Special Seminar on Dynamical Systems and Computer Assisted Proofs

Date: Wednesday 1st October, 2025
Time: 10:00 AM – 12:00 PM
Location: Jussieu Campus, Room 1516-3-09

10:00 AM *Stability of phase portrait for a gradient ODE with memory*
Speaker: Piotr Kalita (Université Jagellonne de Cracovie)
Abstract: We consider a dynamical system governed by the gradient ODE $x' = \nabla F(x)$ in $\mathbb{R}^d$, where we assume a finite number of hyperbolic equilibria whose stable and unstable manifolds intersect transversally. The system is perturbed by a memory term, resulting in the equation $x' = \nabla F(x) + \varepsilon \int_{-\infty}^{t} M(t-s)x(s), ds$, where ε is small and $M(s)$ decays exponentially to zero as $s \to \infty$. The key result is that the structure of connections between the equilibria of the unperturbed system is preserved in the infinite-dimensional dynamical system for sufficiently small ε. The main tool is the Dafermos transform, which allows us to treat, locally, the infinite-dimensional memory variable as a stable one. We demonstrate that this methodology is applicable not only to the analysis of global dynamics but also to study the local dynamics of the problem. This talk is based on a preprint [https://arxiv.org/abs/2406.00910] which is a joint work with Piotr Zgliczyński.

10:55 AM **Break**

11:00 AM *Computer assisted proof of symbolic dynamics for the Kuramoto-Sivashinsky PDE on the line*
Speaker: Piotr Zgliczyński (Université Jagellonne de Cracovie)
Abstract: We discuss a method for rigorous study of dynamics of dissipative PDEs on the torus (i.e. assuming periodic boundary conditions).

As an example we consider the Kuramoto-Sivashinsky PDE on the line

$$u_t = -\nu u_{xxxx} - u_{xx} + (u^2)_x, \qquad \nu > 0,$$

where $x \in \mathbb{R}$, $u(t,x) \in \mathbb{R}$ and we impose odd and periodic boundary conditions

$$u(t,x) = -u(t,-x), \qquad u(t,x) = u(t,x+2\pi),$$

and parameter $\nu = 0.1212$.

On this example we will describe various ingredients of computer assisted proofs in dynamics: interval arithmetics and rigorous numerics for ODEs and dissipative PDEs, geometric tools from dynamics: Smale's horseshoe, symbolic dynamics, cone conditions.

For more information, please contact Jacek Jendrej (`jendrej@imj-prg.fr`)
or Katharina Schratz (`katharina.schratz@sorbonne-universite.fr`).