Aux frontières de la viabilité
Around Viability Boundaries

December 12th - 14th, 2012, Paris

Booklet of Abstracts

December 12th, 2012

Option pricing in a tychastic market model

Pierre Bernhard, INRIA-Sophia Antipolis-Mditerrane, France

We investigate the simple option pricing problem, either for Vanilla or Digital European options. As compared to the work of Saint-Pierre et al., we have a more specific theory we are unable to extend it to exotic options, with more analytical results. In particular, we obtain a representation theorem for the Value function, both in continuous and discrete trading, yielding a fast algorithm for the discrete trading case, together with a convergence theorem of this discrete trading value toward the continuous one as the step size vanishes.

Singularities in dynamic programming

Piermarco Cannarsa, Università di Roma 2, Tor Vergata, Italy

Singularities are usually regarded as points to keep away from - when referred to the value function of an optimal control problem. This viewpoint, however, could be totally reversed if one thought of all the data that are condensed at a singular point. This talk will be devoted to singularities of solutions to Hamilton-Jacobi equations, the dynamics of their propagation, and their invariance properties.

Stabilizing differential inclusions and PDEs without uniqueness by noise

Tomas Caraballo, Universidad de Sevilla, Spain

We prove that the asymptotic behavior of partial differential inclusions and partial differential equations without uniqueness of solutions can be stabilized by adding some suitable Ito noise as an external

perturbation. We show how the theory previously developed for the single-valued case can be successfully applied to handle these set-valued ones. The theory of random dynamical systems is used as an appropriate tool to solve the problem.

Confinement Problems in Individuals - Population Interactions

Rinaldo Colombo, Università degli studi di Brescia, Italy

Various analytical frameworks are able to describe the interaction between agents and a moving population. First, within a pde setting, this presentation describes a well posedness results. Then, a model based on differential inclusions is presented. In this context, recently obtained positive and negative confinement results are discussed.

Hybrid Control: How Set-Valued Analysis Helps Establish Robust Stability Theory

Rafal Goebel, Loyola University, USA

Modern control algorithms, which involve timers or switching and are often implemented with digital components, leads to dynamical systems where some variable evolve continuously and some change instantaneously. Such systems can be conveniently modeled by a combination of tools from continuous-time and discrete-time settings: differential inclusions and difference inclusions, as well as constraints. This combination leads to hybrid inclusions. The talk will outline a recently established robust asymptotic stability theory for hybrid inclusions. The role of set-valued analysis be underlined, even for systems without set-valued dynamics. Limitations of the current theory and future challenges will be discussed.

On the role of Viability in Model Predictive Control

Lars Grüne, Universität Bayreuth, Germany

Model predictive control (MPC) is a popular control method which synthesizes a control law on an infinite time horizon by repeatedly solving finite horizon optimal control problems. One of the main reasons for the popularity of MPC is its ability to incorporate state and control constraints. However, due to the fact that in each step only a state constrained optimal control problem on a finite time horizon is solved, viability of the resulting trajectories on the infinite time horizon is not a priori guaranteed. In this talk we present a selection of approaches which allow for rigorous viability statements. Both MPC schemes employing additional terminal constraints and schemes without such terminal constraints are addressed.

Differential equations in (possibly) nonlinear spaces: A short excursion

Thomas Lorenz, Goethe University Frankfurt am Main, Germany

Ordinary differential equations play a central role in science. Their theory is known to be extended successfully from the finite-dimensional Euclidean space to so-called evolution equations in Banach spaces by means of strongly continuous semigroups. Some further extensions are partial differential equations (PDEs) and stochastic differential equations. For many applications, however, it is difficult to specify a suitable normed vector space. Shapes, for example, do not always have an obvious linear structure if any a priori assumptions about regularity (like convexity or smooth boundaries) are to be avoided and thus analytically, they are described merely as nonempty compact subsets of the Euclidean space. Supplied with the classical Hausdorff distance, they form a metric space instead. This talk focuses on extending (ordinary) differential equations beyond the traditional border of vector spaces such as metric spaces. The long-term goal of this extension is an analytical framework for Cauchy problems in which set-valued evolutions can be handled in the same way as parabolic partial differential equations or semilinear evolution equations, for example. The essential advantage of such a general approach is that we can immediately consider well-posed Cauchy problems of systems although their components are of analytically very different types. In the 1990s, Jean-Pierre Aubin suggested an extension called mutational equations. They are similar to quasidifferential equations introduced by Panasyuk independently. As a key difference from our point of view, Aubin's concept clearly decomposes the full and usually complex problem (related to the fully nonlinear differential equation) into a class of much simpler problems (called transitions) and feedback. This notion applied to problems in function spaces, for example, covers even some functional differential equations directly. His a priori conditions on transitions, however, are too restrictive for applying the original mutational equations to semilinear evolution equations. In this talk, several examples serve as motivation how to extend that framework suitably. The list consists of both deterministic and stochastic examples. Invariance and some viability problems can then be handled similarly to what is well known in vector spaces.

Stability of Nonlinear Systems with Discontinuous Control Laws

Richard Vinter, Imperial College London, UK

Discontinuous control laws are widely used to stabilize control systems in the presence of large disturbances. These control laws typically give rise to rapid switching of the controls and the state trajectories must be interpreted in some generalized sense. We discuss interpretations of these generalized state trajectories, and describe how the stability of such systems can be analysed by means of viability theory and the construction of multiple, non-smooth Lyapunov functions.

December 13th, 2012

Title to be Announced

Jean-Marie Besnier, CREA, Ecole Polytechnique

Developing cerebral architecture for multiple acquisitions and adaptations by young children

Yves Burnod
LIF, Faculté de Médecine Pierre et Marie Curie - Site Pitié-Salpétrière

Young children acquire multiple new sensory-motor abilities and cognitive capacities in a complex and uncertain world. The neural basis of such capacities and abilities have been explored by thousands of brain imaging studies. We have performed a systematic analysis of a database collecting these results to reveal a basic set of brain functional networks which control these abilities and capacities. We explore how the developing connectome which controls the maturation of new functional networks can guide the multiple acquisitions and adaptations by young children during the interactions with a large variety of environments. We will discuss how the theory of Viability can help to model this process.

A model of viability for a monetary economy

Jean Cartelier, Université de Paris X - Nanterre

For at least two centuries the main bone of contention among economists was the stability properties of a market economy. Recurrent crisis, business cycles, long-term oscillations of economic activity were as many occasions to elaborate optimistic or pessimistic arguments. The domination of Arrow-Debreu model of competitive general equilibrium during the 1950s and 1960s allowed economists to make their points more precise. Global asymptotic stability was an area of intensive studies and debates. As it is well-known negative results due to Sonnenschein, Debreu, Mantel, Saari and others put an end to that study during the 1970s. Nowadays, academic theoreticians have no longer concern in out-ofequilibrium positions and in global stability. Self-regulation of a market economy (the traditional and most fundamental question of political economy for centuries) has disappeared on the agenda. Even the recent global financial crisis. although present in innumerable public debates, has not changed anything in the methods of academic theory. Viability approach opens new perspectives in the field. But it raises also difficult problems of interpretation. Equilibrium, a central notion in eonomic theory, has probably to be thought anew. The same is true for the idea of self-regulation of a market economy. In order to illustrate what precedes, a model of a pure market economy is presented and discussed.

Antifragility and tinkering in biology (and in economy). Flexibility is an efficient epigenetic solution for risk management

Antoine Danchin, CEA/Génoscope, Evry

The notion of antifragility, an attribute that allows systems to thrive in an unpredictable world, was recently proposed by Nassim Taleb in the context of financial exchanges. Contrasting reproduction (making a similar copy) with replication (make an identical copy), we show that the management of populations of biological objects similar to each other, but sensitive to the state of the environment, can result in an antifragile behaviour. The discussion will be illustrated by the example of natural ageing, as opposed to the inevitable unfolding of senescence and death.

Application of Viability Theory to Macroeconomics Growth Theory and to Financial Modelling

Vladimir Lozève, Natixis, Paris

Applied Mathematical Modelling in Macroeconomics is based and computed following statistical ethodology. For instance the mainstream Growth models are mainly descriptive, there is no in-built feedback loop regulating the system evolution. Therefore the long term dynamics is divergent. Financial modelling is a branch of stochastic calculus mainly designed to compute the mathematical expectation of the value of an asset which is a measure of central tendency. That kind of models offers very little pertinent information about the borderline of the system as the .Krach. In Macroeconomics Viability Theory could enable us to introduce the long term viable dynamics and the long term cyclicity of the macroeconomics growth.

In Financial Theory Viability Theory could enable us to model the contingent uncertainty of the extreme evolution of financial assets.

Micro/macro viability analysis of individual-based models

Sophie Martin, Irstea

A crucial point for complex systems and the entities that make it up is a viability issue. The mathematical viability theory has been developing for twenty years methods and tools to study the compatibility between dynamics and constraints. This framework has been applied to robotics but also to renewable resources management as well as to broader (eco)-system dynamics or to pure economic ones. In the major part of these works, the systems are described by global variables and the control regulation also operates at the global scale. There are very few viability studies including macro and micro scales. We propose to consider the viability issue in micro-macro systems without assuming a global control possibility. We follow a bottom up approach and study how the superposition of local controls can influence the global viability.

Embryonic development as a viabiity screen for potential evolutionary innovations

Nadine Peyrieras, Institut de Neurobiologie Alfred Fessard, Gif-sur-Yvette

Epistemic viability and the dynamics of acceptances

Joëlle Proust, Institut Jean Nicod, Paris

Intelligent agency requires an ability to control and monitor one's cognitive states, e.g. retrieve memories, check one's perceptions or one's utterances. The aim of cognitive control is to acquire cognitively reliable properties, such as retrieving a correct answer. Intelligent agents in realistic settings, however, need to track other epistemic norms beyond accuracy, such as the comprehensiveness of a list, the intelligibility of a text, the coherence of a story, the relevance of a remark, or the consensuality of a claim. Experimental studies in metacognition suggest that such norms are indeed used by human and some non-human agents to control and monitor their own cognitive performance. Viability theory offers a model of the dynamic constraints that apply to the various forms of self-evaluation, able to explain why sensitivity to epistemic norms differs with ages.

Biodiversity evaluation, modelling and viability

Jean-Philippe Terreaux, Irstea

The biodiversity, if only for the ecosystem services it allows, but not exclusively, is at the source of different values for the present and future generations, values which can be taken into account thanks to the representation of the world which is implicit in our decision models.

So economic considerations in connection with the optimization of the natural resources usage or exploitation can be integrated to estimate the biodiversity value in monetary terms and enable deciding for example between economic development and nature or species conservation.

Another representation of the world and of the decision making, allowed by the theory of the viability, leads to quite another approach, looking for the achievement of objectives which seem to better correspond to the driving of the human affairs, and thereby, which is more acceptable for decision making.

December 14th, 2012

On some non standard control problems

Pierre Cardaliaguet, U. Paris-Dauphine

We discuss some unusual control problems which appear in the analysis of partial differential equations or calculus of variation problems.

Computing and characterizing the cost spread of a financial structure with bid/ask

Bernard Cornet, U. of Paris I and University of Kansas, Lawrence, USA

The aim is to give an explicit formula (easy to compute) of the (super-hedging) cost spread of a financial structure in the case of bid/ask event securities, hence also of bid/ask (incomplete) Arrow securities. First we show that the set of discounting measures (risk-neutral probabilities) can be characterized as the core of its associated risk-neutral capacity (a set-function defined on events, also called characteristic function in TU cooperative games). Second, the risk-neutral capacity is shown to be concave, with an explicit formula, easy to compute for each event. Finally, the super-hedging cost of every payoff is proved to be equal to its Choquet integral with respect to the risk-neutral capacity, thus giving an explicit formula for the cost spread using the Chain formula of the Choquet integral.

Capture basin algorithms and their applications

Anya Désilles[1], Alexandra Fronville[2]
1. ENSTA, 2. European Center for Virtual Reality, Brest

In this talk we focus on the capture basin algorithm on the viability theory. We present some numerical and theoretical aspects of the calculation of capture basins for controlled dynamical systems. In particular, we show how the notion of the capture basin and the viability theory can be applied to solve optimal control problems and some equations of Hamilton-Jacobi type.

This approach is illustrated by an example of a traffic modeling problem. In the macroscopic LWR model the traffic evolution is described by a conservation law with some initial and boundary conditions. It is well known that such problems are difficult to solve analytically and numerically because of presence of shocks. To avoid these difficulties the problem can be stated in an integrated form, as a Hamilton-Jacoby type equation. We use the viability theory framework to define and compute the solution of such problems. The viability theory allows to give sense to a solution of the problem with a very general conditions including boundary, initial and lagrangian (or internal) conditions. The numerical computation of the solution can by made using the capture basin algorithm.

Another example, is the morphogenesis of multicellular organisms. It is a dynamical system of cells (cell multiplication, cell migration, apoptosis) with local interactions between cells and with the extracellular matrix. Cells have the same genome and the orientation or direction of cellular division is not

random. We formalize the cell dynamic in a proper metric space to find conditions (decisions, states) in which operational constraints are always satisfied and therefore in which the system is viable and maintain its shape while renewing. We use the viability theory and the mutational analysis framework to define the cell dynamic in order to think about form as a cellular growth designing a capture basin of the morphological dynamic which drives to a target (the final form) characterized as the fixed point.

Viabilist and Tychastic Approaches to Guaranteed ALM Problem

Olivier Dordan[1] and Luxi Chen[2]
1. U. Bordeaux, 2. Vimades

This study reconsiders the problem of hedging a liability by a portfolio made of a riskless asset and an underlying. Given a process forecasting the lower bounds of the returns of the underlying, the software computes both the Minimum Guaranteed Investment (or Solveny Requirement Capital), and a management rule VPPI (Viabilist Portfolio Performance and Insurance) ensuring that, at each date, the value of the portfolio is "always" exceeding liabilities. Examples of management during the crisis of the summer 2011 are provided.

Sustainability and intergenerational equity

Marie-Hélène Durand, IRD, Montpellier

Economic studies of sustainable development deal with intergenerational equity considered as a conflict of interests arising between present and future generations. To define and to assess intergenerational equity in the setting of an infinite horizon, two separate approaches are undertaken. The first one is axiomatic and purely formal. It is devoted to the establishment of ethical properties that an intertemporal choice criterion needs to fulfill, in order to adequately represent intergenerational equity. The second one is operational and devoted to the construction of choice criteria in long term economic studies or in an infinite horizon setting. All of the proposed criteria are criticized for not fulfilling the prescribed intergenerational equity conditions and because their results are highly dependent of an actualization parameter. Using viability theory, we propose an equity function that is not balancing present and future (and unknown) states as an improbable social planner but that provides each present generation the best way to fix their consumption such that consumption inequalities between generations are minimized. This equity function provides an indicator that respects both the finite anonymity and Pareto efficiency principles and that is computable.

Co-viability modelling for the sustainable management of biodiversity

Luc Doyen, CNRS, Museum national d'histoire naturelle

A fundamental issue for the sustainable management of renewable resources and biodiversity is the reconciliation of ecological and economic requirements with an equity perspective. The presence of

numerous uncertainties in the systems at stake complicates achievement of such a goal. Fisheries and agro-ecological systems constitute challenging case studies in this respect. Viable control under uncertainty is here proposed as a relevant modelling framework to deal with such issues. The approach does not strive to determine optimal or steady-state paths for the joint dynamics of resources and exploitations, but rather aims at maintaining the trajectories of systems within satisfying normative bounds that mix ecological, economic and social requirements. Hence the approach offers a multi-criteria perspective and provides ways to analyse and control the risks and vulnerability of dynamic bio-economic systems. It has major conceptual links with Population Viability Analysis (PVA) and maximin or Rawlsian approaches. Given the flexibility it allows in defining management problems, the approach offers a fruitful modelling framework to address issues of adaptive management and management strategy evaluation. Examples inspired by the management of fisheries or farming worldwide will be proposed to illustrate the interest of such a general approach.

Division Impartiale d'un Euro

Hervé Moulin, Rice University, USA

Chaque participant donne son opinion sur les parts relatives des autres participants, et l'impartialité exige que son message n'ait aucune influence sur sa propre part. Si il est possible de partager l'euro d'une façon compatible avec les messages individuels, la méthode doit entériner ce consensus. S'il y a au moins quatre participants on peut construire une famille naturelle, et en un sens canonique, de méthodes de partage avec ces deux propriétés. Ce n'est pas possible avec deux ou trois agents.

References

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Discrete set-valued interpolation in the Alexandrov topology (with Thierry Géraud)

Laurent Najman
ESIEE, Paris

The main question of this paper is to retrieve continuity on a discrete grid. One possible application, which will guide us, is the construction of the "tree of shapes" (intuitively, the tree of level lines). This tree, which processes in the same way maxima and minima, faces quite a number of theoretical difficulties. These problems exist whether we start from discrete or continuous data, but we will show that an approach both discrete and set-valued makes solving them, with some advantages with respect to continuous ones. For this, two tools are needed:

- The (discrete) Alexandrov spaces. These are topological spaces in which the intersection of any family of open sets is open. In such spaces, we can express classical theorems such as the

Jordan's one, which states that any simple closed curve separates the plane into an interior and an exterior. We will show the interest to build the contours on the Khalimsky grid, a grid equipped with the Alexandrov topology.

- Set-valued analysis. Specifically, we show that it is extremely natural in the discrete case for the value of a function at a given point to be an interval, and this will allow us to express properties of continuity of set-valued functions defined on the Khalimsky grid. Such properties will be essential to obtain a linear algorithm for computing the tree of shapes.

Average long time behaviour for optimal control and differential games

Marc Quincampoix, Université de Bretagne Occidentale

We investigate the long run behaviour for an optimal control or a differential game with a cost which is a Cesaro mean of an integral cost.

The main issue of our approach is the ability to cope with cases where the limit value could depend of the initial condition of the control system as it is usually assumed or proved in the ergodic control literature.

Title to be Announced

Nicolas Seube, ENSTA Bretagne

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