

On the special values of the degree four L -function of $\mathrm{GSp}(4)$

Cours de Francesco Lemma

Abstract : We will be interested in the algebraicity properties of the special values of the degree 4 L -function associated to some cuspidal automorphic representations of $\mathrm{GSp}(4)$, in the conjectural framework of Beilinson and Deligne.

We will first state and explain briefly Beilinson's conjecture on the connection between special (non-critical) values of motivic L -functions and higher regulators. Then we will focus on the case of $\mathrm{GSp}(4)$. The first goal will be to construct 1-extensions of mixed Hodge structures between the trivial Hodge structure and the Betti realization of the motive associated to some cuspidal automorphic representation of $\mathrm{GSp}(4)$, which are in the image of Beilinson's regulator. As a second step we will explain how to relate these 1-extensions to a non-critical special value of the degree 4 L -function, via the integral representation of the L -function defined and studied by Piatetski-Shapiro. Finally we will discuss the connection between our main result and some results of Michael Harris about the critical values of this L -function.

Key words : Shimura varieties, higher regulators, mixed Hodge modules, Rankin-Selberg integral.