# Minimalism in Symmetric Cryptography 

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inzéa

## Minimalism



Credit: Hans Peter Schaefer

Maybe less exciting?


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Why is minimalism interesting?


## Why is minimalism interesting?



Besides (niche) application needs, it helps us understand where security comes from.

## Outline

1. Designing a practical PRP
2. How to make it lightweight?
3. Possible weaknesses coming from "minimal" Sboxes

# Designing a Practical PRP 

## Practical PRP

$$
E_{k}:\{0,1\}^{n} \longrightarrow\{0,1\}^{n}
$$

- indistinguishable from randomly chosen permutations of $\{0,1\}^{n}$ with $n \in\{64,128\}$
- implementable


## Iterated construction



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## Iterated construction



## AES [Daemen-Rijmen 98][FIPS PUB 197]

- blocksize: 128 bits
- 10 rounds for the 128 -bit key version
- Sbox operates on 8 bits
- diffusion layer is linear over $\mathbf{F}_{2}{ }^{8}$
- nonlinear key schedule.

How to make it lightweight?

## Lightweight block ciphers

## AES [Daemen-Rijmen 98][FIPS PUB 197]

- blocksize: 128 bits
- Sbox operates on 8 bits
- diffusion layer is linear over $\mathbf{F}_{2}{ }^{8}$

To make it smaller in hardware:

- blocksize: 64 bits
- smaller Sbox, on $\mathbf{3}$ or $\mathbf{4}$ bits
- linear diffusion layer over a smaller alphabet
- simplified key-schedule

The usual design strategy: PRESENT [Bogdanov et al. 07]


## Lightweight but secure...

## Increase the number of rounds!

- PRESENT [Bogdanov et al. 07]. 31 rounds
- LED [Guo et al. 11]:

LED-64: 32 rounds, LED-128: 48 rounds

- SPECK [Beaulieu et al. 13]:

SPECK64/128: 27 rounds, SPECK128/256: 34 rounds

- SIMON [Beaulieu et al. 13]:

SIMON64/128: 44 rounds, SIMON128/256: 72 rounds

Does lightweight mean "light + wait"? [Knežević et al. 12]

## Lightweight Competitions

CAESAR for authenticated encryption (2014-2019) :
https://competitions.cr.yp.to/caesar.html
Use case 1: Lightweight applications (resource constrained environments)

1. Ascon [Dobraunig, Eichlseder, Mendel, Schläffer 14]
2. Acorn $[\mathrm{Wu} 14]$

NIST Lightweight Cryptography standardization process (2019-2023)
Ascon family (announced in Feb. 2023)

Duplex-Sponge mode for AEAD encryption [Bertoni et al. 12]

where $\mathcal{P}$ is a permutation of $\{0,1\}^{n}$.

## Duplex-Sponge mode in Ascon


where $\mathcal{P}$ is a permutation on 320 bits of which 64 are known/controlled.

## $\mathcal{P}$ in Ascon [Dobraunig, Eichlseder, Mendel, Schläffer 16]



Permutation operating on a 320-bit state:

- 8-bit constant addition;
- Nonlinear Sbox on 5 bits of degree 2 (on the 64 columns);
- 5 simple linear transformations on 64 bits

$$
\Sigma_{i}\left(X_{i}\right)=X_{i} \oplus\left(X_{i} \ggg a_{i}\right) \oplus\left(X_{i} \ggg b_{i}\right)
$$

$\rightarrow 6$ rounds

## Use low-cost Sboxes

## Low-degree Sboxes and algebraic attacks

Algebraic Normal Form of $f: \mathrm{F}_{2}^{n} \rightarrow \mathrm{~F}_{2}$ :
unique polynomial representation in $\mathrm{F}_{2}\left[x_{1}, \ldots, x_{n}\right] /\left(x_{1}^{2}-x_{1}, \ldots, x_{n}^{2}-x_{n}\right)$.

$$
f\left(x_{1}, \ldots, x_{n}\right)=\bigoplus_{u \in \mathbb{F}_{2}^{n}} c_{u} x^{u} \text { with } \boldsymbol{c}_{\boldsymbol{u}} \in \mathbb{F}_{\mathbf{2}}
$$

Evaluation of a monomial:
Evaluation of $x^{(0101)}$ at $x=(0011)$ :

$$
0^{0} 0^{1} 1^{0} 1^{1}=1011=0
$$

$$
x^{u}=1 \text { if and only if } u \preceq x
$$

i.e., $\boldsymbol{u}_{\boldsymbol{i}} \leq \boldsymbol{x}_{\boldsymbol{i}}$ for all $\mathbf{1} \leq \boldsymbol{i} \leq \boldsymbol{n}$.

ANF and values:

$$
f(a)=\bigoplus_{u \preceq a} c_{u} \text { and } c_{u}=\bigoplus_{a \preceq u} f(a)
$$

## Cube-like attacks [Dinur-Shamir 09]

$$
\begin{aligned}
& f: \mathbb{F}_{2}^{64} \times \mathbb{F}_{2}^{256} \rightarrow \mathbb{F}_{2} \\
& (x, k) \quad \mapsto f(x, k) \\
& f(x, k)=\bigoplus_{u \in \mathbb{F}_{2}^{64}} \underbrace{\left(\bigoplus_{u \in \mathbb{F}_{2}^{256}} \alpha_{u, v} k^{v}\right)}_{A_{u}(k)} x^{u}
\end{aligned}
$$

## Attack:

- Offline: determine the polynomial expression of $\boldsymbol{A}_{\boldsymbol{u}}(k)$ for a given $\boldsymbol{u}$.
- Online: for the key used $\boldsymbol{k}^{*}$, compute the value

$$
A_{u}\left(k^{*}\right)=\bigoplus_{v \preceq u} f\left(v, k^{*}\right)
$$

Cube-like attacks on Ascon [Rohit et al. 21][Baudrin-C.-Perrin 22]

$$
S(x, a, b, c, d)=\left\{\begin{aligned}
&(a \oplus 1) x \oplus \\
& a b \oplus a d \oplus a \oplus b \oplus c \\
& x \oplus b \oplus a c \oplus b c \oplus a \oplus b \oplus c \oplus d \\
& c d \oplus a \oplus b \oplus d \oplus 1 \\
&(c \oplus d \oplus 1) x \oplus \\
& a \oplus b \oplus c \oplus d \\
& a x \oplus
\end{aligned}\right.
$$

$\rightarrow$ The degree in $\boldsymbol{x}$ after $\boldsymbol{r}$ rounds is $2^{r-1}$, for $\boldsymbol{r} \leq \mathbf{6}$.

After two rounds:
The coefficient of $x_{0} x_{i}$ is

$$
\left(a_{0} \oplus 1\right) \boldsymbol{P} \oplus \boldsymbol{Q} \oplus\left(c_{0} \oplus d_{0} \oplus 1\right) \boldsymbol{R} \oplus a_{0} S
$$

For some well-chosen $\boldsymbol{i}$, it equals $\left(a_{0} \oplus 1\right) \boldsymbol{P}$ or $\left(c_{0} \oplus d_{0} \oplus 1\right) \boldsymbol{R}$.

## Cube attack on Ascon [Baudrin-C.-Perrin 22]

## After six rounds:

For all 64 outputs, the coefficient of some monomials of degree $2^{5}$ containing $x_{0}$ can be written as

$$
\left(a_{0} \oplus 1\right) \boldsymbol{P} \oplus\left(c_{0} \oplus d_{0} \oplus 1\right) \boldsymbol{R}
$$

$\rightarrow$ If these 64 coefficients do not all vanish, then

$$
a_{0}=0 \text { or } c_{0} \oplus d_{0}=0
$$

+ The converse also holds in practice.

Practical attack in the nonce-misused scenario [Baudrin-C.-Perrin 22]


Recover the full initial state from less than $2^{39.6}$ ciphertexts obtained from the same $(K, N)$ with time complexity $2^{40}$.

Minimalism in cryptography is more fun than in cooking

