Approximation properties and special operator spaces

In this series of lectures we will study several approximation properties of C^*-algebras and linear maps on operator algebras. An important tool is to understand the operator space structure of certain eigenspaces given by a semigroup of completely positive maps on a von Neumann algebra.

Lecture 1: We study approximation properties of C^*-algebras related to nuclearity and in particular factorization of linear maps through finite dimensional C^*-algebras. These factorization can be performed for completely positive maps or with control of the completely bounded norm. We will review result of Connes and Haagerup about injective von Neumann algebras and C^*-algebras having Lance's weak expectation property. If time permits we will compare different approximation properties for the reduced C^*-algebra of discrete groups.

Lecture 2: We study the notion of exactness for C^*-algebras, in particular for reduced C^*-algebra of discrete groups, and linear maps between C^*-algebras. Group von Neumann algebras will be used to show that certain spaces have a large exactness constant which prevents several universal C^*-algebras to be exact.

Lecture 3: The starting point here is Haagerup's proof of the cb-approximation property of the reduced C^*-algebra of the free group. Similar ideas lead to approximation properties for free products of C^*-algebras due to Ricard/Xu which is based the analysis of certain finite eigenspaces of the semigroup given by the length function. If time permits we will discuss recent results of Haagerup concerning the Markov dilation of certain semigroups.