

CORRIGENDUM TO:  
 “QUANTIFIER ELIMINATION IN VALUED ORE MODULES”

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Recall that  $A := K[t; \sigma, \partial]$  is a skew polynomial ring. Lemma 2.4 in [1] should read as follows (but note there is no effect on the rest of the paper):

LEMMA 2.4. For any pair  $\{q_1(t), q_2(t)\}$  of elements of  $A$ , we have the following equivalence in any divisible  $A$ -module  $M$ :

$$\text{ann}_M(q_2(t)) \subseteq \text{ann}_M(q_1(t)) \text{ if and only if}$$

there exists  $q_3(t)$  such that  $(\text{ann}_M(q_2(t)) = \text{ann}_M(q_3(t)))$  and  $q_3(t)$  divides  $q_1(t)$ .

Moreover, if  $q_1(t) = q_2(t) \cdot r(t)$  and if the cardinality  $|\text{ann}_M(q_1(t))/\text{ann}_M(q_2(t))|$  is finite, then  $|\text{ann}_M(q_1(t))/\text{ann}_M(q_2(t))| = |\text{ann}_M(r(t))|$ .

For the convenience of the reader we give a proof below (of the first part), along the lines of Lemma 2.9 and Proposition 2.10 of reference [17] as indicated in [1]. The argument also shows the following: for any pair of elements  $\{q_1(t), q_2(t)\}$  of  $A$ , we have that  $q_2(t)$  divides  $q_1(t)$ , whenever  $\text{ann}_M(q_2(t)) \subseteq \text{ann}_M(q_1(t))$ ,  $\text{deg}(q_1(t)) > \text{deg}(q_2(t))$ , and  $\text{ann}_M(q(t)) \neq \{0\}$  for any  $q(t) \notin K$  which divides  $q_2(t)$  on the right.

PROOF OF THE LEMMA. We will proceed by induction on the sum of the degrees of  $q_1(t)$  and  $q_2(t)$ , assuming that both  $q_1(t), q_2(t)$  are non-zero. Either  $\text{ann}_M(q_2(t)) = \{0\}$ , then take  $q_3(t) = 1$ , or  $\text{ann}_M(q_2(t)) \neq \{0\}$ .

So let  $0 \neq u \in \text{ann}_M(q_2(t))$  and let  $q(t) \in A - \{0\}$  with minimal degree such that  $u \cdot q(t) = 0$ . Note that  $\text{deg}(q(t)) \geq 1$ . Applying the right Euclidean algorithm, we have that  $q_2(t) = q(t) \cdot r_2(t)$  and since  $\text{ann}_M(q_2(t)) \subseteq \text{ann}_M(q_1(t))$ , that  $q_1(t) = q(t) \cdot r_1(t)$  for some  $r_1(t), r_2(t) \in A - \{0\}$ .

Let us show that  $\text{ann}_M(r_2(t)) \subseteq \text{ann}_M(r_1(t))$ . Let  $u' \in \text{ann}_M(r_2(t))$ . Since  $M$  is divisible, there exists  $u''$  such that  $u'' \cdot q(t) = u'$ . So  $u'' \in \text{ann}_M(q_2(t)) \subseteq \text{ann}_M(q_1(t))$  and so  $0 = u'' \cdot q(t) \cdot r_1(t) = u' \cdot r_1(t)$ . So we may apply induction to the pair  $(r_1(t), r_2(t))$  since  $\text{deg}(r_1(t)) + \text{deg}(r_2(t)) < \text{deg}(q_1(t)) + \text{deg}(q_2(t))$ . Therefore, there exists  $r_3(t)$  with  $\text{ann}_M(r_3(t)) = \text{ann}_M(r_2(t))$  and  $r_3(t)$  divides  $r_1(t)$ . It remains to note that  $\text{ann}_M(q(t) \cdot r_3(t)) = \text{ann}_M(q_2(t))$ . So let  $u \in \text{ann}_M(q(t) \cdot r_3(t))$ , then  $u \cdot q(t) \in \text{ann}_M(r_2(t))$  and so  $u \in \text{ann}_M(q_2(t))$ . Conversely let  $u \in \text{ann}_M(q_2(t))$ , so  $u \cdot q(t) \in \text{ann}_M(r_2(t))$ . Since  $\text{ann}_M(r_2(t)) = \text{ann}_M(r_3(t))$ , we have  $u \cdot q(t) \cdot r_3(t) = 0$ . ⊖

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Now suppose that  $\text{ann}_M(q_2(t)) \neq \{0\}$ . Proceeding as in the above proof, either  $\text{deg}(r_2(t)) = 0$  and so  $q_2(t)$  divides  $q_1(t)$ , or by hypothesis  $\text{ann}_M(r_2(t)) \neq \{0\}$ . As before  $\text{ann}_M(r_2(t)) \subseteq \text{ann}_M(r_1(t))$  and so we may apply induction to the pair  $(r_1(t), r_2(t))$  and so  $r_2(t)$  divides  $r_1(t)$  which implies that  $q_2(t)$  divides  $q_1(t)$ .

## REFERENCES

- [1] L. BÉLAIR and F. POINT, *Quantifier elimination in valued Ore modules*, this JOURNAL, vol. 75 (2010), no. 3, pp. 1007–1034.

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