Sino-French Research Program in Diophantine Geometry

Beijing International Center for Mathematical Research
Jing Chun Garden, Room 77201

Abstracts

Yuri Bilu (Université de Bordeaux I), Runge's method

Runge's method is a very old (going back to XIX century) elementary effective method in Diophantine analysis. I will explain how this method works on general curves, and how the general procedure can be adapted to and refined in the case of modular curves, in particular, X_split(p).

I will also briefly speak on modular units (that is, rational functions on modular curves) which are the principal technical tool in "modular Runge's method".

If time permits, I will add a few word's on Baker's method, which is quite similar to Runge's method, but less elementary.

Maria Carrizosa (Université de Lyon I), Introduction to Abelian varieties

Abelian varieties is a wide and interesting topic. We will explain the definition and basic properties of these objects assuming just basic knowledge of algebraic geometry and elliptic curves. We will focus on the ingredients that will be needed in further talks.

The talk main lines are the following :

1- Abelian varieties over the complex numbers : complex torus, Riemann forms, endomorphisms
2- Abelian varieties over general fields : isogenies, decomposing abelian varieties, dual abelian varieties
3- Faltings height : good models, heights

Agnès David (Université de Versailles), Introduction to modular curves

Sinnou David (Université Paris 6), Classical Baker's method and isogeny estimate of Gaudron-Rémond

I will describe the recent result of Gaudron and Rémond giving a sharp, explicit version of the isogeny estimate of Masser-Wüstholz for elliptic curves. The main tool is the period theorem of which we also provide a new explicit version. I will focus mainly on the deduction of the isogeny theorem from the period theorem, explaining how transcendental information (on periods) yields an algebraic result (on isogenies).

Aurélien Galateau (Université Franche-Comté), Serre's theorem on Galois representations

Let E be an elliptic curve and p be a prime. The action of Galois on the p-torsion of E induces a Galois representation: Gal(\bar{Q}/Q)→$GL_2(F_p)$. Serre's Theorem states that if E is not of CM type, this representation is surjective for p large enough. The aim of this short course will be to give a sketch of the original proof. I will start by recalling some classical facts on elliptic curves (torsion points, complex multiplication, Weil's pairing, action of inertia). I shall also need to explain some properties of the subgroups of $GL_2(F_p)$ and describe the maximal subgroups.

Mathilde Herblot (Göthe University, Frankfurt), Complex and p-adic geometric version of the Schneider-Lang theorem

The Schneider-Lang theorem is a classic transcendence criterion for complex numbers. It asserts that there are only finitely many points at which algebraically independent meromorphic functions of finite order of growth can simultaneously take values in a number field, when satisfying a polynomial differential equation with coefficients in this given number field. In this talk, I will give geometrical generalizations of this criterion, holding for both the field of complex numbers and a p-adic field. In dimension one I will state a theorem for formal subschemes admitting a uniformization by an algebraic affine curve. In the higher dimensional case, I will give a theorem which applies to formal subschemes with a uniformization by a product of open subsets of the affine line, under the additional hypothesis that the set of rational points is a Cartesian product. The proofs of these results rely on the slopes method developed by J.-B. Bost and make use of the language of Arakelov geometry.

Hideaki Ikoma (University of Kyoto), How to bound the successive minima on arithmetic varieties

I would like to explain a new method to bound the last successive minima from above that are associated to high powers of a hermitian line bundle $\overline{L}$ on an arithmetic variety X.
As applications, we can prove the following results:
1) the last successive minima are generally bounded from above, provided that X is normal.
2) the sequence defining the sectional capacity of $\overline{L}$ converges.
3) the sectional capacity of $\overline{L}$ is Lipschitz continuous and birationally invariant.
4) necessary and sufficient conditions for $H^0(X,mL)$ to have free basis consisting of strictly small sections for sufficiently large m.
5) a generalization of the theorem of successive minima of S. Zhang. In particular, we can reprove the general equidistribution theorem for rational points of small heights, which was first proved by Berman-Boucksom by using the Monge-Ampere operators.

Jean-François Mestre (Université Paris 7), Curves of genus 3 with S_3 as automorphism group

In this talk, we shall explain how the study of curves of genus 3 with an automorphism group isomorphic to S_3 can be used to obtain many optimal curves of genus 3 over F_q, notably in the case where $q=3^m$ and $q=7^m$.

Pascal Molin (Université Paris 7), Numerical investigation of partially known L functions

We will present some applications of the theory of approximate functional equations for numerical investigations of L-functions. In particular, we will explain how one can take advantage of the (possibly conjectural) Selberg axioms to numerically recover missing parts of a given L-function: its conductor, some Euler factors, or even all Dirichlet coefficients.

Pierre Parent (Université de Bordeaux I), Rational points on $X_0^+(p^r)$

Building on the previous lectures we prove that the modular curves $X_0^+(p^r)$, for p prime and r>1, only have the expected trivial rational points (cusps and CM points) if p=11 or p>13.

The proof (joint work with Yu. Bilu and M. Rebolledo) divides into two parts. We first deal with very large primes $p> 2.10^{11}$, by putting together lower bounds for the height coming from isogeny theorems (Masser-Wüstholz, Pellarin, Gaudron-Rémond), integrality results for rational points (Mazur's method), and upper bounds for the height of integral points (Runge's method).

We then treat small primes, by developping an algorithm relying on the geometry of special fibers of modular curves, and a geometric interpretation of a formula of Gross for special values of modular L-functions.

If time permits, we will discuss some possible generalizations.

Marusia Rebolledo (Université de Clermont-Férrand 2), Results of Mazur, Momose and Merel

Let p=11 or p≥17 be a prime number. Let E/Q be an elliptic curve endowed with two independent subgroups A and B of order p such that the set {A,B} is defined over Q (equivalently the image of the absolute Galois group over Q in Aut(E[p]) is contained in the normalizer of a split Cartan subgroup). Then E has potentially good reduction at *l* for all prime number *l*.

This "integrality theorem" is due to Mazur, Momose and Merel. The goal of my talk is to present with some details the unified proof that Bilu-Parent gave in the article *Serre's uniformity problem in the split Cartan case.*

Abstracts

Fei Xu (Capital Normal University), Counting integral points on certain homogeneous spaces

Chinese Remainder Theorem can be regarded as the simplest example of the strong approximation property. For various arithmetic purposes, Eichler, Kneser, Shimura, Platonov and Prasad established the strong approximation for semi-simple, simply connected algebraic groups. By using Manin's idea for studying the rational points, one can further study the strong approximation with Brauer-Manin obstruction. In this talk, we'll explain the recent progress on the strong approximation with Brauer-Manin obstruction. Several arithmetic applications, especially for the integral points, will be provided.

Gongrong Yang (Peking University), The general Barnes zeta functions

I will build general Barnes zeta function and explore its own properties which include analytic continuation, special values, and functional equations. This is the first step for generalization of Shintani's formula.

Yuancao Zhang (BICMR), *l*-invariants and logarithm derivatives of eigenvalues of Frobenius

Let K be a p-adic local field. In this talk we consider a special kind of its p-adic Galois representations. These representations are similar to the Galois representations occurred in the exceptional zero conjecture for modular forms. In particular, a formula of Colmez can be generalized to our case. This formula relates the derivatives of eigenvalues of Frobenius of a deformation of the Galois representation to its structure invariants.