

cyclic9

```
%sh
lscpu

/tmp/tmp5yYAcP
Architecture:      x86_64
CPU op-mode(s):   32-bit, 64-bit
Byte Order:       Little Endian
CPU(s):           16
On-line CPU(s) list: 0-15
Thread(s) per core: 2
Core(s) per socket: 4
Socket(s):        2
NUMA node(s):    2
Vendor ID:        GenuineIntel
CPU family:       6
Model:            26
Stepping:         5
CPU MHz:          2261.017
BogoMIPS:         4521.83
Virtualization:   VT-x
L1d cache:        32K
L1i cache:        32K
L2 cache:         256K
L3 cache:         8192K
NUMA node0 CPU(s): 0,2,4,6,8,10,12,14
NUMA node1 CPU(s): 1,3,5,7,9,11,13,15
```

```
n=9;R=PolynomialRing(QQ,n,'x');
I = sage.rings.ideal.Cyclic(R,n);
I.gens()

[x0 + x1 + x2 + x3 + x4 + x5 + x6 + x7 + x8, x0*x1 + x1*x2 + x2*x3 +
x3*x4 + x4*x5 + x5*x6 + x6*x7 + x0*x8 + x7*x8, x0*x1*x2 + x1*x2*x3 +
x2*x3*x4 + x3*x4*x5 + x4*x5*x6 + x5*x6*x7 + x0*x1*x8 + x0*x7*x8 +
x6*x7*x8, x0*x1*x2*x3 + x1*x2*x3*x4 + x2*x3*x4*x5 + x3*x4*x5*x6 +
x4*x5*x6*x7 + x0*x1*x2*x8 + x0*x1*x7*x8 + x0*x6*x7*x8 + x5*x6*x7*x8,
x0*x1*x2*x3*x4 + x1*x2*x3*x4*x5 + x2*x3*x4*x5*x6 + x3*x4*x5*x6*x7 +
x0*x1*x2*x3*x8 + x0*x1*x2*x7*x8 + x0*x1*x6*x7*x8 + x0*x5*x6*x7*x8 +
x4*x5*x6*x7*x8, x0*x1*x2*x3*x4*x5 + x1*x2*x3*x4*x5*x6 +
x2*x3*x4*x5*x6*x7 + x0*x1*x2*x3*x4*x8 + x0*x1*x2*x3*x7*x8 +
x0*x1*x2*x6*x7*x8 + x0*x1*x5*x6*x7*x8 + x0*x4*x5*x6*x7*x8 +
x3*x4*x5*x6*x7*x8, x0*x1*x2*x3*x4*x5*x6 + x1*x2*x3*x4*x5*x6*x7 +
x0*x1*x2*x3*x4*x5*x8 + x0*x1*x2*x3*x4*x7*x8 + x0*x1*x2*x3*x6*x7*x8 +
x0*x1*x2*x5*x6*x7*x8 + x0*x1*x4*x5*x6*x7*x8 + x0*x3*x4*x5*x6*x7*x8 +
x2*x3*x4*x5*x6*x7*x8, x0*x1*x2*x3*x4*x5*x6*x7 +
x0*x1*x2*x3*x4*x5*x6*x8 + x0*x1*x2*x3*x4*x5*x7*x8 +
x0*x1*x2*x3*x4*x6*x7*x8 + x0*x1*x2*x3*x5*x6*x7*x8 +
x0*x1*x2*x4*x5*x6*x7*x8 + x0*x1*x3*x4*x5*x6*x7*x8 +
x0*x2*x3*x4*x5*x6*x7*x8 + x1*x2*x3*x4*x5*x6*x7*x8,
x0*x1*x2*x3*x4*x5*x6*x7*x8 - 1]
```

```
%magma
R<x0,x1,x2,x3,x4,x5,x6,x7,x8> := PolynomialRing( RationalField(),9, "grevlex" );
```

```
magma.eval("I:=Ideal(%s);"%(I.gens()));
```

```
%magma
I;

Ideal of Polynomial ring of rank 9 over Rational Field
Order: Graded Reverse Lexicographical
Variables: x0, x1, x2, x3, x4, x5, x6, x7, x8
Basis:
[
x0 + x1 + x2 + x3 + x4 + x5 + x6 + x7 + x8,
x0*x1 + x1*x2 + x2*x3 + x3*x4 + x4*x5 + x5*x6 + x6*x7 + x0*x8 +
x7*x8,
x0*x1*x2 + x1*x2*x3 + x2*x3*x4 + x3*x4*x5 + x4*x5*x6 + x5*x6*x7 +
x0*x1*x8 + x0*x7*x8 + x6*x7*x8,
x0*x1*x2*x3 + x1*x2*x3*x4 + x2*x3*x4*x5 + x3*x4*x5*x6 + x4*x5*x6*x7
+ x0*x1*x2*x8 + x0*x1*x7*x8 + x0*x6*x7*x8 + x5*x6*x7*x8,
x0*x1*x2*x3*x4 + x1*x2*x3*x4*x5 + x2*x3*x4*x5*x6 + x3*x4*x5*x6*x7 +
x0*x1*x2*x3*x8 + x0*x1*x2*x7*x8 + x0*x1*x6*x7*x8 + x0*x5*x6*x7*x8 +
x4*x5*x6*x7*x8,
x0*x1*x2*x3*x4*x5 + x1*x2*x3*x4*x5*x6 + x2*x3*x4*x5*x6*x7 +
x0*x1*x2*x3*x4*x8 + x0*x1*x2*x3*x7*x8 + x0*x1*x2*x6*x7*x8 +
x0*x1*x5*x6*x7*x8 + x0*x4*x5*x6*x7*x8 + x3*x4*x5*x6*x7*x8,
x0*x1*x2*x3*x4*x5*x6 + x1*x2*x3*x4*x5*x6*x7 + x0*x1*x2*x3*x4*x5*x8 +
x0*x1*x2*x3*x4*x7*x8 + x0*x1*x2*x3*x6*x7*x8 + x0*x1*x2*x5*x6*x7*x8 +
x0*x1*x4*x5*x6*x7*x8 + x0*x3*x4*x5*x6*x7*x8 + x2*x3*x4*x5*x6*x7*x8,
x0*x1*x2*x3*x4*x5*x6*x7 + x0*x1*x2*x3*x4*x5*x6*x8 +
x0*x1*x2*x3*x4*x6*x7*x8 + x0*x1*x2*x3*x4*x5*x7*x8 +
x0*x1*x2*x3*x5*x6*x7*x8 + x0*x1*x2*x4*x5*x6*x7*x8 +
x0*x1*x3*x4*x5*x6*x7*x8 + x0*x2*x3*x4*x5*x6*x7*x8 +
x1*x2*x3*x4*x5*x6*x7*x8,
x0*x1*x2*x3*x4*x5*x6*x7*x8 - 1
]
```

```
%magma
time B:=GroebnerBasis(I);
Time: 27973.880
```

```
from giacpy import *
// Giac share root-directory:/home/han/dev/sage/local/share/giac/
// Unable to find keyword file
/home/han/dev/sage/local/share/giac/doc/fr/keywords
// Giac share root-directory:/home/han/dev/sage/local/share/giac/
Help file /home/han/dev/sage/local/share/giac/doc/fr/aide_cas not
```

found
Added 0 synonyms

```
giacsettings.threads=8; # maximum number of threads allowed
giacsettings.proba_epsilon=1e-7; # for probabilistic algorithms
```

```
time BG=libgiac(I.gens()).gbasis([R.gens()])
```

Running a probabilistic check for the reconstructed Groebner basis.
If successful, error probability is less than 1e-07 and is estimated to be less than 10^-119. Use proba_epsilon:=0 to certify (this takes more time).
Time: CPU 176793.69 s, Wall: 24832.38 s

```
libgiac('x0^2+x2*x0').degree('x0')
```

2

```
BG.dim()
```

1344

```
time BG.savegen('/home/han/cyclic9.giacgen')
```

Time: CPU 27.21 s, Wall: 28.30 s

```
%sh
```

```
ls -hl /home/han/cyclic9.giacgen
/tmp/tmp0rIoM5
-rw-r--r-- 1 han 17000 1,16 juin 8 10:01 /home/han/cyclic9.giacgen
```

```
time BGbis=loadgiacgen('/home/han/cyclic9.giacgen')
```

Time: CPU 67.72 s, Wall: 67.86 s

```
BGbis.dim()
```

1344

```
T=True;
for i in range(len(BG)):
    T=T and ((BG[i]-BGbis[i]).ratnormal()==0)
```

```
T
```

True

```
BG[5]
```

x2^2*x3*x4^2+x2*x3^2*x4^2+x2*x3*x4^3+x2*x3^2*x4*x5+x1*x3*x4^2*x5+2*x\
3^2*x4^2*x5+x3*x4^3*x5-2*x1*x3*x4*x5^2-2*x2*x3*x4*x5^2-x3^2*x4*x5^2-\
x3*x4*x5^3+2*x1*x2*x3*x4*x6+2*x2*x3*x4^2*x6+x1*x2*x3*x5*x6-2*x1*x2*x\
4*x5*x6-2*x2*x4*x5*x6+x1*x4^2*x5*x6+2*x3*x4^2*x5*x6+x4^3*x5*x6-2*x1*\
x4*x5^2*x6-2*x2*x4*x5^2*x6-2*x3*x4*x5^2*x6-x4^2*x5^2*x6-x4*x5^3*x6+x\
1*x4*x5*x6^2-2*x2*x4*x5*x6^2+2*x4^2*x5*x6^2-x4*x5^2*x6^2-x2^2*x3*x4*\
x7-2*x3^2*x4*x7-2*x1*x2*x3*x5*x7+x1*x3*x4*x5*x7+x2*x3*x4*x5*x7+x3^2*\
*x4*x5*x7+2*x3*x4^2*x5*x7-x3*x4*x5^2*x7+x1*x2*x3*x6*x7+x2*x3*x4*x6*x\
7+x2^2*x5*x6*x7+x1*x3*x5*x6*x7-2*x2*x3*x5*x6*x7+4*x3*x4*x5*x6*x7-2*x1*\
x5^2*x6*x7-x3*x5^2*x6*x7-6*x4*x5^2*x6*x7-x5^3*x6*x7+x1*x5*x6^2*x7+2*\
x4*x5*x6^2*x7-2*x5^2*x6^2*x7-2*x3*x4*x7^2+x3*x4*x5*x7^2+2*x2*x5*x6*\
x7^2-x3*x5*x6*x7^2+x4*x5*x6*x7^2-2*x5^2*x6*x7^2-2*x1*x2*x3*x4*x8+x2^\
2*x3*x4*x8+x3*x2*x3*x4^2*x8-x3^2*x4^2*x8+2*x1*x2*x3*x5*x8+x2*x3^2*x5*\
x8+x1*x2*x4*x5*x8+x3*x4^2*x5*x8+x4^2*x5^2*x8+2*x1*x2*x3*x6*x8-2*x1*x\
2*x4*x6*x8-2*x2*x3*x4*x6*x8-2*x3*x4^2*x6*x8+x1*x2*x5*x6*x8+x2*x3*x5*x6*\
*x8-2*x1*x4*x5*x6*x8-2*x2*x4*x5*x6*x8-2*x3*x4*x5*x6*x8-x4^2*x5*x6*x8\
+x3*x5^2*x8+x4*x5^2*x6*x8-2*x4*x5*x6^2*x8+x5^2*x6^2*x8-5*x1*x2*x3*\
*x7*x8-2*x2*x3*x7*x8-2*x3*x7*x8+2*x1*x2*x4*x7*x8+x2^2*x4*x7*x8-2*\
x2*x3*x4*x7*x8+x2*x4^2*x7*x8+x3*x4^2*x7*x8+3*x1*x2*x5*x7*x8-x1*x3*x5*\
*x7*x8+2*x3*x5*x7*x8-x3^2*x5*x7*x8+2*x1*x4*x5*x7*x8+3*x2*x4*x5*x7*x\
8+3*x3*x4*x5*x7*x8-2*x2*x3*x4^2*x5*x7*x8-x3*x5^2*x7*x8+5*x4*x5^2*x7*x8-2*x*\
1*x2*x6*x7*x8-x1*x3*x6*x7*x8-4*x2*x3*x6*x7*x8+x1*x4*x6*x7*x8+3*x2*x4*\
*x6*x7*x8-4*x3*x4*x6*x7*x8+x4^2*x6*x7*x8+x2*x5*x6*x7*x8-4*x4*x5*x6*x\
7*x8+2*x5^2*x6*x7*x8+x1*x6^2*x7*x8-2*x6^2*x7*x8-x5*x6^2*x7*x8-2*x1*\
x2*x7^2*x8-x2^2*x3*x7^2*x8+x1*x3*x7^2*x8-2*x2*x3*x7^2*x8+x3^2*x7^2*x8+x\
1*x5*x7^2*x8+3*x5*x7^2*x8+2*x4*x5*x7^2*x8+x5^2*x7^2*x8-2*x1*x6*x7^2*\
*x8-2*x2*x6*x7^2*x8-2*x3*x6*x7^2*x8-3*x5*x6*x7^2*x8-4*x6^2*x7^2*x8-x\
2*x7^3*x8+x3*x7^3*x8+x5*x7^3*x8-3*x6*x7^3*x8+x1*x2*x3*x8^2-x2*x3*x2*x\
8^2+x1*x3*x4*x8^2+4*x2*x3*x4*x8^2+3*x3*x4^2*x8^2-x1*x2*x5*x8^2-x1*x3*x\
5*x8^2-2*x2*x3*x5*x8^2-x3^2*x5*x8^2-x1*x4*x5*x8^2-x2*x4*x5*x8^2-2*x3*\
*x4*x5*x8^2-x4^2*x5*x8^2-x3*x5^2*x8^2-2*x4*x5^2*x8^2-x1*x2*x6*x8^2-x\
2*x3*x6*x8^2+2*x1*x4*x6*x8^2+2*x2*x4*x6*x8^2+2*x3*x4*x6*x8^2+2*x4^2*\
x6*x8^2-x1*x5*x6*x8^2-x2*x5*x6*x8^2-3*x3*x5*x6*x8^2-2*x5^2*x6*x8^2+2*\
*x4*x6^2*x8^2-2*x5*x6^2*x8^2+5*x1*x2*x7*x8^2-x1*x3*x7*x8^2+3*x2*x3*x\
7*x8^2-x3^2*x7*x8^2-x1*x4*x7*x8^2+3*x3*x4*x7*x8^2-x4^2*x7*x8^2-4*x1*\
x5*x7*x8^2-4*x2*x5*x7*x8^2-6*x3*x5*x7*x8^2+2*x4*x5*x7*x8^2-4*x5^2*x7*\
*x8^2+4*x1*x6*x7*x8^2+4*x2*x6*x7*x8^2+3*x3*x6*x7*x8^2+4*x4*x6*x7*x8^2\
+5*x5*x6*x7*x8^2+4*x6^2*x7*x8^2+4*x1*x7^2*x8^2+3*x2*x7^2*x8^2+4*x3*\
x7^2*x8^2+4*x4*x7^2*x8^2+4*x5*x7^2*x8^2+5*x6*x7^2*x8^2+3*x7^3*x8^2-2*\
*x1*x2*x8^3+x1*x3*x8^3-x2*x3*x8^3+x3^2*x8^3+x1*x5*x8^3+x2*x5*x8^3+x3*\
*x5*x8^3-2*x4*x5*x8^3+x5^2*x8^3+x1*x6*x8^3+2*x6*x8^3+2*x3*x6*x8^3+3*\
*x4*x6*x8^3-5*x5*x6*x8^3+x6^2*x8^3-7*x1*x7*x8^3-7*x2*x7*x8^3-8*x3*x7*x\
8^3-8*x4*x7*x8^3-11*x5*x7*x8^3-3*x6*x7*x8^3+2*x7^2*x8^3+2*x1*x8^4+2*\
x2*x8^4+3*x3*x8^4+2*x4*x8^4+3*x5*x8^4+3*x6*x8^4-7*x7*x8^4+2*x8^5

```
BGbis[5]
```

x2^2*x3*x4^2+x2*x3^2*x4^2+x2*x3*x4^3+x2*x3^2*x4*x5+x1*x3*x4^2*x5+2*x\
3^2*x4^2*x5+x3*x4^3*x5-2*x1*x3*x4*x5^2-2*x2*x3*x4*x5^2-x3^2*x4*x5^2-\
x3*x4*x5^3+2*x1*x2*x3*x4*x6+2*x2*x3*x4^2*x6+x1*x2*x3*x5*x6-2*x1*x2*x\
4*x5*x6-2*x2*x4*x5*x6+x1*x4^2*x5*x6+2*x3*x4^2*x5*x6+x4^3*x5*x6-2*x1*\
x4*x5^2*x6-2*x2*x4*x5^2*x6-2*x3*x4*x5^2*x6-x4^2*x5^2*x6-x4*x5^3*x6+x\
1*x4*x5*x6^2-2*x2*x4*x5*x6^2+2*x4^2*x5*x6^2-x4*x5^2*x6^2-x2^2*x3*x4*\
x7-2*x3^2*x4*x7-2*x1*x2*x3*x5*x7+x1*x3*x4*x5*x7+x2*x3*x4*x5*x7+x3^2*\
*x4*x5*x7+2*x3*x4^2*x5*x7-x3*x4*x5^2*x7+x1*x2*x3*x6*x7+x2*x3*x4*x6*x\
7+x2^2*x5*x6*x7+x1*x3*x5*x6*x7-2*x2*x3*x5*x6*x7+4*x3*x4*x5*x6*x7-2*x1*\
x5^2*x6*x7-x3*x5^2*x6*x7-6*x4*x5^2*x6*x7-x5^3*x6*x7+x1*x5*x6^2*x7+2*\
x4*x5*x6^2*x7-2*x5^2*x6^2*x7-2*x3*x4*x7^2+x3*x4*x5*x7^2+2*x2*x5*x6*\
x7^2-x3*x5*x6*x7^2+x4*x5*x6*x7^2-2*x5^2*x6*x7^2-2*x1*x2*x3*x4*x8+x2^\
2*x3*x4*x8+x3*x2*x3*x4^2*x8-x3^2*x4^2*x8+2*x1*x2*x3*x5*x8+x2*x3^2*x5*\
x8+x1*x2*x4*x5*x8+x3*x4^2*x5*x8+x4^2*x5^2*x8+2*x1*x2*x3*x6*x8-2*x1*x\
2*x4*x6*x8-2*x2*x3*x4*x6*x8-2*x3*x4^2*x6*x8+x1*x2*x5*x6*x8+x2*x3*x5*x6*\
*x8-2*x1*x4*x5*x6*x8-2*x2*x4*x5*x6*x8-2*x3*x4*x5*x6*x8-x4^2*x5*x6*x8\
+x3*x5^2*x8+x4*x5^2*x6*x8-2*x4*x5*x6^2*x8+x5^2*x6^2*x8-5*x1*x2*x3*\
*x7*x8-2*x2*x3*x7*x8-2*x3*x7*x8+2*x1*x2*x4*x7*x8+x2^2*x4*x7*x8-2*\
x2*x3*x4*x7*x8+x2*x4^2*x7*x8+x3*x4^2*x7*x8+3*x1*x2*x5*x7*x8-x1*x3*x5*\
*x7*x8+2*x3*x5*x7*x8-x3^2*x5*x7*x8+2*x1*x4*x5*x7*x8+3*x2*x4*x5*x7*x\
8+3*x3*x4*x5*x7*x8-2*x2*x3*x4^2*x5*x7*x8-x3*x5^2*x7*x8+5*x4*x5^2*x7*x8-2*x*\
1*x2*x6*x7*x8-x1*x3*x6*x7*x8-4*x2*x3*x6*x7*x8+x1*x4*x6*x7*x8+3*x2*x4*\
*x6*x7*x8-4*x3*x4*x6*x7*x8+x4^2*x6*x7*x8+x2*x5*x6*x7*x8-4*x4*x5*x6*x\
7*x8+2*x5^2*x6*x7*x8+x1*x6^2*x7*x8-2*x6^2*x7*x8-x5*x6^2*x7*x8-2*x1*\
x2*x7^2*x8-x2^2*x3*x7^2*x8+x1*x3*x7^2*x8-2*x2*x3*x7^2*x8+x3^2*x7^2*x8+x\
1*x5*x7^2*x8+3*x5*x7^2*x8+2*x4*x5*x7^2*x8+x5^2*x7^2*x8-2*x1*x6*x7^2*\
*x8-2*x2*x6*x7^2*x8-2*x3*x6*x7^2*x8-3*x5*x6*x7^2*x8-4*x6^2*x7^2*x8-x\
2*x7^3*x8+x3*x7^3*x8+x5*x7^3*x8-3*x6*x7^3*x8+x1*x2*x3*x8^2-x2*x3*x2*x\
8^2+x1*x3*x4*x8^2+4*x2*x3*x4*x8^2+3*x3*x4^2*x8^2-x1*x2*x5*x8^2-x1*x3*x\
5*x8^2-2*x2*x3*x5*x8^2-x3^2*x5*x8^2-x1*x4*x5*x8^2-x2*x4*x5*x8^2-2*x3*\
*x4*x5*x8^2-x4^2*x5*x8^2-x3*x5^2*x8^2-2*x4*x5^2*x8^2-x1*x2*x6*x8^2-x\
2*x3*x6*x8^2+2*x1*x4*x6*x8^2+2*x2*x4*x6*x8^2+2*x3*x4*x6*x8^2+2*x4^2*\
x6*x8^2-x1*x5*x6*x8^2-x2*x5*x6*x8^2-3*x3*x5*x6*x8^2-2*x5^2*x6*x8^2+2*\
*x4*x6^2*x8^2-2*x5*x6^2*x8^2+5*x1*x2*x7*x8^2-x1*x3*x7*x8^2+3*x2*x3*x\
7*x8^2-x3^2*x7*x8^2-x1*x4*x7*x8^2+3*x3*x4*x7*x8^2-x4^2*x7*x8^2-4*x1*\
x5*x7*x8^2-4*x2*x5*x7*x8^2-6*x3*x5*x7*x8^2+2*x4*x5*x7*x8^2-4*x5^2*x7*\
*x8^2+4*x1*x6*x7*x8^2+4*x2*x6*x7*x8^2+3*x3*x6*x7*x8^2+4*x4*x6*x7*x8^2\
+5*x5*x6*x7*x8^2+4*x6^2*x7*x8^2+4*x1*x7^2*x8^2+3*x2*x7^2*x8^2+4*x3*\
x7^2*x8^2+4*x4*x7^2*x8^2+4*x5*x7^2*x8^2+5*x6*x7^2*x8^2+3*x7^3*x8^2-2*\
*x1*x2*x8^3+x1*x3*x8^3-x2*x3*x8^3+x3^2*x8^3+x1*x5*x8^3+x2*x5*x8^3+x3*\
*x5*x8^3-2*x4*x5*x8^3+x5^2*x8^3+x1*x6*x8^3+2*x6*x8^3+2*x3*x6*x8^3+3*\
*x4*x6*x8^3-5*x5*x6*x8^3+x6^2*x8^3-7*x1*x7*x8^3-7*x2*x7*x8^3-8*x3*x7*x\
8^3-8*x4*x7*x8^3-11*x5*x7*x8^3-3*x6*x7*x8^3+2*x7^2*x8^3+2*x1*x8^4+2*\
x2*x8^4+3*x3*x8^4+2*x4*x8^4+3*x5*x8^4+3*x6*x8^4-7*x7*x8^4+2*x8^5

```
*x8-2*x1*x4*x5*x6*x8-2*x2*x4*x5*x6*x8-2*x3*x4*x5*x6*x8-x4^2*x5*x6*x8\
+x3*x5^2*x6*x8+x4*x5^2*x6*x8-2*x4*x5*x6^2*x8+x5^2*x6^2*x8-5*x1*x2*x3\
*x7*x8-x2^2*x3*x7*x8-x2*x3^2*x7*x8+2*x1*x2*x4*x7*x8+x2^2*x4*x7*x8-2*\
x2*x3^2*x4*x7*x8+x2*x4^2*x7*x8+x3*x4^2*x7*x8+3*x1*x2*x5*x7*x8-x1*x3*x5\
*x7*x8+x2^2*x3*x5*x7*x8-x3^2*x5*x7*x8+2*x1*x4*x5*x7*x8+3*x2*x4*x5*x7*x\
8+3*x3*x4*x5*x7*x8+3*x4^2*x5*x7*x8-x3*x5^2*x7*x8+5*x4*x5^2*x7*x8-2*x\
1*x2*x6*x7*x8-x1*x3*x6*x7*x8-4*x2*x3*x6*x7*x8+x1*x4*x6*x7*x8+3*x2*x4\
*x6*x7*x8-4*x3*x4*x6*x7*x8+x4^2*x6*x7*x8+x2*x5*x6*x7*x8-4*x4*x5*x6*x\
7*x8+2*x5^2*x6*x7*x8+x1*x6^2*x7*x8-x2*x6^2*x7*x8-x5*x6^2*x7*x8-2*x1\
x2*x7^2*x8-x2^2*x7^2*x8+x1*x3*x7^2*x8-2*x2*x3*x7^2*x8+x3^2*x7^2*x8+x\
1*x5*x7^2*x8+x3*x5*x7^2*x8+2*x4*x5*x7^2*x8+x5^2*x7^2*x8-2*x1*x6*x7^2\
*x8-2*x2*x6*x7^2*x8-2*x3*x6*x7^2*x8-3*x5*x6*x7^2*x8-4*x6^2*x7^2*x8-x\
2*x7^3*x8+x3*x7^3*x8+x5*x7^3*x8-3*x6*x7^3*x8+x1*x2*x3*x8^2-x2*x3^2*x\
8^2+x1*x3*x4*x8^2+4*x2*x3*x4*x8^2+x3*x4^2*x8^2-x1*x2*x5*x8^2-x1*x3*x\
5*x8^2-2*x2*x3*x5*x8^2-x3^2*x5*x8^2-x1*x4*x5*x8^2-x2*x4*x5*x8^2-2*x3\
*x4*x5*x8^2-x4^2*x5*x8^2-x3*x5^2*x8^2-2*x4*x5^2*x8^2-x1*x2*x6*x8^2-x\
2*x3*x6*x8^2+2*x1*x4*x6*x8^2+2*x2*x4*x6*x8^2+2*x3*x4*x6*x8^2+2*x4^2*\
x6*x8^2-x1*x5*x6*x8^2-x2*x5*x6*x8^2-3*x3*x5*x6*x8^2-2*x5^2*x6*x8^2+2\
*x4*x6^2*x8^2-2*x5*x6^2*x8^2+5*x1*x2*x7*x8^2-x1*x3*x7*x8^2+3*x2*x3*x\
7*x8^2-x3^2*x7*x8^2-x1*x4*x7*x8^2+3*x3*x4*x7*x8^2-x4^2*x7*x8^2-4*x1\
x5*x7*x8^2-4*x2*x5*x7*x8^2-6*x3*x5*x7*x8^2+2*x4*x5*x7*x8^2-4*x5^2*x\
*x8^2+4*x1*x6*x7*x8^2+4*x2*x6*x7*x8^2+3*x3*x6*x7*x8^2+4*x4*x6*x7*x8\
2+5*x5*x6*x7*x8^2+4*x6^2*x7*x8^2+4*x1*x7^2*x8^2+3*x2*x7^2*x8^2+4*x3\
x7^2*x8^2+4*x4*x7^2*x8^2+4*x5*x7^2*x8^2+5*x6*x7^2*x8^2+3*x7^3*x8^2-2\
*x1*x2*x8^3+x1*x3*x8^3-x2*x3*x8^3+x3^2*x8^3+x1*x5*x8^3+x2*x5*x8^3+x3\
*x5*x8^3-2*x4*x5*x8^3+x5^2*x8^3+x1*x6*x8^3+x2*x6*x8^3+2*x3*x6*x8^3+x\
x4*x6*x8^3-x5*x6*x8^3+x6^2*x8^3-7*x1*x7*x8^3-7*x2*x7*x8^3-8*x3*x7*x\
8^3-8*x4*x7*x8^3-11*x5*x7*x8^3-3*x6*x7*x8^3+2*x7^2*x8^3+2*x1*x8^4+2*\
x2*x8^4+3*x3*x8^4+2*x4*x8^4+3*x5*x8^4+3*x6*x8^4-7*x7*x8^4+2*x8^5
```

(BG[500]).unapply(R.gens())(1,2,3,4,5,6,7,8,9)

```
11212151819618889725369227417425987119826525407678140245400787958735\
31991528287083835393418485985131260131437235615651126092588268819914\
37793671436207821151072742408585376421434632173332587384683728718493\
96023567085144447511599278262168294272446499692424789905109233884425\
14796043252807019214827318274157958045734734120719832492757309950924\
13360510337382127462638789994606302397282064621793572249947021671524\
97257737011026471199553535250713746995060954722051559102174092577524\
40559840511600813031460476706101539254639682773586190329890430355233\
353513429389779357871541870665047995520476212785257441726974
```

(BGbis[500]).unapply(R.gens())(1,2,3,4,5,6,7,8,9)

```
11212151819618889725369227417425987119826525407678140245400787958735\
31991528287083835393418485985131260131437235615651126092588268819914\
37793671436207821151072742408585376421434632173332587384683728718493\
96023567085144447511599278262168294272446499692424789905109233884425\
14796043252807019214827318274157958045734734120719832492757309950924\
13360510337382127462638789994606302397282064621793572249947021671524\
97257737011026471199553535250713746995060954722051559102174092577524\
40559840511600813031460476706101539254639682773586190329890430355233\
353513429389779357871541870665047995520476212785257441726974
```

Time to send the QQ groebner basis of cyclic9 from libgiac to sage:

time L=[R(j) for j in BGbis]

Time: CPU 170.36 s, Wall: 170.70 s