Introduction to dispersive equations

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Introduction

Summary of the course

The aim of this course is to introduce the fundamental concepts of the theory of so-called *dispersive* partial differential equations. These equations model dispersive waves, i.e., waves whose propagation speed depends on the wave number. Dispersion plays a crucial role in describing many physical phenomena, see `https://en.wikipedia.org/wiki/Dispersion_(optics)`.

In the first part of the course, we present a general theory of linear dispersive equations. We then turn to nonlinear dispersive waves, focusing on the case of the Klein-Gordon equation.

Content

- Linear dispersive equations and their solution using the Fourier transform. Dispersion relation. Stationary phase lemma. Notion of group velocity of a wave. Asymptotic description of dispersive waves at large times. Dispersive estimates.
- Nonlinear Klein-Gordon equation. Well-posedness (Cauchy problem). Asymptotic description of solutions at large times in different asymptotic regimes. Nonlinear scattering.
- If time permits: Modified scattering in dimension 1. Normal forms.

Prerequisites

Differential calculus; ordinary differential equations; Fourier transform; basic concepts in functional analysis and partial differential equations.
It is recommended to have completed the "HFE" and "PDE" courses offered in the M2 program.

Schedule

Exam

Homework

List of notations

C	constant
$C(a, b, \ldots)$	constant depending on $a, b, \ldots$

Chapter 1

Linear dispersive waves

1.1 Exercises

Exercise 1.1.

Solution. □

Bibliography

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[4] G. B. Whitham. *Linear and Nonlinear Waves*, John Wiley & Sons, 1974.