

Geometry in its Symplectic, Tropical and Real aspects  
IMJ-PRG, Jussieu, March 14 - 16, 2023

Tuesday, March 14, 2023

Room 15-25-104

**14:00–15:00** Viatcheslav Kharlamov, *Unexpected loss of Smith-Thom maximality: the case of Hilbert squares of real surfaces.*

**15:30–16:30** Kyler Siegel, *On rational curves with cusps and double points.*

Wednesday, March 15, 2023

Room 15-16-413

**10:30–11:30** Erwan Brugallé, *Real bitangents to real plane algebraic curves.*

**14:00–15:00** Denis Auroux, *Functions on curves from Lagrangian Floer theory on their mirrors.*

**15:30–16:30** Tobias Ekholm, *Skein valued curve counts and skein recursion.*

Thursday, March 16, 2023

Rooms 15-25-104 (morning) and 15-16-413 (afternoon)

**10:30–11:30** Thomas Blomme, *Curves in abelian surfaces and multiple cover formula.*

**14:00–15:00** Penka Georgieva, *Signs.*

## Abstracts

### **Viatcheslav Kharlamov**

*Unexpected loss of Smith-Thom maximality: the case of Hilbert squares of real surfaces.*

We explore the maximality of the Hilbert square of maximal real surfaces, and find that in many cases the Hilbert square is maximal if and only if the surface has connected real locus. In particular, the Hilbert square of no maximal K3-surface is maximal. Nevertheless, we exhibit maximal surfaces with disconnected real locus whose Hilbert square is maximal. (The talk is based on joint work with R. Rasdeaconu).

### **Kyler Siegel**

*On rational curves with cusps and double points.*

A classic question in algebraic geometry asks what are the possible singularities for a plane curve of a given degree and genus. This turns out to be closely connected with the theory of symplectic embeddings of stabilized ellipsoids. In this talk I will describe a construction of various new families of rational curves with desirable singularities. Key ingredients include a generalization of Orevkov's birational transformation and scattering diagrams for tropical curves.

### **Erwan Brugallé**

*Real bitangents to real plane algebraic curves.*

The study of bitangents of complex plane algebraic curves is a classical subject, and their number is prescribed by the Plücker formula. Over the real the situation remains mysterious and widely unexplored, mainly due to the lack of methods to tackle this problem. In this talk I will survey what I know about real bitangents of real algebraic curves, and report on some recent work joint with Cristhian Garay.

### **Denis Auroux**

*Functions on curves from Lagrangian Floer theory on their mirrors.*

Mirror symmetry implies an isomorphism between rings of functions or homogeneous coordinate rings on the algebraic side, and certain Floer cohomology groups on the symplectic side. Since the latter Floer groups often admit distinguished bases, this gives rise to so-called 'canonical bases' or 'theta bases' in algebraic geometry, as noted in different settings by Fukaya, Gross-Siebert, Gross-Hacking-Keel, and others. This talk will focus on the case of curves (mostly in the affine setting), reviewing different constructions of mirrors and the extent to which they do or do not give rise to distinguished bases of functions. Some parts of this talk are joint work with Efimov and Katzarkov; other parts are based on Jaewon Chang's PhD thesis.

**Tobias Ekholm***Skein valued curve counts and skein recursion.*

Counting holomorphic curves with Maslov zero Lagrangian boundary conditions in a Calabi-Yau 3-fold by their values in the framed HOMFLY skein module leads to invariant counts. For Lagrangian knot conormals in the cotangent bundle of the 3-sphere the corresponding partition functions encode all colored HOMFLY polynomials of the knot. We demonstrate this by finding skein valued recursion relations for the partitions functions in simple cases. Such skein valued recursions can be seen as generalizations of quantum curves. We introduce a certain skein module for singular 3-manifolds with singularities that are clean intersections along a knot or link and show for the union of a knot conormal and the 3-sphere how the corresponding module is related to the HOMFLY skein recursion.

**Thomas Blomme***Curves in abelian surfaces and multiple cover formula.*

Complex invariants coming from enumeration of complex curves passing through a suitable number of points in an abelian surface vastly differs from the corresponding enumeration in the projective plane. Although the values of the invariants for primitive classes have been known from quite some time, the enumeration for non-primitive classes turns out to be considerably more complicated. The tropical geometry approach enables one to prove a surprisingly short formula giving the values for non-primitive classes, avoiding any concrete (and potentially messy) enumeration.

**Penka Georgieva***Signs.*

I will discuss certain signs appearing in (open) Gromov-Witten theory.