

The Harnack inequality
and fundamental solution for a class of
sub-elliptic operators
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Abstract

In this seminar some recent results concerning Harnack inequalities will be presented for a class of real second-order PDOs \mathcal{L} in divergence form on \mathbb{R}^N , which comprises sub-Laplacians on Carnot groups, sub-elliptic Laplacians on arbitrary Lie groups, elliptic operators in divergence form, as well as the Laplace-Beltrami operator on Riemannian manifolds. In the first part, we will consider a class of sub-elliptic operators, in divergence form, with low-regular coefficients under global doubling and Poincaré assumptions; for these operators a non-homogeneous invariant Harnack inequality will be shown. As a consequence, we will prove the solvability of the Dirichlet problem (in a suitable weak sense). In the second part, we will consider a class of hypoelliptic non-Hörmander operators for which we have been able to construct a Green function; with a completely different approach with respect to the case of doubling metric spaces, we will conclude by showing (by means of techniques of Potential Theory) how the solvability of the Dirichlet problem has been a fundamental tool in order to prove a homogeneous Harnack inequality in the framework of harmonic spaces.