SELMER COMPLEXES – ERRATA

(9.5.6.1) Replace E_∞ on the second line by $E_\infty \otimes \mathbf{Q}_p/\mathbf{Z}_p$.

(11.4.10) The remark is incorrect, as the assumption 11.4.1.5 is not satisfied in this case.

(12.4.4.3) In the supercuspidal case it is the group G_v, but not necessarily I_v, which acts absolutely irreducibly on $V(f)_v$. In the monomial case, the quadratic extension E_w/F_v can be unramified.

(12.6.4.9) The sentence "If v splits in K/F ..." in the proof of (iii) is incorrect, in general. As a result, one has to add the following assumption to 12.6.4.9(iii): no prime $v \mid p$ that splits in K/F divides $\mathfrak{n}(g)_{\mathrm{St}}^{(P_1 \cdots P_s)}$. This assumption also has to be added to **(12.6.4.12)**, but not to (12.6.4.11).

(12.6.4.12) In the proof of 12.6.4.12(iii), the inequality $m \geq m'$ should be replaced by $m \leq m'$.

(12.7.14.2)(iii) The statement and the proof are correct only in the case when I_v acts absolutely irreducibly on V_v. If this is not the case, then $V_v \xrightarrow{\sim} \mathrm{Ind}_{G_w}^{G_v}(\mu)$ and $V_v' \xrightarrow{\sim} \mathrm{Ind}_{G_w}^{G_v}(\mu')$, where $\mu, \mu' : E_w^* \longrightarrow \overline{L}_{\mathfrak{p}}^*$ with μ'/μ unramified.

(12.7.14.3)(iii), (12.7.14.5)(i) By the previous remark, the proofs (which refer to (12.7.14.2)(iii)) are incomplete in the supercuspidal case. In fact, for $p > 2$, both statements follow from [De2, Thm. 6.5] and (12.7.14.2.2). In general, these are special cases of (the second half of the proof of) Prop. 2.2.4 in [Ne5, Doc. Math. **12** (2007), 243–274]. Both references use Galois-theoretical ε-factors, which coincide with the automorphic ones.

(12.9.6) In fact, if $2 \nmid [F:\mathbf{Q}]$, then there is no exceptional extension K'/F, either. If it existed, put $\eta' = \eta_{K'/F}$. As K'/F is unramified outside ∞P, we have $1 = \prod_v \eta_v'(-1) = \prod_{v \mid \infty P} \eta_v'(-1) = (-1)^{[F:\mathbf{Q}]}\eta_P'(-1)$, hence $\eta_P'(-1) = -1$. In particular, P is ramified in K'/F. As f is p-ordinary, $\pi_P(g)$ is not supercuspidal, which implies (by 12.6.1.2.3) that $\pi(g)_P = \pi(\mu, \mu\eta_P')$ for some $\mu : F_P^* \longrightarrow \mathbf{C}^*$. The central character of $\pi(g)_P$ is trivial; thus $\mu^2\eta_P' = 1$ and $\eta_P'(-1) = \mu(-1)^{-2} = 1$, contradiction.

As a result, we can omit the assumption "Assume that g does not have CM by ..." in **(12.9.5)**, **(12.9.8)**, **(12.9.9)**, **(12.9.11)** and **(12.9.12)**. Similarly, in **(12.9.7)**, we can omit the assumptions (i), (ii) in the case $2 \nmid [F:\mathbf{Q}]$.

(12.9.8), (12.9.9) Add the following assumption: no prime $v \mid p$ that splits in K/F divides $\mathfrak{n}(g)_{\mathrm{St}}^{(P)}\mathfrak{n}(g')_{\mathrm{St}}^{(P)}$.

(12.9.13) In the proof, we choose $\mathcal{P}'$ such that g' also satisfies $(\mathfrak{n}(g')_{\mathrm{St}}, (p)) = (1)$.

(12.11.12) In the proof of (iii), $SL_2(\mathbf{F}_p) \cap h^{-1}Ch$ is contained in a *not necessarily split* Cartan subgroup, which implies that its order is less than or equal to $p+1$, hence the number of elements of order 2 in PH is equal at least to

$$\left|\mathrm{Im}\left(\{gcg^{-1} \mid g \in G\} \longrightarrow PGL_2(k)\right)\right| \geq \frac{|SL_2(\mathbf{F}_p)|}{2\,|SL_2(\mathbf{F}_p) \cap h^{-1}Ch|} \geq \frac{|SL_2(\mathbf{F}_p)|}{2(p+1)} = \frac{p(p-1)}{2}.$$

If $PH \xrightarrow{\sim} S_4$ or A_5, then *(assuming that H is not large enough)* $p \nmid |PH|$ ([Dic], §260). On the other hand, the inequalities

$$(\forall p \nmid |S_4|) \qquad |\{\text{elements of order 2 in } S_4\}| = 9 < \frac{p(p-1)}{2}$$

$$(\forall p \nmid |A_5|) \qquad |\{\text{elements of order 2 in } A_5\}| = 15 < \frac{p(p-1)}{2}$$

imply that $PH \neq S_4, A_5$. In fact, one can show more: if G is not solvable and $p > 3$, then H is not solvable. Indeed, non-solvability of G implies that, either (1) G contains $hSL_2(\mathbf{F}_p)h^{-1}$ (which is not solvable, as $p > 3$), hence so does H, by Lemma; or (2) $PG \xrightarrow{\sim} A_5$. In the latter case the image of c in $PG \xrightarrow{\sim} A_5$ is an element of order 2, and the subgroup of A_5 generated by all conjugates of such an element is equal to A_5; thus $PH = PG \xrightarrow{\sim} A_5$, which implies that H is not solvable.