The integrated density of states for an interacting multielectron homogeneous model

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Abstract

For a system of n interacting electrons moving in the background of a "homogeneous" potential, we show that, if the single particle Hamiltonian admits a density of states, so does the interacting Hamiltonian. Moreover this integrated density of states coincides with that of the free electron Hamiltonian.

1 Introduction

We consider n interacting electrons moving in a "homogeneous" electric field in the d-dimensional configuration space $\mathbb{R}^d$. A typical example of what we mean by a "homogeneous" potential is an Anderson or alloy-type random potential. The goal of the present short note is to prove that, if the Hamiltonian of the single particle in the "homogeneous" media admits an integrated density of states (IDS), then, so does the interacting n-particle Hamiltonian. Moreover, this IDS is equal to that of the interacting n-particle Hamiltonian. Heuristically, this is easily understood : it follows from the fact that the electron-electron interaction essentially lives on a strict sub-manifold of the total configuration space, whereas the IDS is a thermodynamic limit over the whole space and, as such, measures "bulk" phenomena.

1.1 The interacting multi-electron model

The non-interacting n-electron Hamiltonian satisfies $H_n = -\triangle + V_n$ where the Laplacian $-\triangle$ on $\mathbb{R}^{nd}$ describes the free kinetic energy of the n electrons. As all the electrons are in the same background, the potential V_n is of the form

$$V_n = \sum_{k=1}^{n} \mathbf{1}_{L^2(\mathbb{R}^d)}^{\otimes(k-1)} \otimes V_1 \otimes \mathbf{1}_{L^2(\mathbb{R}^d)}^{\otimes(n-k)}. \tag{1.1}$$

Hence, the noninteracting n-electron Hamiltonian takes the form

$$H_n := \sum_{k=1}^{n} \mathbf{1}_{L^2(\mathbb{R}^d)}^{\otimes(k-1)} \otimes H_1 \otimes \mathbf{1}_{L^2(\mathbb{R}^d)}^{\otimes(n-k)} \text{ where } H_1 = -\Delta + V_1. \tag{1.2}$$

On the one particle potential V_1, we assume that

(H.1.a) $(V_1)_+ := \max\{V_1, 0\}$ is locally square integrable and $(V_1)_- := \max\{-V_1, 0\}$ is an infinitesimally $-\triangle$-bounded potential that is $\mathcal{D}((V_1)_-) \supseteq \mathcal{D}(-\triangle)$ and for all $\alpha > 0$, there exists $\gamma(\alpha) < \infty$, such that for all $\phi \in \mathcal{D}(-\triangle)$

$$\|(V_1)_-\phi\| \leq \alpha\| \triangle \phi\| + \gamma(\alpha)\|\phi\| \tag{1.3}$$

(H.1.b) the operator H_1 admits an integrated density of states, say N_1, that is, if $H_{1|\Lambda(0,L)}$ denotes the Dirichlet restriction of H_1 to a cube $\Lambda(0, L)$ centered at 0 of side-length, L, then the following limit exists

$$N_1(E) := \lim_{L\to+\infty} L^{-d}\text{Trace}(\mathbf{1}_{]-\infty,E]}(H_{1|\Lambda(0,L)})).$$

Assumption (H.1.a) implies essential self-adjointness of $-\triangle + V_1$ on $C_0^\infty(\mathbb{R}^d)$ by [RS2] Theorem X.29. Indeed,

$$V_n = V_n^+ - V_n^-, \quad V_n^\pm(x_1, ..., x_n) := \sum_{j=1}^{n} (V_1)_\pm(x_j) \tag{1.4}$$

where

- V_n^- is infinitesimally $-\triangle$-bounded i.e. (1.3) holds for the same constants and the Laplacian over $\mathbb{R}^{nd}$,
- V_n^+ is non-negative locally square integrable.

The self-adjoint extensions of $-\triangle + V_1$ and $-\triangle + V_n$ are again denoted by H_1 and H_n; they are bounded from below.
Classical models for which the IDS is known to exist include periodic, quasi-periodic and ergodic random Schrödinger operators (see e.g. [PaFi]).
In the definition of the density of states, we could also have considered the case of Neumann or other boundary conditions.

The interacting n-particle Hamiltonian is of the form

$$H := -\triangle + V_i + V_n, \tag{1.5}$$

where

$$V_i(x_1, ..., x_n) := \sum_{1\leq k<l\leq n} V(x_k - x_l) \tag{1.6}$$

is a localized repulsive interaction potential generated by the electrons; so we assume

(H.2) $V : \mathbb{R}^d \to \mathbb{R}$ is measurable non-negative, locally square integrable and V tends to 0 at infinity.

The standard repulsive interaction in three dimensional space is of course the Coulomb interaction $V(x) = 1/|x|$. In some cases, due to screening, it must be replaced by the Yukawa interaction $V(x) = e^{-|x|}/|x|$.

Finally, we make one more assumption on both V_1 and V : we assume that

(H.3) the operator $V_i(H_n - i)^{-1}$ is bounded.

Assumption (H.3) is satisfied in the case of the Coulomb and Yukawa potential for those V_1 satisfying (H.1.a): H_n is self-adjoint on $\mathcal{D}(H_n) \subseteq \mathcal{D}(-\triangle)$, hence $\|V_i(H_n - i)^{-1}\| \leq \|V_i(-\triangle - i)^{-1}\| \cdot \|(-\triangle - i)(H_n - i)^{-1}\|$, where $\|(-\triangle - i)(H_n - i)^{-1}\| < \infty$ due to closed graph theorem and $\|V_i(-\triangle - i)^{-1}\| < \infty$ for Coulomb and Yukawa interaction potentials V_i, see [RS2] Theorem X.16.

2 The integrated density of states

We now compute the IDS for the n-electron model. Let $\Lambda_L = \Lambda(0, L)$ be the cube in $\mathbb{R}^d$ centered at 0 with side-length L and write $\Lambda_L^n = \Lambda_L \times \cdots \times \Lambda_L$ for the product of n copies of Λ_L. We denote the restriction of the interacting n-particle Hamiltonian H to Λ_L^n with Dirichlet boundary conditions by $H_{|\Lambda_L^n}$. Clearly assumptions (H.2) and (H.1.a) guarantee that $H_{|\Lambda_L^n}$ is bounded from below with compact resolvent. Hence, for any $E \in \mathbb{R}$, one defines the normalized counting functions

$$N_L(E) := L^{-nd}\mathrm{Trace}(\mathbf{1}_{]-\infty,E]}(H_{|\Lambda_L^n})).$$

As usual, N, the IDS of H is defined as the limit of $N_L(E)$ when $L \to +\infty$. Equivalently, one can define the density of states measure applied to a test function φ as the limit of $L^{-nd}\mathrm{Trace}[\varphi(H_{|\Lambda_L^n})]$. If the limit exists, it defines a non-negative measure. It is a classical result that the existence of that limit (for all test functions) or that of $N_L(E)$ are equivalent ([PaFi]).

2.1 The IDS for the noninteracting n-electron system

Recall that, by assumption (H.1.b), the single particle model H_1 admits an IDS (see [PaFi]) and a density of states measure denoted respectively by N_1 and ν_1.
Let $H_{n|\Lambda_L^n}$ be the restriction of H_n to Λ_L^n with Dirichlet boundary conditions. One has

Lemma 2.1. *The IDS for the noninteracting n-electron model given by*

$$N_n(E) := \lim_{L\to\infty} \frac{1}{L^{nd}}\mathrm{Trace}(\mathbf{1}_{]-\infty,E]}H_{n|\Lambda_L^n}) \tag{2.1}$$

exists and satisfies

$$N_n = N_1 * \nu_1 * \cdots * \nu_1. \tag{2.2}$$

Let us comment on this result. First, the convolution product in (2.2) makes sense as all the measures and functions are supported on half-axes of the form $[a, +\infty)$; this results from assumption (H.1.a) . When the field V_1 is not bounded from below, one will need some estimate on the decay of N_1 and ν_1 near $-\infty$ to make sense of (2.2) (and to prove it); such estimates are known for some models (see e.g. [PaFi, KlPa]).

Proof of Lemma 2.1. The operator H_n is the sum of n commuting Hamiltonians, each of which is unitarily equivalent to H_1; so is $H_{n|\Lambda_L^n}$, its restriction to the cube Λ_L^n. As the sum decomposition of H_n commutes with the restriction to Λ_L^n, the eigenvalues of $H_{n|\Lambda_L^n}$ are exactly the sum of n eigenvalues of H_1 restricted to Λ_L. This immediately yields that

$$\mathrm{Trace}(\mathbf{1}_{]-\infty,E]}(H_{n|\Lambda_L^n})) = (\hat{N}_1^L * \hat{\nu}_1^L * \cdots * \hat{\nu}_1^L)(E)$$

where $\hat{N}_1^L(E)$ is the eigenvalue counting function for H_1 restricted to Λ_L, and $\hat{\nu}_1^L$ its counting measure (i.e. $d\hat{N}_1^L$). The normalized counting function and measure, N_1^L and ν_1^L, are defined as

$$N_1^L = \frac{1}{L^d}\hat{N}_1^L \quad \text{and} \quad \nu_1^L = \frac{1}{L^d}\hat{\nu}_1^L.$$

The existence of the density of states of H_1 then exactly says that N_1^L and ν_1^L converge respectively to N_1 and ν_1. The convergence of $N_1^L * \nu_1^L * \cdots * \nu_1^L$ to $N_1 * \nu_1 * \cdots * \nu_1$ is then guaranteed as the convolution is bilinear bi-continuous operation on distributions. This completes the proof of Lemma 2.1. □

Let us now say a word on the boundary conditions chosen to define the IDS. Here, we chose to define it as a thermodynamic limit of the normalized counting for Dirichlet eigenvalues. Clearly, if we know that the single particle Hamiltonian has a IDS defined as the thermodynamic limit of the normalized counting for Neumann eigenvalues, so does the non-interacting n-body Hamiltonian. Moreover, in the case when the two limits coincide for the one-body Hamiltonian, they also coincide for the non-interacting n-body Hamiltonian. Using Dirichlet-Neumann bracketing, one then sees that the integrated densities of states for both the one-body and non-interacting n-body Hamiltonian for positive mixed boundary conditions also exist and coincide with that defined with either Dirichlet or Neumann boundary conditions.

2.2 Existence of the IDS for the interacting n-electron system

Let $H_{|\Lambda_L^n}$ denote the restriction of H to the box Λ_L^n with Dirichlet boundary conditions. Our main result is

Theorem 2.2. *Assume (H.1), (H.2) and (H.3) are satisfied. For any $\varphi \in C_0^\infty(\mathbb{R})$, one has*

$$\frac{1}{L^{nd}}\mathrm{Trace}[\varphi(H_{|\Lambda_L^n}) - \varphi(H_{n|\Lambda_L^n})] \overset{L\to\infty}{\longrightarrow} 0. \tag{2.3}$$

As the density of states measure of H is defined by

$$\langle \varphi, dN \rangle = \lim_{L\to+\infty} \frac{1}{L^{nd}}\mathrm{Trace}[\varphi(H_{|\Lambda_L^n})].$$

we immediately get the

Corollary 2.3. *Assume (H.1), (H.2) and (H.3) are satisfied. The IDS for the interacting n-electron model H exists and coincides with that of the noninteracting model H_0; hence, it satisfies*

$$N = N_n = N_1 * \nu_1 * \cdots * \nu_1.$$

Note that, in view of the remark concluding section 2.1, we see that the integrated density of states of the interacting n-body Hamiltonian is independent of the boundary conditions if that of the one-body Hamiltonian is.

One of the interesting properties of the integrated density of states is its regularity; it is well known to play an important role in the theory of localization for random one-particle models (see e.g. [Stl]). Usually, it comes into play through a Wegner estimate i.e. an estimate of the type

$$\mathbb{E}(\mathrm{Trace}\mathbf{1}_{]E_0-\eta,E_0+\eta[}(H_{|\Lambda_L^n})) \leq C_W \eta |\Lambda_L^n|$$

For a specific model of random one-particle Hamiltonian, a Wegner estimate for the IDS of the interacting Hamiltonian was proved in [Ze]. This estimate yields Lipshitz continuity of the IDS for that model.

On the other hand, Corollary 2.3 directly relates the regularity of the IDS of the interacting system to that of the IDS of the single particle Hamiltonian. The regularity of the IDS of the single particle has been the subject of a lot of interest recently (see e.g. [CHK, Stz]).

Proof of Theorem 2.2. We take some $q > \frac{nd}{2}$ and specify the appropriate choice later on. By assumption (H.1.a) and (H.2), there exists $\zeta > 0$ such that

$$-\infty < -\zeta \leq \min\left(\inf_{L\geq 1}\{\inf[\sigma(H_{n|\Lambda_L^n}) \cup \sigma(H_{|\Lambda_L^n})]\}, \inf[\sigma(H_n)\cup\sigma(H)]\right). \tag{2.4}$$

Let $\gamma = \gamma(1/2)$ be given by (1.3) for $\alpha = 1/2$. Fix $\lambda_0 > \zeta + 2\gamma + 1$.
By (2.4), we only need to prove (2.3) for $\varphi \in C_0^\infty(\mathbb{R})$ supported in $(-\zeta-1, +\infty)$. For such a function, let $\tilde{\varphi}$ be an almost analytic extension of the function $x \mapsto (x+\lambda_0)^q\varphi(x) \in C_0^\infty(\mathbb{R})$ i.e. $\tilde{\varphi}$ satisfies

- $\tilde{\varphi} \in \mathcal{S}(\{z \in \mathbb{C} : |\Im z| < 1\}$
- for any $k \in \mathbb{N}$, the family of functions $(x \mapsto \frac{\partial\tilde{\varphi}}{\partial\bar{z}}(x+iy)|y|^{-k})_{0<|y|<1}$ is bounded in $\mathcal{S}(\mathbb{R})$.

The functional calculus based on the Helffer-Sjöstrand formula implies

$$\begin{aligned}&\varphi(H_{|\Lambda_L^n}) - \varphi(H_{n|\Lambda_L^n}) = \\ &= \frac{i}{2\pi}\int_{\mathbb{C}} \frac{\partial\tilde{\varphi}}{\partial\bar{z}}(z)[(H_{|\Lambda_L^n}+\lambda_0)^{-q}(H_{|\Lambda_L^n}-z)^{-1} - \\ &\qquad (H_{n|\Lambda_L^n}+\lambda_0)^{-q}(H_{n|\Lambda_L^n}-z)^{-1}]dz\wedge d\bar{z}.\end{aligned} \tag{2.5}$$

In the following, we apply an idea, which has already been used in [Kl] and [KlPa] and which simplifies in this situation. Using resolvent equality, the integrand in (2.5) is written as

$$
\begin{aligned}
(H_{|\Lambda_L^n} + \lambda_0)^{-q}(H_{|\Lambda_L^n} - z)^{-1} - (H_{n|\Lambda_L^n} + \lambda_0)^{-q}(H_{n|\Lambda_L^n} - z)^{-1} = \\
= (H_{n|\Lambda_L^n} + \lambda_0)^{-q}[(H_{|\Lambda_L^n} - z)^{-1} - (H_{n|\Lambda_L^n} - z)^{-1}] \\
+ [(H_{|\Lambda_L^n} + \lambda_0)^{-q} - (H_{n|\Lambda_L^n} + \lambda_0)^{-q}](H_{|\Lambda_L^n} - z)^{-1} \\
= -(H_{n|\Lambda_L^n} + \lambda_0)^{-q}(H_{n|\Lambda_L^n} - z)^{-1} V_i (H_{|\Lambda_L^n} - z)^{-1} \\
- \sum_{l=1}^{q} (H_{n|\Lambda_L^n} + \lambda_0)^{l-q-1} V_i (H_{|\Lambda_L^n} + \lambda_0)^{-l}(H_{|\Lambda_L^n} - z)^{-1}
\end{aligned}
\tag{2.6}
$$

Estimating the trace of (2.6), we choose $\varepsilon > 0$ and write

$$
V_i = V_i \cdot \mathbf{1}_{\{|V_i| \le \varepsilon\}} + V_i \cdot \mathbf{1}_{\{|V_i| > \varepsilon\}} \tag{2.7}
$$

and note, that $V_i \cdot \mathbf{1}_{\{|V_i| \le \varepsilon\}}$ is bounded by $\|V_i \cdot \mathbf{1}_{\{|V_i| \le \varepsilon\}}\| \le \varepsilon$. As V is non-negative, one has

$$
\operatorname{supp}(V_i \cdot \mathbf{1}_{\{|V_i| > \varepsilon\}}) \subseteq \bigcup_{j=1}^{n} \bigcup_{\substack{i=1 \\ i \neq j}}^{n} \left\{ (x_1, \ldots, x_n) \in \mathbb{R}^{nd} : \, V(x_i - x_j) \ge \frac{\varepsilon}{n(n-1)} \right\}. \tag{2.8}
$$

As, by assumption (H.2), V tends to 0 at infinity, (2.8) implies that there exists $0 < C(n; \varepsilon)$ (independent of L) such that

$$
\mu(\{|V_i| > \varepsilon\} \cap \Lambda_L^n) \le C(n, \varepsilon) L^{(n-1)d}, \tag{2.9}
$$

where $\mu(\cdot)$ denotes the Lebesgue measure. Using decomposition (2.7) of V_i, we obtain

$$
\begin{aligned}
|\mathrm{Trace}(H_{n|\Lambda_L^n} + \lambda_0)^{-q}(H_{n|\Lambda_L^n} - z)^{-1} V_i (H_{|\Lambda_L^n} - z)^{-1}| \\
\le \frac{\varepsilon}{|\Im z|^2} \mathrm{Trace}|(H_{n|\Lambda_L^n} + \lambda_0)^{-q}| \\
+ \frac{1}{|\Im z|} \|V_i (H_{|\Lambda_L^n} - z)^{-1}\| \cdot \mathrm{Trace}|(H_{n|\Lambda_L^n} + \lambda_0)^{-q} \mathbf{1}_{\{|V_i| > \varepsilon\} \cap \Lambda_L^n}| \\
\le \frac{\varepsilon}{|\Im z|^2} \|(H_{n|\Lambda_L^n} + \lambda_0)^{-1}\|_{\mathcal{I}_q}^q + \frac{1}{|\Im z|^2} \|(H_{n|\Lambda_L^n} + \lambda_0)^{-1}\|_{\mathcal{I}_q}^{q-1} \\
\cdot \|(H_{n|\Lambda_L^n} + \lambda_0)^{-1} \mathbf{1}_{\{|V_i| > \varepsilon\} \cap \Lambda_L^n}\|_{\mathcal{I}_q} \cdot \|V_i (H_{n|\Lambda_L^n} + \lambda_0)^{-1}\|
\end{aligned}
\tag{2.10}
$$

where $\|\cdot\|_{\mathcal{I}_q}$ denotes the q-th Schatten class norm (see [Si]) and we used Hölder's inequality. In the same way, the cyclicity of the trace yields

$$
\begin{aligned}
|\mathrm{Trace}(H_{n|\Lambda_L^n} + \lambda_0)^{l-q-1} V_i (H_{|\Lambda_L^n} + \lambda_0)^{-l}(H_{|\Lambda_L^n} - z)^{-1}| \\
\le \mathrm{Trace}|(H_{|\Lambda_L^n} + \lambda_0)^{-l}(H_{n|\Lambda_L^n} + \lambda_0)^{l-q-1} V_i (H_{|\Lambda_L^n} - z)^{-1}| \\
\le \|(H_{|\Lambda_L^n} + \lambda_0)^{-l}(H_{n|\Lambda_L^n} + \lambda_0)^{l}\| \cdot \mathrm{Trace}|(H_{n|\Lambda_L^n} + \lambda_0)^{-q-1} V_i (H_{|\Lambda_L^n} - z)^{-1}| \\
\le \frac{C}{|\Im z|} \|(H_{n|\Lambda_L^n} + \lambda_0)^{-1}\|_{\mathcal{I}_q}^{q-1} \cdot \|(H_{n|\Lambda_L^n} + \lambda_0)^{-1} \mathbf{1}_{\{|V_i| > \varepsilon\} \cap \Lambda_L^n}\|_{\mathcal{I}_q} \cdot \|V_i (H_{n|\Lambda_L^n} + \lambda_0)^{-1}\| \\
+ C \frac{\varepsilon}{|\Im z|} \|(H_{n|\Lambda_L^n} + \lambda_0)^{-1}\|_{\mathcal{I}_q}^q.
\end{aligned}
\tag{2.11}
$$

We are now left with estimating $\|(H_{n|\Lambda_L^n}+\lambda_0)^{-1}\|_{\mathcal{I}_q}$ and $\|(H_{n|\Lambda_L^n}+\lambda_0)^{-1}\mathbf{1}_{\{|V_i|>\varepsilon\}\cap\Lambda_L^n}\|_{\mathcal{I}_q}$ for q sufficiently large, depending on nd. Therefore, we compute

$$\begin{aligned}\|(H_{n|\Lambda_L^n}+\lambda_0)^{-1}\mathbf{1}_{\{|V_i|>\varepsilon\}\cap\Lambda_L^n}\|_{\mathcal{I}_q} &\leq \|(H_{n|\Lambda_L^n}+\lambda_0)^{-1}(-\triangle_{|\Lambda_L^n}+\lambda_0)^{\frac{1}{2}}\|_{\mathcal{I}_{2q}} \\ &\qquad \cdot \|(-\triangle_{|\Lambda_L^n}+\lambda_0)^{-\frac{1}{2}}\mathbf{1}_{\{|V_i|>\varepsilon\}\cap\Lambda_L^n}\|_{\mathcal{I}_{2q}} \end{aligned} \tag{2.12}$$

We use the decomposition (1.4). As the Laplacians are positive, the infinitesimal $-\Delta$-boundedness on V_n^-, Theorem X.18 of [RS2] and the definition of γ imply the following form bound

$$|\langle\phi, V^-_{n|\Lambda_L^n}\phi\rangle| \leq \frac{1}{2}\langle\phi, -\triangle_{|\Lambda_L^n}\phi\rangle + \gamma\|\phi\|^2.$$

As $\lambda_0 > 2\gamma+1$, one has

$$H_{n|\Lambda_L^n}+\lambda_0 \geq -\triangle_{|\Lambda_L^n}+V^-_{n|\Lambda_L^n}+\lambda_0 \geq \frac{1}{2}(-\triangle_{|\Lambda_L^n}-2\gamma+2\lambda_0) \geq \frac{1}{2}(-\triangle_{|\Lambda_L^n}+\lambda_0).$$

Thus, the operator $H_{n|\Lambda_L^n}+\lambda_0$ is invertible and

$$(H_{n|\Lambda_L^n}+\lambda_0)^{-1} \leq 2(-\triangle_{|\Lambda_L^n}+\lambda_0)^{-1}.$$

Let $(\mu_{\underline{j}})_{\underline{j}}$ and $(\phi_{\underline{j}})_{\underline{j}}$ respectively denote the eigenvalues and eigenfunctions of the Dirichlet Laplacian $-\triangle_{|\Lambda_L^n}$ (the index $\underline{j}$ runs over $(\mathbb{N}^{nd})^*$). For $q\in\mathbb{N}$ such that $2q>nd$, we compute

$$\begin{aligned}\|(H_{n|\Lambda_L^n}+\lambda_0)^{-1}(-\triangle_{|\Lambda_L^n}+\lambda_0)^{\frac{1}{2}}\|^{2q}_{\mathcal{I}_{2q}} &= \sum_{\underline{j}\in\mathbb{N}^{nd}}(\mu_{\underline{j}}(-\triangle_{|\Lambda_L^n})+\lambda_0)^q\langle\phi_{\underline{j}},(H_{n|\Lambda_L^n}+\lambda_0)^{-1}\phi_{\underline{j}}\rangle^{2q} \\ &\leq 2^{2q}\sum_{\underline{j}\in\mathbb{N}^{nd}}(\mu_{\underline{j}}(-\triangle_{|\Lambda_L^n})+\lambda_0)^q\langle\phi_{\underline{j}},(-\triangle_{|\Lambda_L^n}+\lambda_0)^{-1}\phi_{\underline{j}}\rangle)^{2q} \\ &= 2^{2q}\sum_{\underline{j}\in\mathbb{N}^{nd}}(\mu_{\underline{j}}(-\triangle_{|\Lambda_L^n})+\lambda_0)^{-q} \leq CL^{nd}.\end{aligned}$$

The last estimate is a direct computation using the explicit form of the Dirichlet eigenvalues.
By Lemma 2.2 in [KlPa], we know that, for $q\in\mathbb{N}$ such that $2q>nd$, there exists $C_q>0$ such that, for any measurable subset $\Lambda'\subseteq\Lambda_L^n$, one has

$$\|(-\triangle_{|\Lambda_L^n}+\lambda_0)^{-\frac{1}{2}}\mathbf{1}_{\Lambda'}\|^{2q}_{\mathcal{I}_{2q}} \leq C_q\mu(\Lambda'). \tag{2.13}$$

Choosing $\Lambda' = \{|V_i|>\varepsilon\}\cap\Lambda_L^n$ and taking (2.9) into account, then by combining estimates (2.10)–(2.13), we get that there exists c, depending only on q (and the bound in assumption (H.3)), such that

$$\begin{aligned}&\mathrm{Trace}|(H_{|\Lambda_L^n}+\lambda_0)^{-q}(H_{|\Lambda_L^n}-z)^{-1}-(H_{n|\Lambda_L^n}+\lambda_0)^{-q}(H_{n|\Lambda_L^n}-z)^{-1}| \\ &\qquad\qquad \leq c\Big(\frac{\varepsilon}{|\Im z|^2}L^{nd}+\frac{1}{|\Im z|^2}L^{nd-\frac{d}{2q}}+\frac{\varepsilon}{|\Im z|}L^{nd}+\frac{1}{|\Im z|}L^{nd-\frac{d}{2q}}\Big).\end{aligned}$$

Using this inequality in (2.5), we get (2.3) as, $\tilde\varphi$ being almost analytic, $\overline{\partial}\tilde\varphi(z)$ vanishes to any order in $\Im z$ as z approaches the real line. Thus, we completed the proof of Theorem 2.2. □

References

[CHK] Jean-Michel Combes, Peter Hislop, Frédéric Klopp: Local and Global Continuity of the Integrated Density of States; Contemporary Mathematics; **327**, 61-74 (2003)

[Kl] Frédéric Klopp: Internal Lifshits tails for random perturbations of periodic Schrödinger operators; Duke Mathematical Journal; **98**(2) 335-396, (1999)

[KlPa] Frédéric Klopp, Leonid Pastur: Lifshitz tails for random Schrödinger operators with negative Poisson potential; Communications in Mathematical Physics; **206**, 57-103 (1999)

[PaFi] Leonid Pastur, Alexander Figotin: Spectra of Random and Almost-Periodic Operators; Springer Verlag (1992)

[RS2] Michael Reed, Barry Simon: Methods of Modern Mathematical Physics; Volume II: Fourier Analysis, Self-Adjointness, Academic Press, San Diego 1975

[RS4] Michael Reed, Barry Simon: Methods of Modern Mathematical Physics; Volume IV: Analysis of Operators, Academic Press, San Diego 1978

[Si] Barry Simon: Trace ideals and their application; Cambridge University Press, Cambridge 1979

[Stl] Peter Stollman: Caught by disorder; Birkhäuser 2001

[Stz] Günter Stolz: Strategies in localization proofs for one-dimensional random Schrödinger operators; Proc. Indian Acad. Sci. (Math. Sci.); **112**, 229-243 (2002)

[Ze] Heribert Zenk: An interacting multielectron Anderson model; Preprint mp_arc 03-410