Decision Problems For Turing Machines

Olivier Finkel
Equipe de Logique Mathématique
CNRS et Université Paris Diderot Paris 7
UFR de Mathématiques case 7012, site Chevaleret,
75205 Paris Cedex 13, France.
finkel@logique.jussieu.fr

Dominique Lecomte
Institut de Mathématiques de Jussieu
Projet Analyse Fonctionnelle
Université Paris 6, tour 46-0, boîte 186,
4, place Jussieu, 75 252 Paris Cedex 05, France.
dominique.lecomte@upmc.fr
and
Université de Picardie, I.U.T. de l'Oise, site de Creil,
13, allée de la faïencerie, 60 107 Creil, France.

Abstract

We answer two questions posed by Castro and Cucker in [CC89], giving the exact complexities of two decision problems about cardinalities of ω-languages of Turing machines. Firstly, it is $D_2(\Sigma_1^1)$-complete to determine whether the ω-language of a given Turing machine is countably infinite, where $D_2(\Sigma_1^1)$ is the class of 2-differences of Σ_1^1-sets. Secondly, it is Σ_1^1-complete to determine whether the ω-language of a given Turing machine is uncountable.

Keywords. Theory of computation; computational complexity; formal languages; ω-languages; Turing machines; decision problems; analytical hierarchy.

1 Introduction

Many classical decision problems arise naturally in the fields of Formal Language Theory and of Automata Theory.
Castro and Cucker studied decision problems for ω-languages of Turing machines in [CC89]. Their motivation was, on the one hand, to classify problems about Turing machines and, on the other hand, to "give natural complete problems for the lowest levels of the analytical hierarchy which

constitute an analog of the classical complete problems given in recursion theory for the arithmetical hierarchy." They studied the degrees of many classical decision problems like : "Is the ω-language recognized by a given machine non empty ?", "Is it finite ?" "Do two given machines recognize the same ω-language ?" In particular, they proved that the non-emptiness and the infiniteness problems for ω-languages of Turing machines are Σ_1^1-complete, and that the universality problem, the inclusion problem, and the equivalence problem are Π_2^1-complete. Thus these problems are located at the first or the second level of the analytical hierarchy and are "highly undecidable."

Notice that Staiger studied also in [Sta93] the verification property, which is in fact the inclusion problem, for many classes of ω-languages located at one of the first three levels of the arithmetical hierarchy. Cenzer and Remmel studied in [CR03] some decision problems for classes of ω-languages accepted by some computable deterministic automata. These classes of ω-languages are located at one of the first three levels of the arithmetical hierarchy, while the class of ω-languages of Turing machines considered by Castro and Cucker and in this paper is actually the class of effective analytic sets. Thus the class we consider in this paper is much larger than the classes studied by Cenzer and Remmel in [CR03]. Cenzer and Remmel studied also various *approximate* verification properties, determining the index sets for pairs of languages (V, W) such that $W - V$ is finite, is a set of measure zero or contains only finitely many computable sequences. The verification property is also studied by Klarlund in [Kla94].

The following questions were left open by Castro and Cucker in [CC89]. What is the complexity of the following decision problems: "Is the ω-language recognized by a given Turing machine countably infinite ?", "Is the ω-language recognized by a given Turing machine uncountable?"

We answer here these questions, giving the exact complexities of these two decision problems about cardinalities of ω-languages of Turing machines. Firstly, it is $D_2(\Sigma_1^1)$-complete to determine whether the ω-language of a given Turing machine is countably infinite, where $D_2(\Sigma_1^1)$ is the class of 2-differences of Σ_1^1-sets. Secondly, it is Σ_1^1-complete to determine whether the ω-language of a given Turing machine is uncountable.

This can be compared with this corresponding result of [CR03, CR99]. It is Π_3^0-complete to determine whether a given Π_1^0 ω-language is infinite. It is Σ_1^1-complete to determine whether a given Π_1^0 ω-language is uncountable, and it is Π_1^1-complete to determine whether a given Π_1^0 ω-language is countably infinite, see [CR99, Theroem 4.5]. We refer the reader to [CR03] for results about other classes of ω-languages, like the class of Σ_1^0 ω-languages, or the class of Π_2^0 ω-languages.

2 Recall of basic notions

The set of natural numbers is denoted by $\mathbb{N}$. We assume the reader to be familiar with the arithmetical and analytical hierarchies on subsets of $\mathbb{N}$, these notions may be found in the textbooks on computability theory [Rog67] [Odi89, Odi99].
We now recall the notions of 1-reduction and of Σ_n^1-completeness (respectively, Π_n^1-completeness). Given two sets $A, B \subseteq \mathbb{N}$ we say A is 1-reducible to B and write $A \leq_1 B$ if there exists a total computable injective function f from $\mathbb{N}$ to $\mathbb{N}$ with $A = f^{-1}[B]$. A set $A \subseteq \mathbb{N}$ is said to be Σ_n^1-complete (respectively, Π_n^1-complete) iff A is a Σ_n^1-set (respectively, Π_n^1-set) and for each Σ_n^1-set (respectively, Π_n^1-set) $B \subseteq \mathbb{N}$ it holds that $B \leq_1 A$. An important fact is that, for each integer $n \geq 1$, there exist some Σ_n^1-complete subset of $\mathbb{N}$. Examples of such sets are precisely described in [Rog67] or [CC89]. In the sequel E_1 denotes a Σ_1^1-complete subset of $\mathbb{N}$. The set $E_1^- = \mathbb{N} - E_1 \subseteq \mathbb{N}$ is a Π_1^1-complete set.

We assume now the reader to be familiar with the theory of formal (ω)-languages [Tho90, Sta97]. We recall some usual notations of formal language theory.
When Σ is a finite alphabet, a *non-empty finite word* over Σ is any sequence $x = a_1 \ldots a_k$, where $a_i \in \Sigma$ for $i = 1, \ldots, k$, and k is an integer ≥ 1. $\Sigma^\star$ is the *set of finite words* (including the empty word) over Σ.
The *first infinite ordinal* is ω. An *ω-word* over Σ is an ω-sequence $a_1 \ldots a_n \ldots$, where for all integers $i \geq 1$, $\quad a_i \in \Sigma$. When σ is an ω-word over Σ, we write $\sigma = \sigma(1)\sigma(2) \ldots \sigma(n) \ldots$, where for all i, $\sigma(i) \in \Sigma$.
The usual concatenation product of two finite words u and v is denoted $u \cdot v$ and sometimes just uv. This product is extended to the product of a finite word u and an ω-word v: the infinite word $u \cdot v$ is then the ω-word such that:
$(u \cdot v)(k) = u(k)$ if $k \leq |u|$, and $(u \cdot v)(k) = v(k - |u|)$ if $k > |u|$.
The *set of ω-words* over the alphabet Σ is denoted by Σ^ω. An *ω-language* over an alphabet Σ is a subset of Σ^ω.

Recall now the notion of acceptance of infinite words by Turing machines considered by Castro and Cucker in [CC89].

Definition 2.1 *A non deterministic Turing machine $\mathcal{M}$ is a 5-tuple $\mathcal{M} = (Q, \Sigma, \Gamma, \delta, q_0)$, where Q is a finite set of states, Σ is a finite input alphabet, Γ is a finite tape alphabet satisfying $\Sigma \subseteq \Gamma$, q_0 is the initial state, and δ is a mapping from $Q \times \Gamma$ to subsets of $Q \times \Gamma \times \{L, R, S\}$. A configuration of $\mathcal{M}$ is a triple (q, σ, i), where $q \in Q$, $\sigma \in \Gamma^\omega$ and $i \in \mathbb{N}$. An infinite sequence of configurations $r = (q_i, \alpha_i, j_i)_{i \geq 1}$ is called a run of $\mathcal{M}$ on $w \in \Sigma^\omega$ iff:*

(a) $(q_1, \alpha_1, j_1) = (q_0, w, 1)$, *and*

(b) for each $i \geq 1$, $(q_i, \alpha_i, j_i) \vdash (q_{i+1}, \alpha_{i+1}, j_{i+1})$,

where $\vdash$ is the transition relation of $\mathcal{M}$ defined as usual. The run r is said to be complete if $(\forall n \geq 1)(\exists k \geq 1)(j_k \geq n)$. The run r is said to be oscillating if $(\exists k \geq 1)(\forall n \geq 1)(\exists m \geq n)(j_m = k)$.

Definition 2.2 *Let $\mathcal{M} = (Q, \Sigma, \Gamma, \delta, q_0)$ be a non deterministic Turing machine and $F \subseteq Q$. The ω-language accepted by $(\mathcal{M}, F)$ is the set of ω-words $\sigma \in \Sigma^\omega$ such that there exists a complete non oscillating run $r = (q_i, \alpha_i, j_i)_{i \geq 1}$ of $\mathcal{M}$ on σ such that, for all $i, q_i \in F$.*

The above acceptance condition is denoted $1'$-acceptance in [CG78]. Other usual acceptance conditions are the now called Büchi or Muller acceptance conditions, respectively denoted 2-acceptance and 3-acceptance in [CG78]. Cohen and Gold proved the following result in [CG78, Theorem 8.2].

Theorem 2.3 (Cohen and Gold [CG78]) *An ω-language is accepted by a non deterministic Turing machine with $1'$-acceptance condition iff it is accepted by a non deterministic Turing machine with Büchi (respectively, Muller) acceptance condition.*

Notice that this result holds because Cohen's and Gold's Turing machines accept infinite words via *complete non oscillating runs*, while $1'$, Büchi or Muller acceptance conditions refer to the sequence of states entered during an infinite run.
For other approaches, acceptance is based only on the sequence of states entered by the machine during an infinite computation [Sta97], or one requires also that the machine reads the whole infinite tape [EH93]. We refer the reader to [SW78, Sta99, FS00, Sta00] for a study of these different approaches.

We recall the existence of the arithmetical and analytical hierarchies of ω-languages, see [SW78, Sta97]; see also [LT94] about logical specifications for infinite computations. The first class of the analytical hierarchy is the class Σ_1^1 of *effective analytic sets* which are obtained by projection of arithmetical sets. By [Sta99, Theorem 16] (see also [Sta00, Theorem 5.2]) we have the following characterization of the class of ω-languages accepted by non deterministic Turing machines via acceptance by complete runs (i.e., not necessarily non oscillating).

Theorem 2.4 ([Sta99]) *The class of ω-languages accepted by non deterministic Turing machines with $1'$ (respectively, Büchi, Muller) acceptance condition is the class Σ_1^1 of effective analytic sets.*

We return now to Cohen's and Gold's non deterministic Turing machines accepting via *complete non oscillating runs*. The following result follows from [CG78, Note 2 page 12] and from Theorem 2.4.

Theorem 2.5 *The class of ω-languages accepted by Cohen's and Gold's non deterministic Turing machines with $1'$ (respectively, Büchi, Muller) acceptance condition is the class Σ^1_1 of effective analytic sets.*

3 Decision problems about Turing machines

In the sequel we consider, as in [CC89], that the alphabet Σ contains only two letters a and b, and we shall denote $\mathcal{M}_z$ the non deterministic Turing machine of index z, reading words over Σ, equipped with a $1'$-acceptance condition. We now recall the results of Castro and Cucker giving the exact complexity of the non-emptiness problem and of the infiniteness problem for ω-languages of Turing machines.

Theorem 3.1

1. *$\{z \in \mathbb{N} \mid L(\mathcal{M}_z) \neq \emptyset\}$ is Σ^1_1-complete.*
2. *$\{z \in \mathbb{N} \mid L(\mathcal{M}_z)$ is infinite$\}$ is Σ^1_1-complete.*

We now state our first new result.

Lemma 3.2 *$\{z \in \mathbb{N} \mid L(\mathcal{M}_z)$ is countably infinite$\}$ is in the class $D_2(\Sigma^1_1)$.*

Proof. We first show that $\{z \in \mathbb{N} \mid L(\mathcal{M}_z)$ is countable$\}$ is in the class Π^1_1. Notice that here "countable" means "finite or countably infinite."

We know that an ω-language $L(\mathcal{M}_z)$ accepted by a Turing machine $\mathcal{M}_z$ is a Σ^1_1-subset of Σ^ω. But it is known that a Σ^1_1-subset L of Σ^ω is countable if and only if for every $x \in L$ the singleton $\{x\}$ is a Δ^1_1-subset of Σ^ω, see [Mos80, page 243].

On the other hand the following result is proved in [HKL90, Theorem 3.3.1]. There esists a Π^1_1-set $W \subseteq \mathbb{N}$ and a Π^1_1-set $C \subseteq \mathbb{N} \times \Sigma^\omega$ such that, if we denote $C_n = \{x \in \Sigma^\omega \mid (n, x) \in C\}$, then $\{(n, \alpha) \in \mathbb{N} \times \Sigma^\omega \mid n \in W$ and $\alpha \notin C_n\}$ is a Π^1_1-subset of the product space $\mathbb{N} \times \Sigma^\omega$ and the Δ^1_1-subsets of Σ^ω are the sets of the form C_n for $n \in W$.

We can now first express $(\exists n \in W \; C_n = \{x\})$ by the sentence $\phi(x)$:

$$\exists n \; [n \in W \text{ and } (n, x) \in C \text{ and } \forall y \in \Sigma^\omega \; [(n \in W \text{ and } (n, y) \notin C) \text{ or } (y = x)]]$$

But we know that C is a Π^1_1-set and that $\{(n, \alpha) \in \mathbb{N} \times \Sigma^\omega \mid n \in W$ and $\alpha \notin C_n\}$ is a Π^1_1-subset of $\mathbb{N} \times \Sigma^\omega$. Moreover the quantification $\exists n$ in the above formula is a first-order quantification therefore the above formula $\phi(x)$ is a Π^1_1-formula.

We can now express that $L(\mathcal{M}_z)$ is countable by the sentence $\psi(z)$:

$$\forall x \in \Sigma^\omega \quad [(x \notin L(\mathcal{M}_z)) \text{ or } (\exists n \in W \ C_n = \{x\})]$$

that is,

$$\forall x \in \Sigma^\omega \quad [(x \notin L(\mathcal{M}_z)) \text{ or } \phi(x)]$$

We know from [CC89] that $x \notin L(\mathcal{M}_z)$ is expressed by a Π^1_1-formula. Thus the above formula $\psi(z)$ is a Π^1_1-formula. This proves that the set $\{z \in \mathbb{N} \mid L(\mathcal{M}_z) \text{ is countable}\}$ is in the class Π^1_1.

On the other hand the set $\{z \in \mathbb{N} \mid L(\mathcal{M}_z) \text{ is infinite}\}$ is in the class Σ^1_1. Finally the set $\{z \in \mathbb{N} \mid L(\mathcal{M}_z) \text{ is countably infinite}\}$ is the intersection of a Σ^1_1-set and of a Π^1_1-set, i.e. it is in the class $D_2(\Sigma^1_1)$. □

We now give the exact complexity for this decision problem.

Theorem 3.3 *$\{z \in \mathbb{N} \mid L(\mathcal{M}_z)$ is countably infinite$\}$ is $D_2(\Sigma^1_1)$-complete.*

Proof. Recall that Castro and Cucker proved in [CC89, Proof of Proposition 3.1] that there is a computable injective function φ from $\mathbb{N}$ into $\mathbb{N}$ such that there are two cases:

First case: $z \in E_1$ and $L(\mathcal{M}_{\varphi(z)}) = \Sigma^\omega$.
Second case: $z \in E_1^-$ and $L(\mathcal{M}_{\varphi(z)}) = \emptyset$.

We can easily define injective computable functions g and h from $\mathbb{N}$ into $\mathbb{N}$ suh that for every integer $z \in \mathbb{N}$ it holds that :

$$L(\mathcal{M}_{g(z)}) = L(\mathcal{M}_z) \cup a^\star \cdot b^\omega$$

and

$$L(\mathcal{M}_{h(z)}) = L(\mathcal{M}_z) \cap a^\star \cdot b^\omega$$

We can see that there are now two cases:
First case: In this case $z \in E_1$ and $L(\mathcal{M}_{\varphi(z)}) = \Sigma^\omega$. Thus $L(\mathcal{M}_{g\circ\varphi(z)}) = \Sigma^\omega$ is uncountable and $L(\mathcal{M}_{h\circ\varphi(z)}) = a^\star \cdot b^\omega$ is countable.
Second case: In this case $z \in E_1^-$ and $L(\mathcal{M}_{\varphi(z)}) = \emptyset$. Thus $L(\mathcal{M}_{g\circ\varphi(z)}) = a^\star \cdot b^\omega$ is countable and $L(\mathcal{M}_{h\circ\varphi(z)}) = \emptyset$.

We define now the following simple operation over ω-languages. For two ω-words $x, x' \in \Sigma^\omega$ the ω-word $x \oplus x'$ is defined by : for every integer $n \geq 1$ $(x \oplus x')(2n-1) = x(n)$ and $(x \oplus x')(2n) = x'(n)$. For two ω-languages $L, L' \subseteq \Sigma^\omega$, the ω-language $L \oplus L'$ is defined by $L \oplus L' = \{x \oplus x' \mid x \in L \text{ and } x' \in L'\}$.

It is easy to see that there is a computable injective function Φ from $\mathbb{N}^2$ into $\mathbb{N}$ suh that for every integer $z, z' \in \mathbb{N}$ it holds that

$$L(\mathcal{M}_{\Phi(z,z')}) = L(\mathcal{M}_z) \oplus L(\mathcal{M}_{z'})$$

We want now to show that every subset of $\mathbb{N}$ in the class $D_2(\Sigma_1^1)$ is 1-reducible to the set $\{z \in \mathbb{N} \mid L(\mathcal{M}_z)$ is countably infinite$\}$.

Let E be a $D_2(\Sigma_1^1)$-subset of $\mathbb{N}$. Then there are some sets $A \subseteq \mathbb{N}$ and $B \subseteq \mathbb{N}$ such that A is a Σ_1^1-set and B is a Π_1^1-set and $E = A \cap B$. But the set E_1 is Σ_1^1-complete and the set E_1^- is Π_1^1-complete. Thus there are some injective computable mappings f_A and f_B from $\mathbb{N}$ into $\mathbb{N}$ suh that $A = f_A^{-1}(E_1)$ and $B = f_B^{-1}(E_1^-)$.

It is easy to see that there is an injective computable function Ψ from $\mathbb{N}$ into $\mathbb{N}$ suh that for every $z \in \mathbb{N}$ it holds that :

$$L(\mathcal{M}_{\Psi(z)}) = L(\mathcal{M}_{\Phi(h \circ \varphi \circ f_A(z), g \circ \varphi \circ f_B(z))}) = L(\mathcal{M}_{h \circ \varphi \circ f_A(z)}) \oplus L(\mathcal{M}_{g \circ \varphi \circ f_B(z)})$$

We next show that Ψ is a reduction. We divide into cases.

First case: $z \in E = A \cap B$. Then $f_A(z) \in E_1$ and $L(\mathcal{M}_{h \circ \varphi \circ f_A(z)}) = a^\star \cdot b^\omega$ is countably infinite. Moreover $f_B(z) \in E_1^-$ and $L(\mathcal{M}_{g \circ \varphi \circ f_B(z)}) = a^\star \cdot b^\omega$ is countably infinite. Thus the ω-language $L(\mathcal{M}_{\Psi(z)}) = L(\mathcal{M}_{h \circ \varphi \circ f_A(z)}) \oplus L(\mathcal{M}_{g \circ \varphi \circ f_B(z)})$ is also countably infinite.

Second case: $z \notin E = A \cap B$. Then either $z \notin A$ or $z \notin B$. Assume first that $z \notin A$. Then $f_A(z) \notin E_1$, i.e. $f_A(z) \in E_1^-$. Thus $L(\mathcal{M}_{h \circ \varphi \circ f_A(z)}) = \emptyset$. Assume now that $z \notin B$, i.e. $f_B(z) \in E_1$. Then $L(\mathcal{M}_{g \circ \varphi \circ f_B(z)}) = \Sigma^\omega$. We can see that if either $z \notin A$ or $z \notin B$ the ω-language $L(\mathcal{M}_{\Psi(z)}) = L(\mathcal{M}_{h \circ \varphi \circ f_A(z)}) \oplus L(\mathcal{M}_{g \circ \varphi \circ f_B(z)})$ can not be countably infinite because it can only be either empty or uncountable.

Finally, using the reduction Ψ we have proved that

$$E \leq_1 \{z \in \mathbb{N} \mid L(\mathcal{M}_z) \text{ is countably infinite}\}$$

so this latter set is $D_2(\Sigma_1^1)$-complete. □

Remark 3.4 *Castro and Cucker noticed in [CC89] that the set $\{z \in \mathbb{N} \mid L(\mathcal{M}_z)$ is countably infinite$\}$ is in the class Σ_2^1 but they asked whether this set is Σ_2^1-complete. Our result shows that the answer is "no" because a $D_2(\Sigma_1^1)$-set is actually much less complex than a Σ_2^1-complete set.*

Recall that an ω-language accepted by a Turing machine $\mathcal{M}$ is a Σ^1_1-subset of Σ^ω. Then it is well known that such a set is either countable or has the cardinal $2^{\aleph_0}$ of the continuum, see [Mos80]. Therefore an ω-language accepted by a Turing machine has cardinal $2^{\aleph_0}$ iff it is not a countable set. We can now state the following result.

Theorem 3.5

1. *$\{z \in \mathbb{N} \mid L(\mathcal{M}_z)$ is uncountable$\}$ is Σ^1_1-complete.*
2. *$\{z \in \mathbb{N} \mid L(\mathcal{M}_z)$ is countable$\}$ is Π^1_1-complete.*

Proof. We first prove item (1).
We have already seen that $\{z \in \mathbb{N} \mid L(\mathcal{M}_z)$ is countable$\}$ is in the class Π^1_1. Thus the set $\{z \in \mathbb{N} \mid L(\mathcal{M}_z)$ is uncountable$\}$ is a Σ^1_1-set.

To prove the completeness result we can use an already cited result of Castro and Cucker. There is a computable injective function φ from $\mathbb{N}$ into $\mathbb{N}$ for which one of the two following cases hold:
First case: $z \in E_1$ and $L(\mathcal{M}_{\varphi(z)}) = \Sigma^\omega$.
Second case: $z \in E_1^-$ and $L(\mathcal{M}_{\varphi(z)}) = \emptyset$.

The reduction φ shows that :

$$E_1 \leq_1 \{z \in \mathbb{N} \mid L(\mathcal{M}_z) \text{ is uncountable}\}$$

so this latter set is Σ^1_1-complete.

Item (2) follows directly from Item (1). $\square$

References

[CC89] J. Castro and F. Cucker. Nondeterministic ω-computations and the analytical hierarchy. *Z. Math. Logik Grundlag. Math.*, 35:333–342, 1989.

[CG78] R.S. Cohen and A.Y. Gold. ω-computations on Turing machines. *Theoretical Computer Science*, 6:1–23, 1978.

[CR99] D. A. Cenzer and J. B. Remmel. Index sets in computable analysis. *Theoretical Computer Science*, 219(1–2):111–150, 1999.

[CR03] D. A. Cenzer and J. B. Remmel. Index sets for omega-languages. *Mathematical Logic Quaterly*, 49(1):22–33, 2003.

[EH93] J Engelfriet and H. J. Hoogeboom. X-automata on ω-words. *Theoretical Computer Science*, 110(1):1–51, 1993.

[FS00] R. Freund and L. Staiger. Acceptance of ω-languages by communicating deterministic turing machines. In *Words, Sequences, Grammars, Languages: where Biology, Computer Science, Linguistics and Mathematics Meet*, volume 1, pages 115–125. 2000.

[HKL90] L.A. Harrington, A.S. Kechris, and A. Louveau. A Glimm-Effros dichotomy for Borel equivalence relations. *Journal of the American Mathematical Society*, 3:903–928, 1990.

[Kla94] N. Klarlund. The limit view of infinite computations. In B. Jonsson and J.Parrow, editors, *Proceedings of the 5th International Conference on Concurrency Theory, CONCUR 94, Uppsala, Sweden, August 22-25, 1994*, volume 836 of *Lecture Notes in Computer Science*, pages 351–366. Springer, 1994.

[LT94] H. Lescow and W. Thomas. Logical specifications of infinite computations. In J. W. de Bakker, W. P. de Roever, and G. Rozenberg, editors, *A Decade of Concurrency*, volume 803 of *Lecture Notes in Computer Science*, pages 583–621. Springer, 1994.

[Mos80] Y. N. Moschovakis. *Descriptive set theory.* North-Holland Publishing Co., Amsterdam, 1980.

[Odi89] P.G. Odifreddi. *Classical Recursion Theory, Vol I*, volume 125 of *Studies in Logic and the Foundations of Mathematics.* North-Holland Publishing Co., Amsterdam, 1989.

[Odi99] P.G. Odifreddi. *Classical Recursion Theory, Vol II*, volume 143 of *Studies in Logic and the Foundations of Mathematics.* North-Holland Publishing Co., Amsterdam, 1999.

[Rog67] H. Rogers. *Theory of Recursive Functions and Effective Computability.* McGraw-Hill, New York, 1967.

[Sta93] L. Staiger. Recursive automata on infinite words. In P. Enjalbert, A. Finkel, and K. W. Wagner, editors, *Proceedings of the 10th Annual Symposium on Theoretical Aspects of Computer Science, STACS 93, Würzburg, Germany, February 25-27, 1993*, volume 665 of *Lecture Notes in Computer Science*, pages 629–639. Springer, 1993.

[Sta97] L. Staiger. ω-languages. In *Handbook of formal languages, Vol. 3, edited by G. Rozenberg and A. Salomaa*, pages 339–387. Springer, Berlin, 1997.

[Sta99] L. Staiger. On the power of reading the whole infinite input tape. *Grammars*, 2(3):247–257, 1999.

[Sta00] L. Staiger. On the power of reading the whole infinite input tape. In *Finite Versus Infinite: Contributions to an Eternal Dilemma*, Discrete Mathematics and Theoretical Computer Science, pages 335–348. Springer-Verlag London, 2000.

[SW78] L. Staiger and K. Wagner. Rekursive folgenmengen I. *Z. Math Logik Grundlag. Math.*, 24:523–538, 1978.

[Tho90] W. Thomas. Automata on infinite objects. In J. van Leeuwen, editor, *Handbook of Theoretical Computer Science*, volume B, Formal models and semantics, pages 135–191. Elsevier, 1990.