

CORRIGENDUM TO:
 “QUANTIFIER ELIMINATION IN VALUED ORE MODULES”

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Recall that $A := K[t; \sigma, \partial]$ is a skew polynomial ring. Lemma 2.4 in [1] should read as follows (but note there is no effect on the rest of the paper):

LEMMA 2.4. For any pair $\{q_1(t), q_2(t)\}$ of elements of A , we have the following equivalence in any divisible A -module M :

$$\text{ann}_M(q_2(t)) \subseteq \text{ann}_M(q_1(t)) \text{ if and only if}$$

there exists $q_3(t)$ such that $(\text{ann}_M(q_2(t)) = \text{ann}_M(q_3(t)))$ and $q_3(t)$ divides $q_1(t)$.

Moreover, if $q_1(t) = q_2(t) \cdot r(t)$ and if the cardinality $|\text{ann}_M(q_1(t))/\text{ann}_M(q_2(t))|$ is finite, then $|\text{ann}_M(q_1(t))/\text{ann}_M(q_2(t))| = |\text{ann}_M(r(t))|$.

For the convenience of the reader we give a proof below (of the first part), along the lines of Lemma 2.9 and Proposition 2.10 of reference [17] as indicated in [1]. The argument also shows the following: for any pair of elements $\{q_1(t), q_2(t)\}$ of A , we have that $q_2(t)$ divides $q_1(t)$, whenever $\text{ann}_M(q_2(t)) \subseteq \text{ann}_M(q_1(t))$, $\text{deg}(q_1(t)) > \text{deg}(q_2(t))$, and $\text{ann}_M(q(t)) \neq \{0\}$ for any $q(t) \notin K$ which divides $q_2(t)$ on the right.

PROOF OF THE LEMMA. We will proceed by induction on the sum of the degrees of $q_1(t)$ and $q_2(t)$, assuming that both $q_1(t), q_2(t)$ are non-zero. Either $\text{ann}_M(q_2(t)) = \{0\}$, then take $q_3(t) = 1$, or $\text{ann}_M(q_2(t)) \neq \{0\}$.

So let $0 \neq u \in \text{ann}_M(q_2(t))$ and let $q(t) \in A - \{0\}$ with minimal degree such that $u \cdot q(t) = 0$. Note that $\text{deg}(q(t)) \geq 1$. Applying the right Euclidean algorithm, we have that $q_2(t) = q(t) \cdot r_2(t)$ and since $\text{ann}_M(q_2(t)) \subseteq \text{ann}_M(q_1(t))$, that $q_1(t) = q(t) \cdot r_1(t)$ for some $r_1(t), r_2(t) \in A - \{0\}$.

Let us show that $\text{ann}_M(r_2(t)) \subseteq \text{ann}_M(r_1(t))$. Let $u' \in \text{ann}_M(r_2(t))$. Since M is divisible, there exists u'' such that $u'' \cdot q(t) = u'$. So $u'' \in \text{ann}_M(q_2(t)) \subseteq \text{ann}_M(q_1(t))$ and so $0 = u'' \cdot q(t) \cdot r_1(t) = u' \cdot r_1(t)$. So we may apply induction to the pair $(r_1(t), r_2(t))$ since $\text{deg}(r_1(t)) + \text{deg}(r_2(t)) < \text{deg}(q_1(t)) + \text{deg}(q_2(t))$. Therefore, there exists $r_3(t)$ with $\text{ann}_M(r_3(t)) = \text{ann}_M(r_2(t))$ and $r_3(t)$ divides $r_1(t)$. It remains to note that $\text{ann}_M(q(t) \cdot r_3(t)) = \text{ann}_M(q_2(t))$. So let $u \in \text{ann}_M(q(t) \cdot r_3(t))$, then $u \cdot q(t) \in \text{ann}_M(r_2(t))$ and so $u \in \text{ann}_M(q_2(t))$. Conversely let $u \in \text{ann}_M(q_2(t))$, so $u \cdot q(t) \in \text{ann}_M(r_2(t))$. Since $\text{ann}_M(r_2(t)) = \text{ann}_M(r_3(t))$, we have $u \cdot q(t) \cdot r_3(t) = 0$. ⊖

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Now suppose that $\text{ann}_M(q_2(t)) \neq \{0\}$. Proceeding as in the above proof, either $\text{deg}(r_2(t)) = 0$ and so $q_2(t)$ divides $q_1(t)$, or by hypothesis $\text{ann}_M(r_2(t)) \neq \{0\}$. As before $\text{ann}_M(r_2(t)) \subseteq \text{ann}_M(r_1(t))$ and so we may apply induction to the pair $(r_1(t), r_2(t))$ and so $r_2(t)$ divides $r_1(t)$ which implies that $q_2(t)$ divides $q_1(t)$.

REFERENCES

- [1] L. BÉLAIR and F. POINT, *Quantifier elimination in valued Ore modules*, this JOURNAL, vol. 75 (2010), no. 3, pp. 1007–1034.

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