CALABI-YAU ALGEBRAS AND TQFT

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A Calabi-Yau (CY) algebra of dimension d is a compact A_∞ algebra $(A, m = \sum_{i \geq 1} m_i)$ with a non-degenerate cyclically invariant pairing of degree d. In the sense of formal non-commutative geometry, it is the analogue of a symplectic structure. Compactness means that the cohomology of (A, m_1) is finite dimensional. It is well known (Kontsevich-Soibelman, Costello) that the Hochschild homology of a CY algebra has the structure of a topological quantum field theory (TQFT), namely it is an algebra over a dg-PROP of chains in the moduli space $\mathcal{M}_{g,m,n}$ of smooth Riemann surfaces with m incoming and n outgoing marked points, where $m, n \geq 1$. There is a need though to relax the compactness condition, in particular with respect to applications related to path spaces as in the string topology of Chass Sullivan and the Fykaya category. We will review the main theory and introduce the new notion of a *weak CY structure* (a formal, non-commutative Poisson structure) on a non-necessarily compact algebra. We show that a weak CY algebra also defines a TQFT. A special case of a weak CY structure is a CY_∞ structure on a smooth algebra. As an example we will show that there is a CY_∞ structure on a dg-algebra of homotopy types of paths in a smooth compact manifold. It is expected to be an algebraic model for the string topology of Chass -Sulivan. These results are joint work with Maxim Kontsevich.