Seminar on Dispersive PDEs

Thursday, March 12, 2026
Room 15-16-309, Jussieu Campus

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12:00–13:30 Seminar Lunch

13:30–14:15 Neba Polneau, *Long-time behavior of small solutions of NLS with non-generic potentials in 1D*

Abstract. The goal of this talk is to present an almost global result on the long-time behavior of small solutions of the cubic nonlinear Schrödinger equation in the presence of an external non-generic potential. Roughly speaking, non-generic potentials are potentials that exhibit a resonance at the bottom of the spectrum of the associated linear Schrödinger operator. These resonances make the analysis more involved, as we will see. To handle these issues, we use the distorted Fourier transform and introduce a modified eigenfunction basis tailored to the non-generic setting. We then discuss the nonlinear analysis through what we call the Nonlinear Spectral Distribution (NSD), a tempered distribution that encodes the nonlinear interactions in the (distorted) Fourier space. Finally, we examine some nonlinear estimates specific to the non-generic setting.

This talk is based on joint work with Jacek Jendrej (IMJ–Sorbonne University) and Gong Chen (Georgia Tech).

14:15–15:00 Xuemei Li, *Dynamics of the focusing, energy-critical Hartree equation with radial data*

Abstract. We study the long time dynamics of the radial solutions for the focusing, energy-critical Hartree equation. The nondegeneracy of the positive bubble solutions plays a key role in the spectral analysis of the linearized operator, the construction of the special threshold solutions, and the classification of the threshold solutions. The main arguments are the spectral theory of the linearized operator, the modulational analysis, and the concentration compactness rigidity argument pioneered by T. Duyckaerts, F. Merle, and C. Kenig to classify the threshold solutions for the energy-critical NLS and NLW. This is joint work with C. Liu, X. Tang, and G. Xu.

15:00–15:15 Break

15:15–16:00 Xuanying Li, *Non-radial multi-bubbles blow up for the energy-supercritical Schrödinger equation*

Abstract. In this talk, we present the existence of multi-bubble, finite-time blow-up solutions to the focusing energy-supercritical Schrödinger equation. For any fixed integers $K \geq 1$, $L \geq 2$, and exponent $p > 0$, we show that provided the dimension d is sufficiently large ($d \geq d_0(p, K)$), there exists a K-bubble blow-up solution concentrating at any set of distinct points $\{x_j\}_{j=1}^K \subset \mathbb{R}^d$. The solution takes the asymptotic form

$$\psi(t,x) \sim \frac{1}{t^{\frac{1}{p}(1/2+\nu)}} \sum_{j=1}^{K} Q\left(\frac{x-x_j}{t^{1/2+\nu}}\right),$$

where Q denotes the ground state and $\nu > 0$ is determined by

$$\nu = -\frac{1}{2} - \frac{L}{\lambda_+}, \qquad \lambda_+ = \frac{1}{p} - \frac{d-2-\sqrt{\text{Discr}}}{2}.$$

Here Discr is a positive irrational constant depending on d and p. A fundamental step in our analysis is the determination of a precise blow-up rate that eliminates the oscillatory phenomena inherent in the self-similar regime. We discuss how this cancellation enables the construction of a finite-term polynomial approximate solution with quantized blow-up rates, which constitutes the heart of the proof.

16:00–16:45 Jingyuan Gu, *Construction of multi-bubble solutions for the energy-critical wave equation in dimension four*

Abstract. For any $K \geq 2$, we construct a global solution of the energy-critical focusing wave equation in dimension four blowing up in infinite time at K points $z_k \in \mathbb{R}^4$. Here the points z_k can be any configuration satisfying that for each $j \in \{1, \ldots, K\}$,

$$\sum_{\substack{1 \leq k \leq K \\ k \neq j}} \frac{1}{|z_j - z_k|^2}$$

is a positive constant independent of j, which we denote by c. The concentration rate of each bubble is asymptotic to

$$te^{-\xi t^{2/3}}$$

as $t \to \infty$, where ξ is a positive constant depending only on c.

16:45 Closing