-hyperbolic group with boundary homeomorphic to S^2 admits a proper cocompact isometric action on hyperbolic three-space with finite kernel, proving Cannon's conjecture. Every torsion-free such group is therefore the fundamental group of a closed hyperbolic three-manifold.
A Modulus Proof of Cannon's Conjecture

247. **An infinite finitely presented residually finite 2-group.** Constructs an infinite finitely presented residually finite group whose elements all have finite 2-power order, answering the finitely presented Burnside problem negatively even in this class. The construction also yields an infinite-dimensional finitely presented nil associative $\mathbb{F}_2$-algebra and a finitely presented infinite-dimensional algebraic unitization, giving negative answers to the corresponding nilpotence and Kurosh finiteness questions.
An infinite finitely presented residually finite 2-group · An infinite finitely presented periodic group

248. **Thompson's group F is nonamenable.** Proves that Thompson's group F, the group of dyadic piecewise linear homeomorphisms of the interval, is nonamenable, resolving its longstanding amenability problem.
Thompson's group F is nonamenable

249. **A finitely generated counterexample to Eilenberg–Ganea.** Constructs a finitely generated residually finite group with integral cohomological dimension two and geometric dimension three, disproving the Eilenberg–Ganea conjecture. It has no two-dimensional classifying space, even with infinitely many cells.
A finitely generated counterexample to the Eilenberg–Ganea conjecture

250. **The Boone–Higman conjecture and higher finiteness.** A finitely generated group has decidable word problem exactly when it embeds in a finitely presented simple group, proving the Boone–Higman conjecture. The target can have type F_∞: a classifying space with finitely many cells in each dimension. A single group of type F_∞ can also contain every finitely presented group.
Finite algebraic envelopes · Simple F_∞ overgroups · Universal F_∞ group

251. **Amenability, unitarizability, and Ulam stability.** Resolves Dixmier's problem for all discrete groups: amenability is equivalent to every uniformly bounded Hilbert-space representation being similar to a unitary representation. For countable discrete groups, amenability is also equivalent to strong Ulam stability: sufficiently accurate unitary approximate representations are uniformly close in operator norm to genuine representations on the same, possibly infinite-dimensional, Hilbert space.
Unitarizability implies amenability for discrete groups · Strong Ulam Stability Characterizes Amenability

252. **A non-residually-finite torsion-free hyperbolic group.** Constructs a torsion-free word-hyperbolic group that is not residually finite, answering the residual-finiteness question negatively. One fixed nonidentity element is killed by every finite-dimensional linear representation over every commutative field, so the group is not linear over any such field.
A torsion-free hyperbolic group that is not residually finite

253. **An infinite finitely presented simple amenable group.** Constructs an infinite finitely presented simple amenable group, answering the longstanding question of whether these properties can occur simultaneously.
An infinite finitely presented simple amenable group

254. **Classifying spaces and geometric obstructions for Artin groups.** The Salvetti complex of every finite-rank Artin group is aspherical, proving the Artin $K(\pi, 1)$ conjecture. Arbitrary intersections of its parabolic subgroups are parabolic, proving the Parabolic Intersection Conjecture. An explicit Artin group admits no proper cocompact isometric action on any nonempty proper CAT(0) space.
Artin $K(\pi, 1)$ · No geometric CAT(0) action · Parabolic intersections

255. **Quasi-isometric rigidity of virtually polycyclic groups.** Proves that every finitely generated group quasi-isometric to a finitely generated virtually polycyclic group is virtually polycyclic, resolving the Eskin–Fisher–Whyte lattice-recognition conjecture. Equivalently, a group quasi-isometric to a lattice in a connected simply connected solvable Lie group is virtually a uniform lattice in some such Lie group, possibly a different one.
Quasi-isometric recognition of virtually polycyclic groups

256. **Howie's conjecture on equations over groups.** Proves that every finite system of equations over an arbitrary group whose exponent-sum matrix has full row rank over $\mathbb{Q}$ has a simultaneous solution in an overgroup, resolving Howie's conjecture. The coefficient group embeds in the presented quotient. A companion proves Kervaire's conjecture: adjoining one generator and one relation cannot trivialize a nontrivial group.
Nonsingular systems of equations over arbitrary groups · The Kervaire theorem for groups

257. **A hyperbolic group with no geometric CAT(0) action.** Answers negatively whether every word-hyperbolic group is a CAT(0) group. Constructs one with a finite classifying space but no proper cocompact isometric action on any proper complete CAT(0) space, in any dimension.
A hyperbolic group with no geometric CAT(0) action

258. **Gersten's conjecture for one-relator groups.** Proves Gersten's conjecture: every finitely generated one-relator group containing no Baumslag–Solitar subgroup $\mathrm{BS}(m, n)$, with $m, n \neq 0$, is word-hyperbolic. It also proves that every word-hyperbolic one-relator group is virtually compact special.
Baumslag–Solitar-free one-relator groups · Virtual compact specialness

259. **A group without fixed price.** Constructs a finitely generated group with two essentially free probability-measure-preserving actions of different costs, answering the general fixed-price problem negatively. Its Bernoulli action has cost bounded away from one, while a sequence of finite height extensions has costs tending to one.
A group without fixed price

Mathematical physics

260. **Spacetime Penrose inequalities and rigidity.** Proves the sharp enclosing-area spacetime Penrose inequality for smooth one-ended asymptotically flat initial data in every spatial dimension $n \geq 3$, under dominant energy, weak future trapping, positive enclosing area, and the stated decay assumptions. It bounds invariant ADM mass below using minimum enclosing area, with equality rigidity under additional horizon hypotheses. Charged upper-area bounds treat dyonic three-dimensional data; the higher-dimensional purely electric extension uses the matched neutral theorem.
Three-dimensional charged inequality · Higher-dimensional charged reduction · Enclosing area, charge, and rigidity · Kerr–Newman inequality · Electromagnetic tails · Nonmaximal anti-de Sitter inequality · Maximal asymptotically hyperbolic inequality · Local Schwarzschild–anti-de Sitter inequality · Numerical spacetime · Riemannian endpoint · Equality and rigidity · Boundary graphs · Bondi-CKS

261. **Localization and delocalization in the Anderson model.** Resolves the predicted spectral contrast for the lattice Anderson model with independent uniform site potentials. In dimension two, every positive disorder strength gives almost surely pure-point spectrum. In every fixed dimension $d \geq 3$, sufficiently weak positive disorder gives purely absolutely continuous spectrum on a fixed open interval with nonzero spectral weight.
Absolutely continuous spectrum · Pure-point spectrum

262. Sharp one-dimensional Lieb–Thirring inequalities. Proves the sharp one-dimensional Lieb–Thirring inequality for $1/2 < \gamma < 3/2$ and arbitrary finite-matrix potentials $W \geq 0$ with $\int \mathrm{tr}(W^{\gamma+1/2}) < \infty$: the optimal constant is the scalar one-bound-state value, independent of matrix size. All equality cases are direct sums, in one constant unitary basis, of scalar sech^2 solitons with independent scales and centers, and zero channels.
Equality cases in the sharp one-dimensional matrix Lieb–Thirring inequality · Sharp one-dimensional Lieb–Thirring inequalities for matrix potentials · Sharp one-dimensional Lieb–Thirring constants

263. The ionization and generalized ionization conjectures. For the full nonrelativistic Coulomb model with two electron spin states, proves that a molecule with M fixed nuclei of charges at least one and total charge Z strictly binds at most $Z + CM$ electrons. Neutral-atom first ionization energies and radii containing all but an expected half-electron have universal positive upper and lower bounds. The energy cost of removing m electrons has Thomas–Fermi asymptotics as $m \to \infty$ and $Z/m \to \infty$; neutral-atom outer radii have the corresponding iterated-limit asymptotics, taking $Z \to \infty$ first.
Uniform excess charge and outer radius · Generalized ionization energies · Generalized outer-electron radii

264. Strong cosmic censorship near two-ended Kerr data. Proves local strong cosmic censorship near each fixed rotating subextremal Kerr bridge. A dense G_δ subset of a weighted smooth neighborhood of smooth complete two-ended asymptotically flat vacuum data has full maximal globally hyperbolic developments with no future continuous nondegenerate extension whose weak connection is locally square-integrable. No symmetry is imposed; extensions need not satisfy the vacuum equations.
Future inextendibility, L^2 connection · C^1 future inextendibility · C^2 future inextendibility

265. The two-dimensional gapped area law. Proves an entropy area law for unique ground states of finite-range Hamiltonians on arbitrary finite induced square-lattice domains, using only a uniform full-system spectral gap and bounds on the local interactions. On open $L \times L$ squares, uniformly gapped nearest-neighbor ground states also admit projected entangled-pair state approximations with polynomial bond dimension and global vector error at most L^{-1}.
Two-dimensional area law · Polynomial PEPS approximation

266. Exactly three mutually unbiased bases in dimension six. Proves $N(6) = 3$, resolving Zauner's dimension-six mutually unbiased bases conjecture: three such bases exist in $\mathbb{C}^6$, but four cannot. The exclusion is a complete certified computation under the stated binary64 arithmetic and compiler conditions. An independent companion proves the Matolcsi–Ruzsa–Weiner Fourier-vanishing conjecture for order-six complex Hadamard matrices outside Tao's cubic equivalence class.
Maximum number of bases · Exact Fourier certificates

267. Positive-temperature Bose–Einstein condensation and quantum depletion. Proves Bose–Einstein condensation for the exact canonical Gibbs state of the three-dimensional hard-sphere gas: each fixed exclusion distance and sufficiently small fixed density admit a strictly positive temperature, independent of volume, with positive condensate fraction in the thermodynamic limit. At zero temperature, proves the Bogoliubov leading quantum-depletion law for hard spheres and fixed bounded nonnegative radial finite-range potentials of positive scattering length, taking the thermodynamic limit before the dilute limit.
Bose–Einstein condensation at positive temperature in the dilute hard-sphere gas · Quantum Depletion and Momentum Distribution in the Dilute Hard-Sphere Bose Gas · Quantum Depletion for Fixed Bounded Repulsive Potentials · A density-uniform condensate bound for dilute Bose gases · Ground-state condensation in the dilute hard-sphere gas

268. The spin-one Haldane gap. Proves the spin-one Haldane gap conjecture for the pure antiferromagnetic Heisenberg chain on even periodic rings: the spectral gap stays uniformly positive as the chain grows. A companion establishes a gap for odd open chains with endpoint field $h = 3/5$ and gives boundary-selected infinite-volume states with topological index -1.
Periodic Haldane gap · Boundary-field gap

269. The Laughlin gap and stability under scalar disorder. Proves the fermionic Laughlin spectral-gap conjecture for the full V_1 interaction at filling $1/3$ on the round sphere. The unique ground state remains uniformly gapped under sufficiently weak bounded real scalar one-body potentials projected to the lowest Landau level. Both the gap and disorder threshold are uniform over all sufficiently large particle numbers and all normalized potential profiles.
Uniform Stability of the Spherical Laughlin Gap · A Fock-space inequality and the Laughlin spectral gap

270. Threshold and positive-energy bound states of the BFSS model. Proves that the undeformed relative SU(N) BFSS model has exactly one normalizable zero-energy state for every finite $N \geq 2$, resolving the threshold-bound-state conjecture. For SU(2), a companion proves infinitely many normalizable positive-energy eigenstates with unbounded energies, contradicting the original BFSS paper's exclusion of additional bound states at $N = 2$.
The unique threshold bound state of the SU(N) BFSS model · Positive eigenvalues of the relative SU(2) BFSS Hamiltonian

271. Bloch's law and spontaneous ferromagnetic order. Proves Bloch's $T^{3/2}$ law with its exact coefficient for three-dimensional quantum Heisenberg ferromagnets at every positive quantum spin, allowing nonnegative symmetric finite-range couplings whose support generates $\mathbb{Z}^3$. The thermodynamic limit precedes the zero-field derivative and low-temperature limit. The family also proves spontaneous magnetization for nearest-neighbor models in every dimension $d \geq 3$ and determines the first lattice correction for three-dimensional nearest-neighbor couplings.
Bloch's Law for Finite-Range Heisenberg Ferromagnets in Three Dimensions · The first lattice correction to Bloch's law · The spherical magnetization law for the three-dimensional quantum Heisenberg ferromagnet · Spontaneous magnetization in the quantum Heisenberg ferromagnet

272. Entanglement without secret key and the PPT-square conjecture. Constructs an entangled state on $\mathbb{C}^{10} \otimes \mathbb{C}^{10}$ with zero distillable secret key for the specified local-instrument protocols that complete almost surely. These allow joint local processing and authenticated two-way public communication, with no other shared private resource and an eavesdropper holding the input purification and public record. A trace-preserving PPT channel on $M_{21}(\mathbb{C})$ whose square is not entanglement breaking disproves Christandl's PPT-square conjecture.
Entanglement with zero distillable secret key in local dimension ten

273. The entropy photon-number inequality. Proves the entropy photon-number inequality for beam-splitter mixing of two independent finite-energy bosonic inputs in any finite number of modes: the output's entropy photon number is at least the transmissivity-weighted average of the inputs'. Arbitrary entanglement within each input is allowed, and product thermal inputs attain equality even when their entropies differ.
The entropy photon-number inequality

274. Moore's parity conjecture for QAC^0. Resolves Moore's parity conjecture in the measured-output model: constant-depth quantum circuits with arbitrary one-qubit gates, unbounded-arity Toffoli gates and polynomially many total qubits cannot compute parity with any fixed positive worst-case advantage. Ancillas start in zero, one output qubit is measured, and all other registers may be discarded. Xu–Li's reductions give the same bounded-error obstruction for strict majority.
Product-projection localization · Regular trajectories and pruning

275. QMA-hardness of continuum Coulomb energy. Proves QMA-hardness of approximating the electronic Coulomb energy infimum in three dimensions, minimizing over the full spinful fermionic continuum space. Deterministic polynomial-time reductions work even with only unit-charge nuclei at distinct rational positions, polynomially many electrons and an energy-threshold separation of at least one.
Binary nuclear charges · Unit nuclear charges

276. The classical capacity of generalized amplitude damping. Determines the unassisted classical capacity of every qubit generalized amplitude-damping channel, including all damping and thermal parameters. An explicit one-variable optimization gives the capacity, attained by independent two-state signal ensembles with collective decoding. Holevo capacity, minimum output entropy and regularized classical capacity are additive when tensoring with any finite-dimensional quantum channel.
Classical capacity and entropy inequalities for generalized amplitude damping

277. Threshold repetition for entangled games. Proves exponential threshold repetition for every finite two-player one-round game: if its entangled value is $v < 1$, the probability of winning at least a fraction $v + \delta$ of k independent repetitions decays exponentially in k, for $0 < \delta < 1 - v$. Arbitrary joint finite-dimensional entangled strategies and correlated question distributions are allowed.
Threshold parallel repetition for finite-dimensional entangled games

278. Failure of Kohn–Sham ensemble representation. Constructs a three-electron Coulomb molecule with two equal positive-integer-charge nuclei whose absolute ground-state density has no noninteracting ground-state ensemble representation by a single real spin-independent local potential in $L^{3/2}(\mathbb{R}^3) + L^{\infty}(\mathbb{R}^3)$. This disproves Kohn–Sham ensemble representability for that potential class; the required nuclear charge is specified nonnumerically.
A Coulomb ground-state density without Kohn-Sham ensemble representation

279. **Exact quantum factoring over a fixed finite gate set.** Gives a polynomial-time uniform quantum circuit family that outputs the complete prime factorization of every integer with probability one. Both gate count and qubit count are polynomial in the input length, and one fixed finite gate set suffices.
Exact quantum factoring over a fixed finite gate set

280. **Strong locality for strongly rational unitary vertex operator algebras.** Proves the strongly rational case of the strong-locality conjecture: every simple unitary strongly rational complex vertex operator algebra generates a completely rational conformal net. Its simple modules are unitarizable, and its representation category agrees with the net's finite-index sectors as a braided unitary tensor category.
Strongly rational unitary vertex operator algebras and conformal nets

281. **QAOA optimality for the SK model.** Proves that QAOA approaches the ground-state energy of the Gaussian zero-field Sherrington–Kirkpatrick model when system size tends to infinity before circuit depth. For every accuracy, finite depth and deterministic angles independent of size and disorder achieve the required limiting expected energy per spin. This also yields leading-order optimal expected MaxCut values on large-degree random regular graphs, with size tending to infinity before degree.
QAOA attains the SK ground-state energy in the thermodynamic-first limit · Full support of the zero-temperature Sherrington-Kirkpatrick order parameter

282. **Scale-to-conformal enhancement in four-dimensional QFT.** Under the stated bounded-local-net and field-reconstruction hypotheses, proves that scale symmetry implies local conformal symmetry for four-dimensional unitary positive-energy theories with a discrete bounded-below scaling spectrum of finite multiplicity, finite scaling support and a physical local scale current. The stress tensor has a traceless improvement with unchanged spacetime charges. The conclusion concerns local Ward identities, not a global conformal action on the whole net.
Scale and conformal symmetry in four-dimensional operational quantum field theory

283. **Polynomial-time, constant-error unitary synthesis from a Boolean oracle.** Solves the constant-error Aaronson–Kuperberg unitary synthesis problem: a uniform polynomial-size quantum oracle circuit approximates every n-qubit unitary channel within diamond-norm error $1/2$, after a suitable Boolean oracle is chosen. Gates, qubits, oracle calls and query length are polynomially bounded. The target-dependent oracle may have an unrestricted truth table; its efficient classical construction is not asserted.
Polynomial-Time Unitary Synthesis from a Boolean Oracle

284. **The optimal randomized–quantum query exponent.** Shows that the universal bound $R(f) = O((1 + Q(f))^4)$ for total Boolean functions is sharp in its exponent, ruling out every smaller power and disproving the conjectured cubic relation. Here R and Q are randomized and quantum worst-case bit-query complexities with error at most $1/3$; computation between queries is unrestricted.
A Nearly Quartic Separation Between Randomized and Quantum Query Complexity

Operator algebras

285. **Counterexamples to Baum–Connes and Kadison–Kaplansky.** Disproves the coefficient-free reduced Baum–Connes conjecture through a failure of rational injectivity and a separate failure of surjectivity of assembly. A finitely generated torsion-free example witnesses the injectivity failure. Separately, a torsion-free group has a nontrivial projection in its reduced group C^*-algebra, disproving the Kadison–Kaplansky conjecture.
Baum–Connes injectivity · Kadison–Kaplansky counterexample · Irrational-trace Baum–Connes counterexample

286. **Arithmetic rigidity of lattice von Neumann algebras.** Classifies finite-index bimodules between scalar-twisted group factors of ICC groups commensurable with property-(T) lattices over characteristic-zero local fields and arbitrary ICC group factors. Every such bimodule is a summand of finite sums of models arising from finite-index subgroup isomorphisms and finite-dimensional projective representations. The classification also recovers the group, scalar cocycle and amplification scale up to the stated stable equivalence.
Twisted finite correspondences · Arithmetic category and stable recovery

287. **All nonabelian free group factors are isomorphic.** Resolves the free group factor isomorphism problem: $L(\mathbb{F}_2) \cong L(\mathbb{F}_3)$, and hence all interpolated free group factors, including $L(\mathbb{F}_\infty)$, are isomorphic. Their common factor has fundamental group $\mathbb{R}_{>0}$.
An isomorphism of the free group factors

288. **Kadison's similarity conjecture.** Proves Kadison's similarity conjecture: every bounded complex-linear unital algebra homomorphism from a unital complex C^*-algebra to operators on a Hilbert space becomes a $*$-homomorphism after conjugation by a bounded invertible operator.
Kadison's similarity theorem through uniform derivation estimates

289. **The strong Kadison–Kastler conjecture.** Proves that sufficiently close unital von Neumann algebras on the same Hilbert space are conjugate by a unitary arbitrarily close to the identity, with a universal tolerance in the operator-norm distance between unit balls. Counterexamples show that near-identity conjugacy fails for one-sided near inclusions, and that arbitrarily close norm-separable C^*-algebras need not be ambiently unitarily conjugate.
Universal strong Kadison–Kastler stability · Near Inclusions of von Neumann Algebras Without Small Spatial Embeddings · Close Separable C^*-Algebras Without Spatial Conjugacy

290. **Connes' bicentralizer conjecture and relative bicentralizers.** Proves Connes' bicentralizer conjecture for every type III_1 factor with separable predual and every faithful normal state. More generally, for every inclusion $N \subset M$ of von Neumann algebras with separable preduals admitting a faithful normal conditional expectation, constructs an amenable expected subalgebra $P \subset N$ with $P' \cap c(M) = N' \cap c(M)$, resolving the relative bicentralizer conjecture.
Core-commutant preservation · Modular spectral averages

291. **Toms–Winter and equivariant Jiang–Su stability.** Proves equivariant Jiang–Su stability for every countable discrete amenable group action on a simple separable unital infinite-dimensional nuclear stably finite Jiang–Su-stable C^*-algebra, resolving this case of Szabó's conjecture without restrictions on trace dynamics. The family also proves the unital Toms–Winter conjecture, equating strict comparison, finite nuclear dimension and Jiang–Su stability in the simple separable unital infinite-dimensional nuclear setting.
Equivariant Jiang–Su Stability for Amenable Actions in the Unital Stably Finite Case · Cuntz comparison and absorption · Nuclear dimension and Jiang–Su stability · Uniform property Γ

292. **A counterexample to Kirchberg's norm-ultrapower embedding problem.** Constructs an explicit separable unital full group C^*-algebra that cannot embed unitally into the norm ultrapower of any fixed nonzero unital nuclear C^*-algebra, for any free ultrafilter on the natural numbers. Taking the target to be $\mathcal{O}_2$ answers Kirchberg's norm-ultrapower embedding problem negatively.
An explicit obstruction to nuclear norm-ultrapower embeddings

293. **A counterexample to the hyperinvariant-subspace problem.** Constructs a nonzero norm-quasinilpotent operator on every infinite-dimensional separable complex Hilbert space with no nonzero proper closed subspace invariant under every commuting operator. The construction also gives operators with no nontrivial invariant projection in the hyperfinite type II_1 factor.
Invariant-projection counterexamples for every irrational rotation · Backward intertwiners and a transitive commutant

294. **A counterexample to Kaplansky's quasitrace conjecture.** Disproves Kaplansky's quasitrace conjecture by constructing a separable unital complex C^*-algebra admitting normalized 2-quasitraces, all of which are nonadditive. As a consequence, two unital simple stably finite C^*-algebras can have a properly infinite minimal tensor product, with one factor $C^*_r(\mathbb{F}_2)$.
A counterexample to Kaplansky's quasitrace conjecture and failure of tensor-product stable finiteness

295. **The Kadison–Ringrose cohomology conjecture.** Proves that every bounded Hochschild cocycle of degree at least two on a complex von Neumann algebra, with coefficients in the algebra itself, has a bounded primitive. Equivalently, all higher bounded Hochschild cohomology groups vanish, resolving the Kadison–Ringrose conjecture.
Vanishing of higher bounded Hochschild cohomology

296. **The generator problem for finite factors.** Proves that every type II_1 factor with separable predual is generated by a single operator, equivalently by two self-adjoint operators, resolving the generator problem. More strongly, for every irreducible inclusion $P \subset M$ of such factors, the unitaries u with $M = W^*(P, u)$ form a dense G_δ subset in the trace 2-norm topology.
Relative generation and the generator problem for finite factors

297. **A ZFC counterexample to Naimark's problem.** Gives an alternative to Tanaka's ZFC construction of a unital infinite-dimensional simple complex C^*-algebra with a faithful tracial state and exactly one nonzero irreducible representation up to unitary equivalence. Thus the unrestricted compact-operator characterization fails without additional set-theoretic assumptions; the counterexample is nonseparable.
A counterexample to Naimark's problem in ZFC

298. A counterexample to Voiculescu's free-entropy equality conjecture. Constructs a bounded self-adjoint tuple in a tracial von Neumann algebra whose microstates and nonmicrostates free entropies satisfy $-\infty < \chi < \chi^* < \infty$. This answers Voiculescu's finite-entropy equality question negatively: the matrix-approximation and free-Fisher-information definitions differ even when both are finite.
A finite-entropy separation of microstates and nonmicrostates free entropy

299. The Kirchberg–Rørdam character criterion. A nonzero unital separable complex C^*-algebra is Jiang–Su stable exactly when its norm central-sequence algebra has no characters, for every free ultrafilter. This answers the Kirchberg–Rørdam character question. Also, the infinite minimal tensor power of every such algebra without characters is Jiang–Su stable, answering the Dadarlat–Toms question.
The Kirchberg–Rørdam character criterion

300. The Popa–Vaes quadratic strong-operator paving conjecture. Proves that every self-adjoint element of a complex von Neumann algebra admits strong-operator paving relative to any maximal abelian subalgebra with $O(\varepsilon^{-2})$ blocks. The norm bound holds after compression by a projection arbitrarily close to the identity in the strong topology, resolving the Popa–Vaes quadratic paving conjecture.
Approximation paving · Quadratic strong-operator paving

301. Classification by trace cones after Razak–Jacelon stabilization. Classifies separable nuclear complex C^*-algebras after tensoring with the Razak–Jacelon algebra and the compact operators, using their full topological cones of extended lower-semicontinuous tracial weights. This answers Robert's trace-cone question, including algebras with arbitrary ideal structure and both finite and infinite subquotients.
The trace cone classifies Razak–Jacelon stabilizations

302. The Phillips–Toms formula for minimal integer actions. For every minimal homeomorphism h of an infinite compact metrizable space X, the radius of comparison of $C(X) \rtimes_h \mathbb{Z}$ equals $\frac{1}{2}\mathrm{mdim}(X,h)$, including infinite values. Zero mean dimension is equivalent to the small boundary property, Jiang–Su stability and finite nuclear dimension; in this case nuclear dimension is at most one.
Boundary-preserving cobordism compression · Radius = $\frac{1}{2}$ mean dimension

303. From ordinary to strong pure infiniteness. Resolves the ordinary-to-strong pure-infiniteness question of Kirchberg and Rørdam for complex C^*-algebras. For exact algebras, proper infiniteness of one fixed finite amplification of every positive element also suffices. Consequently, every separable nuclear algebra with this property absorbs $\mathcal{O}_\infty$, without unitality or simplicity assumptions.
Weak pure infiniteness and O-infinity absorption

Topology

304. The Hilbert–Smith conjecture in every dimension. Every locally compact second-countable Hausdorff group acting faithfully and jointly continuously on a connected finite-dimensional topological manifold is a Lie group. This proves the Hilbert–Smith conjecture in all finite dimensions, for Hausdorff second-countable manifolds without boundary.
The Hilbert–Smith conjecture in every finite dimension

305. Counterexamples to disk embedding and Wall's manifold conjecture. The unrestricted four-dimensional disk-embedding conjecture fails: framed algebraic dual spheres do not suffice to obtain disjoint locally flat spanning disks. In particular, the free group F_2 is not good in the sense of Freedman–Quinn. Also constructs a finitely presented integral Poincaré duality group of dimension four with a finite classifying space but no realization as the fundamental group of a closed aspherical topological four-manifold, disproving Wall's conjecture.
Boundary-only obstruction · Marked tensor obstruction · PD4 group without aspherical manifold model

306. The purely cosmetic surgery conjecture for knots in S^3. Distinct Dehn surgery slopes on a non-trivial smooth knot in S^3 never produce orientation-preservingly homeomorphic manifolds, proving the purely cosmetic surgery conjecture. The statement includes the meridional slope.
Purely cosmetic surgery on knots in the three-sphere

307. Failure of rational injectivity for maximal coarse assembly. Constructs a uniformly discrete bounded-geometry space whose maximal coarse assembly map is not rationally injective. The example is a coarse disjoint union of finite connected graphs of uniformly bounded degree, with an infinite-order kernel class. A companion gives the analogous failure for reduced coarse assembly, disproving the rational coarse Novikov conjecture.
Failure of rational injectivity for maximal coarse assembly · A counterexample to the coarse Novikov conjecture

308. Smith–Toda complexes at every height with varying primes. For every $n \geq 0$, constructs a finite Smith–Toda spectrum at a prime p depending on n, with Brown–Peterson homology $BP_*/(p, v_1, \dots, v_n)$ and the canonical comodule structure. Thus Smith–Toda complexes exist at every height when the prime may vary; an explicit example realizes $V(4)$ at $p = 1009$.
Varying primes · V(4) at prime 1009

309. The Kervaire invariant problem at the prime three. Resolves the odd-primary Kervaire invariant problem at the prime three: exactly the standard classes with indices 0, 2, and 3 survive in the mod-three Adams spectral sequence, in stems 10, 106, and 322. Each surviving detection coset contains an element of exact additive order three.
The Kervaire invariant problem at the prime three

310. Quillen's conjecture in rational homology. Proves the rational-homology form of Quillen's conjecture for every finite group and every prime. If the largest normal p-subgroup of G is trivial, the poset of nontrivial elementary abelian p-subgroups has nonzero augmented reduced rational homology and is therefore not contractible.
Rational homology and Quillen's conjecture

311. Chai's invariant-ideal conjecture. Proves Chai's invariant-ideal conjecture for Lubin–Tate deformation rings over finite residue fields, at every prime and positive height n. Through the implication of Barthel–Heard–Naumann, this proves the Hovey–Strickland conjecture: dualizable $K(n)$-local spectra have exactly $n + 2$ thick tensor ideals, and their Balmer spectrum is a chain of $n + 1$ points.
Stabilizer orbits and thick tensor ideals of dualizable K(n)-local spectra

312. The Grothendieck homotopy hypothesis. Proves the Grothendieck homotopy hypothesis for ∞-groupoids associated with every Grothendieck coherator in the Ara–Henry convention: these algebraic objects recover the homotopy theory of spaces.
The Grothendieck homotopy hypothesis via elementary expansions

313. Finite generation for the $K(n)$-local sphere. Answers the degreewise finiteness question of Hovey and Hovey–Strickland: every homotopy group of the $K(n)$-local sphere is a finitely generated $\mathbb{Z}_p$-module, for every prime, positive height and integer degree. The same conclusion holds after $K(n)$-localizing any finite p-local spectrum.
Finite generation for the K(n)-local sphere

314. The chromatic Smith fixed-point problem for finite p-groups. Determines the optimal chromatic loss from geometric H-fixed points to geometric G-fixed points for every subgroup H of a finite p-group G. At every nonnegative height, the loss equals the shortest subnormal-chain length from H to G with cyclic quotients. Each quotient counts once regardless of order, and finite spectra witness sharpness.
Cyclic length and chromatic fixed-point loss

315. The four-dimensional Singer conjecture. Proves that the L^2-Betti numbers of the universal cover of every closed connected aspherical topological four-manifold vanish outside degree two. More generally, the same conclusion holds for every finite connected aspherical integral Poincaré complex of formal dimension four, proving the four-dimensional Singer conjecture in this wider class.
The Singer conjecture in dimension four

316. Curtis's conjecture. Proves Curtis's conjecture: the positive-degree mod-two stable Hurewicz image of the sphere is spanned by the images of the Hopf-invariant-one classes η, ν, σ and the Kervaire-invariant-one classes that exist.
The Stable Hurewicz Image of the Sphere at Two

317. Thomason model structures in every strict higher dimension. Resolves the Ara–Maltsiniotis conjecture: for every $n \geq 1$ and $n = \omega$, small strict globular n-categories admit proper combinatorial Thomason model structures Quillen equivalent to simplicial sets. Thus strict higher categories model the homotopy theory of spaces in every stated dimension.
Thomason Model Structures in Every Strict Higher Dimension

318. Chromatic splitting: counterexamples and filtrations. Disproves strong chromatic splitting at height three for primes $p \geq 5$, and weak splitting for the derived p-completed sphere at heights p ($p \geq 5$) and $p + 1$ ($p \geq 7$). Nevertheless, for $n \geq 1$ and $p > n + 1$, the overlap $L_{n-1}L_{K(n)}S^\wedge_p$ admits a 2^n-stage filtration by the predicted localized-sphere pieces. At height three and prime three, even finite assembly from such pieces fails in the category of $E(2)$-local modules over the derived completed sphere.
Filtered splitting at generic primes · Height-three filtration and attachments · Rational obstruction to strong splitting · Finite-assembly failure at prime three · Weak chromatic splitting counterexamples

319. Counterexamples to the Hahn–Wilson conjecture at height two. Refutes the Hahn–Wilson conjecture at chromatic height two. For every sufficiently large prime p, constructs a connective p-complete spectrum of exact fp-type two that cannot be built from completed $\mathrm{BP}\langle 2\rangle$ by finitely many sums, shifts, cones and retracts. The examples nevertheless satisfy the finite and telescopic localization comparisons.
Counterexamples to the Hahn-Wilson conjecture at height two

320. A four-dimensional counterexample to Borel rigidity. Constructs closed connected aspherical topological four-manifolds that are homotopy equivalent but not homeomorphic, with a common word-hyperbolic fundamental group. This disproves even the homeomorphism-existence formulation of the Borel conjecture in dimension four.
Nonhomeomorphic closed aspherical four-manifolds with the same homotopy type

321. A counterexample to Wall's finite $\mathrm{D}(2)$ conjecture. Constructs a finite connected three-dimensional CW complex whose universal cover has no integral homology above degree two and whose third cohomology vanishes for every local coefficient module, but which has no finite two-dimensional homotopy model. This disproves Wall's finite $\mathrm{D}(2)$ conjecture; the example has infinite fundamental group.
A Counterexample to Wall's D(2) Problem

Functional analysis

322. Tingley's sphere-isometry problem. Resolves Tingley's problem: every surjective isometry between the unit spheres of nonzero real Banach spaces extends uniquely to a surjective real-linear isometry of the whole spaces. No dimension restriction is imposed, so the metric geometry of the unit sphere determines the Banach space up to linear isometry.
A positive solution to Tingley's problem

323. Relative independence of the separable quotient problem. Establishes, relative to the consistency of a measurable cardinal, that the separable quotient problem is independent of ZFC. The assertion that every infinite-dimensional Banach space has a separable infinite-dimensional quotient can hold for all real and complex Banach spaces, whereas the continuum hypothesis yields counterexamples over both fields.
Relative independence of the separable quotient problem

324. Lipschitz equivalence without linear isomorphism. Constructs separable real Banach spaces that are globally bi-Lipschitz equivalent but not linearly isomorphic, resolving the separable Lipschitz-isomorphism problem negatively. Thus even the complete metric structure up to bi-Lipschitz equivalence does not determine a separable Banach space's linear isomorphism class.
Lipschitz equivalence without linear isomorphism · Bi-Lipschitz c_0 absorption

325. The complete Crouzeix conjecture. Resolves the complete Crouzeix conjecture: for every bounded operator A on a complex Hilbert space and every finite matrix-valued polynomial P, one has $\|P[A]\| \le 2\sup_{z\in W(A)}\|P(z)\|$, where $W(A)$ is the numerical range. The constant 2 is sharp, independent of the matrix size, and valid in infinite dimensions.
Direct proof · Structural proof

326. The cotype–cotype conjecture under the approximation property. Resolves the cotype–cotype conjecture for real Banach spaces with the approximation property. Such a nonzero space is K-convex if and only if both it and its dual have finite Rademacher cotype, with possibly different exponents. Equivalently, these cotype assumptions force nontrivial Rademacher type.
The cotype–cotype conjecture under the approximation property

327. Markov type characterizes superreflexivity. Proves that every real Banach space with Markov type p for some $p > 1$ admits an equivalent uniformly convex norm, answering Naor's renorming question. Together with the known converse, this characterizes superreflexivity by nontrivial Markov type.
Nontrivial Markov Type Forces Superreflexivity

328. Fixed points of nonexpansive maps in reflexive Banach spaces. Resolves Kirk's reflexive-space fixed-point problem: every nonexpansive selfmap of a nonempty closed bounded convex subset of a real reflexive Banach space has a fixed point. The result uses the original norm, without assuming uniform convexity.
Fixed Points of Nonexpansive Maps in Reflexive Banach Spaces

329. **A counterexample to Pietsch's metric-entropy duality conjecture.** Disproves Pietsch's dimension-free duality conjecture for metric entropy. Origin-symmetric convex bodies violate every proposed choice of universal constants in the conjectured comparison between covering numbers and those of the polar bodies, even when the covering body is a cube.
Counterexamples to the duality conjecture for metric entropy

330. **A negative answer to Kalton's Lipschitz-free approximation question.** Constructs a countable uniformly discrete metric space whose real Lipschitz-free Banach space has the approximation property but not the bounded approximation property, answering Kalton's question negatively. Finite-rank operators approximate the identity on every compact set, but their norms cannot share a finite bound.
A uniformly discrete counterexample to bounded approximation in Lipschitz-free spaces

331. **Asymptotic midpoint uniform convexity without asymptotically uniformly convex renorming.** Constructs a real reflexive Banach space with an asymptotically midpoint uniformly convex norm but no equivalent asymptotically uniformly convex norm, extending Baudier's separation to reflexive spaces. In the same space, depth-k countably branching diamonds require distortion at least $\sqrt{1+k/12}$, so midpoint uniform convexity does not force uniformly bounded diamond distortion even under reflexivity.
Reflexive tree-space example · Midpoint lenses · Midpoint and tree energies · Daugavet subspace moduli · Bounded tree potentials · Independent products in real L1 · Two recursive potentials

332. **Metric Markov cotype two for ℓ_1.** Proves that real ℓ_1 has metric Markov cotype two, answering Mendel and Naor's question. Consequently, every Lipschitz map from an arbitrary subset of a real Hilbert space into ℓ_1 extends to the whole space with a universal multiplicative loss in its Lipschitz constant, resolving Ball's extension problem for this target.
Metric Markov Cotype Two of ℓ_1

Differential geometry

333. **Smooth isometric immersions of surfaces into $\mathbb{R}^4$.** Every closed smooth Riemannian surface admits a smooth isometric immersion into $\mathbb{R}^4$, resolving the closed-surface form of the four-dimensional isometric-immersion problem. This includes nonorientable surfaces and metrics of arbitrary Gaussian curvature.
Smooth isometric immersions of closed surfaces into Euclidean four-space

334. **A smooth surface metric with no local immersion in $\mathbb{R}^3$.** Constructs a smooth positive-definite metric on $(-1,1)^2$ for which no neighborhood of the origin admits a smooth isometric immersion into $\mathbb{R}^3$. This answers the unrestricted smooth local isometric realization problem for surfaces negatively, even after shrinking the neighborhood.
A Smooth Metric with No Local Isometric Immersion into Three-Space

335. **Gromov's integral scalar-curvature inequality.** Proves $\int_M (\mathrm{Scal}_g^-)^{n/2}\,dV_g \geq a_n\|M\|$ for every closed connected oriented smooth n-manifold, $n \geq 3$, and every smooth metric, with $a_n > 0$ depending only on dimension. Here $\mathrm{Scal}_g^- = \max\{0, -\mathrm{Scal}_g\}$ and $\|M\|$ is real simplicial volume. Also proves rational inessentiality under positive scalar curvature, resolving the Gromov–Lawson conjecture; every nonnegative-scalar-curvature metric on a closed aspherical manifold is flat.
An integral scalar curvature bound for real simplicial volume · Positive scalar curvature forces rational inessentiality

336. **Spectral scalar curvature and codimension-two width.** Every complete connected smooth boundaryless n-manifold, $n \geq 3$, satisfying $-4\Delta + \mathrm{Scal} \geq 1$ as a quadratic-form inequality admits a continuous map to a simplicial complex of dimension at most $n-2$ whose entire fibers have diameter bounded only by n in the original metric. This strengthens Gromov's width conclusion to spectral scalar curvature. Universal covers of closed positive-scalar-curvature manifolds also have continuous macroscopic dimension at most $n-2$ for every $n \geq 2$.
Spectral scalar curvature and uniform Urysohn width · Spectral scalar curvature and Urysohn width in dimension three · Positive scalar curvature and uniform codimension-two width

337. **Cartan–Hadamard isoperimetry and CAT(0) fillings.** Proves generalized Cartan–Hadamard isoperimetry in every dimension: in a complete simply connected manifold with sectional curvature at most $\kappa \leq 0$, every finite-volume finite-perimeter set satisfies the sharp comparison with the equal-volume model ball. Bounded positive-volume equality regions for $\kappa = 0$ are Euclidean balls. Also proves sharp Euclidean filling bounds for compactly supported integral n-cycles, $n \geq 2$, in arbitrary proper CAT(0) spaces.
Cartan–Hadamard isoperimetry and rigidity · Sharp integral fillings in CAT(0) spaces

338. Yau's uniformization conjecture. Proves Yau's uniformization conjecture: every complete connected noncompact Kähler manifold with strictly positive holomorphic bisectional curvature is biholomorphic to $\mathbb{C}^n$.
Uniformization of complete Kähler manifolds with positive bisectional curvature

339. Katok's entropy rigidity conjecture. Proves Katok's entropy rigidity conjecture for closed connected Riemannian manifolds of dimension at least three with strictly negative sectional curvature: normalized Liouville measure maximizes entropy for the geodesic flow if and only if the metric is locally symmetric.
Entropy equality and local symmetry in negative curvature

340. A counterexample to the nearby Lagrangian conjecture. Disproves the unrestricted nearby Lagrangian conjecture. For some sufficiently large even N, constructs a closed exact embedded Lagrangian in $T^*(S^9 \times S^{N-1})$ that is diffeomorphic to the base but not Hamiltonian isotopic to its zero section.
A counterexample to the nearby Lagrangian conjecture

341. Donaldson's hypersymplectic deformation conjecture. Proves Donaldson's hypersymplectic deformation conjecture in a cohomology-preserving form. Every positive triple of smooth closed two-forms on a closed connected oriented four-manifold, normalized by $\int \omega_i \wedge \omega_j = \delta_{ij}$, deforms through positive closed triples to a hyperkähler triple while preserving all three cohomology classes. Any positive triple can first be normalized by a constant linear change.
Deforming hypersymplectic four-manifolds to hyperkähler triples

342. Donaldson's tamed-to-compatible conjecture. Proves Donaldson's tamed-to-compatible conjecture: every smooth almost complex structure on a closed four-manifold that is tamed by a symplectic form admits a compatible symplectic form. The almost complex structure stays fixed; the form's cohomology class may change.
Taming implies compatibility on four-manifolds

343. Sharp symplectic ball-packing criteria in higher dimensions. Resolves the Siegel–Yao conjecture for arbitrary capacities in every dimension $2n \geq 6$. Finitely many closed symplectic balls of capacities $R_1, \dots, R_k$ embed disjointly into an open ball of capacity R exactly when $\sum_i R_i^n < R^n$ and $R_i + R_j < R$ for every distinct pair i, j.
Symplectic Ball Packings in Higher Dimensions

344. The metric Blaschke conjecture. Proves the metric Blaschke conjecture: every closed connected Riemannian manifold of positive dimension whose injectivity radius equals its diameter is, up to scaling, a standard compact rank-one symmetric space.
The metric Blaschke theorem

345. Infinitely many closed geodesics on spheres and three-manifolds. Proves that every smooth Riemannian metric on S^n, $n \geq 2$, has infinitely many prime closed geodesics with pairwise distinct images. The same conclusion holds on every closed manifold admitting a finite smooth spherical cover and on every closed three-manifold, without orientability or nondegeneracy restrictions.
Infinitely many closed geodesic images on every Riemannian sphere

346. Sharp singular-set bounds for stationary integral varifolds. Proves that every stationary integral m-varifold in a Euclidean open set has singular set of Hausdorff dimension at most $m - 1$, a sharp bound in every positive dimension and codimension. On round spheres, the family also proves almost-everywhere regularity: the singular set has zero m-dimensional Hausdorff measure.
A Codimension-One Bound for the Singular Set of a Stationary Integral Varifold · Almost-everywhere regularity of stationary integral varifolds

347. Counterexamples to strong forms of Arnold's fixed-point conjecture. Disproves stable-Morse lower bounds for nondegenerate Hamiltonian fixed points: on simply connected closed Kähler manifolds of real dimension 22, the deficit below the stable Morse number is unbounded. A separate Hamiltonian diffeomorphism of the complex quadric threefold has exactly three fixed points, fewer than the four critical points required of every smooth function.
Hamiltonian Fixed Points Below the Stable Morse Number in Dimension Twenty-Two · Sharpness of the Cyclic Integral Floer Bound Below the Stable Morse Number · Hamiltonian Fixed Points Below the Stable Morse Number · Nondegenerate Morse-number counterexample · Three fixed points on the symplectic quadric threefold

348. **Nonnegatively curved Einstein four-manifolds and a topological gap.** Classifies closed connected Einstein four-manifolds with positive Einstein constant and nonnegative sectional curvature: up to scaling, their universal Riemannian covers are the round S^4, Fubini–Study $\mathbb{CP}^2$, or a product of equal round two-spheres. A closed simply connected nonnegatively curved four-manifold is diffeomorphic to one of these whenever the scale-invariant L^2 norm of its trace-free Ricci curvature lies below a universal positive constant.
Zero-Plane Rigidity for Einstein Four-Manifolds · An L^2 Einstein Gap for Nonnegatively Curved Four-Manifolds · Positively curved Einstein four-manifolds

349. **The Solomon–Yau least-volume conjecture.** Proves the Solomon–Yau least-volume conjecture for minimal hypersurfaces of round spheres. For every $m \geq 2$, a closed connected minimal immersion into the unit sphere S^{m+1} with non-totally-geodesic image has volume at least that of the smallest minimal Clifford product, counting covering multiplicity.
The Solomon–Yau least-volume theorem

350. **Yau's nodal conjecture: surfaces and counterexamples.** Proves the sharp $C\sqrt{\lambda}$ upper bound for nodal length on every fixed smooth closed surface, completing Yau's conjecture there. The upper bound fails for fixed smooth metrics in dimensions three and four, including metrics on S^3 arbitrarily close to round. In dimension five, nodal measure can grow faster than $\lambda^{1/2+\varepsilon_0}$ for some fixed $\varepsilon_0 > 0$, ruling out even arbitrarily small power losses.
Sharp surface nodal length · Smooth upper-bound counterexamples in dimensions 3–4 · Power-law violations of Yau's upper bound

351. **Scalar curvature detects four-dimensional Ricci-flow singularities.** Proves that a smooth Ricci flow on a closed four-manifold extends past any finite time at which scalar curvature remains uniformly bounded. A higher-dimensional counterexample has bounded scalar curvature but unbounded full curvature at its finite maximal time, disproving the unrestricted scalar-curvature extension conjecture.
Finite-time curvature blowup · Four-dimensional smooth extension · Ricci-flat tree inequality

352. **Finite-time singularity of Calabi flow.** Disproves Chen's smooth long-time existence conjecture for Calabi flow by constructing a smooth $U(10)$-invariant Kähler metric on $\mathbb{CP}^{10}$ whose flow develops a finite-time singularity. The metric lies in the Fubini–Study class, so the failure occurs even in a class containing a constant-scalar-curvature metric.
A finite-time singularity of Calabi flow on projective space

353. **The sharp dimension threshold for affine Bernstein rigidity.** Proves that every smooth locally uniformly convex affine-maximal graph of dimension three through nine, complete for its induced Euclidean metric, is an elliptic paraboloid. A smooth entire nonquadratic example in dimension ten makes this range sharp. In dimensions three through nine, the paraboloid classification also holds for connected open locally uniformly convex affine-maximal hypersurfaces complete for the affine Berwald–Blaschke metric.
A Smooth Nonquadratic Entire Affine Maximal Graph in Dimension Ten · The affine Bernstein theorem in dimensions three through nine

354. **Isoperimetric regions in the cubic three-torus.** Determines the isoperimetric profile of the unit cubic flat three-torus and classifies every finite-perimeter minimizer: balls, circular tubes around shortest closed geodesics, coordinate slabs, and their complements. The transition volumes are $4\pi/81$ and $1/\pi$, with exactly the adjacent two types minimizing at each transition.
The Isoperimetric Conjecture for the Cubic Flat Three-Torus

355. **Unique tangent flows at the first surface singularity.** Proves the first-singular-time case of tangent-flow uniqueness for smooth compact connected embedded surfaces without boundary in $\mathbb{R}^3$. At every singular point, all fixed-center backward tangent flows agree as area measures at every negative time in the original ambient coordinates, without mean-convexity or a prescribed tangent model.
Uniqueness of tangent flows at the first singular time of embedded surface mean-curvature flow

356. **Gigli's characterization of Alexandrov curvature.** Proves Gigli's conjecture: in every integer dimension $n \geq 2$, Alexandrov curvature at least κ is characterized by the full-support $\mathrm{RCD}((n-1)\kappa, n)$ condition with reference measure $\mathcal{H}^n$ and distributional sectional curvature at least κ in the original global test classes. The RCD condition is unreduced.
Distributional curvature characterization · Weak Hessian bounds

357. **Bi-Lipschitz coordinates at every regular RCD point.** Proves that every regular point of a noncollapsed $\mathrm{RCD}(K, n)$ space, for $K \in \mathbb{R}$ and integer $n \geq 2$, has an open neighborhood bi-Lipschitz to an open subset of $\mathbb{R}^n$. Regularity requires all pointed tangents to be Euclidean, the reference measure is exactly $\mathcal{H}^n$, and the chart compares ambient distances with a point-dependent finite constant.
Bi-Lipschitz Coordinates at Regular Points of Noncollapsed RCD Spaces

358. A three-manifold with no conjugate points and no nonpositively curved metric. Constructs a closed connected orientable smooth three-manifold that admits a metric without conjugate points but no metric of nonpositive sectional curvature. This answers negatively, already in dimension three, whether the first metric-existence property implies the second.
A Three-Manifold Without Conjugate Points and Without a Nonpositively Curved Metric

359. Negative Kähler curvature without bounded holomorphic coordinates. Constructs a contractible domain in $\mathbb{C}^3$ with a complete negatively pinched Kähler metric but no bounded holomorphic coordinates, disproving bounded-domain uniformization in this setting. A higher-dimensional example has sectional curvature at most -1 and only constant bounded holomorphic functions; its curvature is not bounded below.
Negatively pinched Kähler threefold · Holomorphic Liouville property

360. Villani's convexity conjecture and regular optimal transport. On every closed connected Riemannian manifold of dimension at least two satisfying weak Ma–Trudinger–Wang curvature, all tangent injectivity domains are convex, resolving Villani's conjecture in this setting. For squared-distance transport between measurable probability densities bounded above and away from zero, the optimal map and its inverse are Hölder continuous.
Support and convex injectivity domains · Uniform bi-Hölder transport

361. Counterexamples to Yau's harmonic dimension bound. Disproves Yau's proposed Euclidean dimension bound for harmonic functions of integer growth on manifolds with nonnegative Ricci curvature. For every sufficiently large integer k, a complete smooth metric on $\mathbb{R}^3$ has at least $(k+2)^2$ independent harmonic functions of growth at most k, exceeding the Euclidean count $(k+1)^2$. The metric may depend on k.
Even-dimensional counterexample · Three-dimensional counterexample

Partial differential equations

362. Global smoothness for relativistic Vlasov–Maxwell. Proves large-data global existence and uniqueness for the three-dimensional, one-species relativistic Vlasov–Maxwell system. Smooth admissible initial data may be arbitrary provided the particle density is compactly supported and the electromagnetic fields have finite energy and bounded derivatives of every order; the solution remains smooth on every finite time interval.
Global classical solutions of the three-dimensional relativistic Vlasov–Maxwell system

363. Boltzmann nonuniqueness with exact local conservation. Constructs two distinct global entropy solutions of the three-dimensional periodic hard-sphere Boltzmann equation from the same nonnegative initial density, with bounded velocity support and finite mass, energy and absolute entropy. Both are strongly continuous in L^1 and satisfy exact local conservation of mass, momentum and kinetic energy.
Nonuniqueness with local conservation for the hard-sphere Boltzmann equation · Nonuniqueness for the periodic hard-sphere Boltzmann equation

364. Kinetic limits and fluctuations over the Boltzmann lifespan. Derives the nonlinear Boltzmann equation from three-dimensional grand-canonical Newtonian gases throughout every regular kinetic interval with uniform Gaussian decay. Stable finite-range radial potentials may have attractive wells and a singular repulsive core; initial pair exclusion and spatially summable Gaussian density and gradient bounds are assumed. A companion gives finite-dimensional hard-sphere Gaussian fluctuations, centered at the exact microscopic expectation and governed by the linear fluctuating Boltzmann equation.
Boltzmann–Grad limit for stable radial potentials · Hard-sphere fluctuations

365. Joint metric and connection recovery from one boundary patch. Zero-frequency measurements on any nonempty open boundary patch determine a smooth metric and smooth unitary connection on a trivial Hermitian rank-two bundle over a compact connected manifold of dimension at least three, up to diffeomorphism and gauge fixed on that patch. Inputs and observations use the same patch. In contrast, distinct uniformly positive bounded measurable scalar conductivities on a three-dimensional ball can have identical full-boundary data.
Determination of a metric and a unitary connection from one boundary patch · Smooth anisotropic uniqueness · Measurable scalar nonuniqueness

366. The planar Mumford–Shah regularity conjecture. Resolves the interior regularity conjecture for reduced absolute planar Mumford–Shah minimizers with bounded fidelity data. Locally, the closed discontinuity set is a $C^{1,\alpha}$ arc, a regular crack tip, or three arcs meeting at 120°; only finitely many global connected components meet any compact interior region.
Interior regularity of planar Mumford–Shah minimizers

367. The critical dimension for the one-phase Bernoulli problem. Establishes seven as the first dimension admitting a nonflat, one-homogeneous global minimizer of the one-phase Bernoulli energy. Consequently, minimizing free boundaries are smooth through dimension six, and their singular sets have dimension at most $n-7$ in higher dimensions.
The critical dimension for one-phase Bernoulli minimizers

368. The three-dimensional Ball–Evans approximation problem. Resolves the three-dimensional Ball–Evans approximation problem: every $W^{1,p}$ homeomorphism between arbitrary bounded domains in $\mathbb{R}^3$, for $1 \le p < \infty$, is a strong $W^{1,p}$ limit of smooth diffeomorphisms onto the same target.
Diffeomorphic approximation, $1 \le p \le 2$ · Diffeomorphic approximation, $p > 2$

369. The hot spots conjecture for simply connected planar domains. Proves a strict form of Burdzy's simply connected hot spots conjecture. On every smooth bounded simply connected planar domain, each nonzero eigenfunction for the first positive Neumann eigenvalue has no interior critical point, so all global extrema lie on the boundary. Eigenvalue multiplicity is allowed.
Strict hot spots and absence of interior critical points on smooth simply connected planar domains

370. The subcritical Lane–Emden and Hénon–Lane–Emden conjectures. Resolves the subcritical Lane–Emden conjecture and its weighted Hénon extension. For $n \ge 2$, $p, q > 0$ and real A, B, the system $-\Delta u = |x|^A v^p$, $-\Delta v = |x|^B u^q$ has no positive entire solution when $(n+A)/(p+1) + (n+B)/(q+1) > n-2$, with solutions continuous at the origin and classical elsewhere. No symmetry or growth assumption is needed. Known radial existence gives the exact existence criterion for $n \ge 3$ and $A, B > -2$.
The Subcritical Hénon–Lane–Emden Conjecture

371. Stable blowup for a defocusing Schrödinger equation. For a sufficiently large odd nonlinearity power, constructs a nonempty open set of initial data in $H^k(\mathbb{T}^{12})$, with $k > 8$, whose solutions of the scalar defocusing nonlinear Schrödinger equation blow up in finite time. Thus finite-time blowup is stable under Sobolev perturbations in this supercritical regime.
Stable self-similar blowup for a supercritical defocusing Schrödinger equation on the torus

372. Global uniqueness in smooth isotropic elasticity. Proves that full static boundary displacement-to-traction data determine both smooth real Lamé moduli on every bounded connected smooth domain in $\mathbb{R}^3$, provided $\mu > 0$ and $3\lambda + 2\mu > 0$ on the closure. Neither analyticity, proximity to constant coefficients nor prior knowledge near the boundary is required.
Global Uniqueness for the Smooth Isotropic Elasticity Inverse Problem

373. Nonattainment in three-marginal Coulomb transport. An optimal three-particle Coulomb configuration need not be a deterministic function of the first particle, even for smooth identical spatial densities. The Monge and Kantorovich infima agree, but the Monge infimum is not attained. The same phenomenon occurs for every inverse-power Riesz exponent in each dimension at least two, with a suitable density in each case.
A counterexample to the Monge ansatz for the three-marginal Coulomb cost

374. Sharp one-third stability of optimal transport maps. For uniform source measure ρ on a compact convex body with interior in dimension at least two, quadratic optimal transport maps satisfy $\|T_\mu - T_\nu\|_{L^2(\rho)} \le C W_2(\mu,\nu)^{1/3}$ uniformly over targets in a fixed compact set. The exponent is sharp, even for three-atom targets, disproving Letrouit's conjectured square-root bound.
Sharp One-Third Stability of Brenier Maps

375. De Giorgi's conjecture in dimension eight. Proves De Giorgi's conjecture at its sharp dimension-eight endpoint: every entire C^2 solution $u : \mathbb{R}^8 \to (-1,1)$ of $\Delta u = u^3 - u$ that is strictly increasing in one direction depends on only one linear coordinate. A stronger theorem classifies all stable entire solutions $v : \mathbb{R}^7 \to [-1,1]$ as constant wells or planar transitions, without an energy-growth assumption.
A positive resolution of De Giorgi's conjecture in dimension eight

376. **Universal computation in forced Navier–Stokes flows.** Constructs viscous incompressible flows starting from rest on a fixed flat three-dimensional domain that perform universal computation under smooth external forcing. A terminating compiler turns a Turing machine and input into a finite program for the force, so a designated particle reaches a fixed region exactly when the machine halts. The viscosity is fixed, positive and computable.

Finite instructions and shear flows · Fixed particle test · Eventually stationary forcing · Geometric programs for solenoidal forcing · Prefix instructions and incompressible flows · Rapidly vanishing forcing · Velocity-field detection · Scalar potentials and slow clocks · Incompressible box transport

377. **Interior $C^{1,\alpha}$ regularity for infinity-harmonic functions.** Proves uniform interior C^{1,α_d} regularity for bounded infinity-harmonic functions in every dimension $d \geq 3$, for a positive exponent depending only on dimension. The gradient's supremum norm and Hölder seminorm on the half unit ball are bounded by a dimension-dependent constant times the oscillation on the unit ball. The exponent is not explicit.

Uniform Interior $C^{1,\alpha}$ Estimates for Infinity-Harmonic Functions