Erratum to ”A lemma on nearby cycles and its application to the tame Lubin-Tate space”

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Weizhe Zheng has pointed out that the statement of Theorem 2.2 is incorrect in some situations where the divisors $Y_i$ are not regular. This occurs in the simplest situation of an elliptic curve with multiplicative reduction, taking $J = I$ (a singleton).

In order to fix the problem, one has to assume in Theorem 2.2 that $Y_J$ is regular (with its reduced subscheme structure), or equivalently that $Y_J^0 = Y_J \setminus \bigcup_{i \notin J} Y_i$.

We note that this property is fulfilled in our application to Yoshida’s model, since in this case $Y_J \setminus \bigcup_{i \notin J} Y_i$ is a Deligne-Lusztig variety.

The flaw in the proof lies in the reduction to the local situation. Indeed, let $x \in Y_J$ be a closed point and denote by $I_x \subset I$ the subset of components such that $x \in Y_i$. Let $(X', x') \xrightarrow{f} (X, x)$ be etale and such that $X'_x = \sum_{i \in I'} e'_i Y'_i$ with each $Y'_i$ regular, and let $I'_x \subset I'$ be the corresponding subset for $x'$. The map $I'_x \xrightarrow{f} I_x$ induced by $f$ is such that $f^{-1}(Y_i) = \sum_{i \in f^{-1}(i)} Y'_i$ in a neighborhood of $x'$. Therefore it is surjective and the fiber $f^{-1}(i)$ is a singleton if and only if $Y_i$ is regular at $x$. Similarly, we have $f^{-1}(Y_J) = \sum_{J' \subset f^{-1}(J), |J'|=|J|} Y'_J$ (equality of $|J|$-cycles), so that $f^{-1}(Y_J)$ is of the form $Y_{J'}$ if and only if $Y_J$ is regular at $x$. In this case, we see that $i'_x^! i^*_J (R\Psi(\Lambda)) = 0$ if and only if $i'_x, i^*_J (R\Psi(\Lambda)) = 0$, which allows to reduce to the local computation of the paper.

If $Y_J$ is not regular, then the singular locus of $f^{-1}(Y_J)$ is $\bigcup_{J' \subset f^{-1}(J), |J'|=|J|+1} Y'_J$, which is not contained in $\bigcup_{i \notin f^{-1}(J)} Y'_i$, so that the singular locus of $Y_J$ is not contained $\bigcup_{i \notin J} Y_i$ either. So we see that $Y_J$ is regular if and only if $Y_J^0 = Y_J \setminus \bigcup_{i \notin J} Y_i$. 

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