Erratum to "A lemma on nearby cycles and its application to the tame Lubin-Tate space"

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July 3, 2012

Weizhe Zheng has pointed out that the statement of Theorem 2.2 is incorrect in some situations where the divisors Y_i are not regular. This occurs in the simplest situation of an elliptic curve with multiplicative reduction, taking $J = I$ (a singleton).

In order to fix the problem, one has to assume in Theorem 2.2 that Y_J is regular (with its reduced subscheme structure), or equivalently that $Y_J^0 = Y_J \setminus \bigcup_{i \notin J} Y_i$.

We note that this property is fulfilled in our application to Yoshida's model, since in this case $Y_J \setminus \bigcup_{i \notin J} Y_i$ is a Deligne-Lusztig variety.

The flaw in the proof lies in the reduction to the local situation. Indeed, let $x \in Y_J$ be a closed point and denote by $I_x \subset I$ the subset of components such that $x \in Y_i$. Let $(X', x') \xrightarrow{f} (X, x)$ be etale and such that $X'_s = \sum_{i \in I'} e'_i Y'_i$ with each Y'_i *regular*, and let $I'_{x'} \subset I'$ be the corresponding subset for x'. The map $I'_{x'} \xrightarrow{f_*} I_x$ induced by f is such that $f^{-1}(Y_i) = \sum_{i' \in f_*^{-1}(i)} Y'_{i'}$ in a neighborhood of x'. Therefore it is surjective and the fiber $f_*^{-1}(i)$ is a singleton if and only if Y_i is regular at x. Similarly, we have $f^{-1}(Y_J) = \sum_{J' \subset f_*^{-1}(J), |J'| = |J|} Y'_{J'}$ (equality of $|J|$-cycles), so that $f^{-1}(Y_J)$ is of the form $Y_{J'}$ if and only if Y_J is regular at x. In this case, we see that $i_x^! i_J^*(R\Psi(\Lambda)) = 0$ if and only if $i_{x'}^! i_{J'}^*(R\Psi(\Lambda)) = 0$, which allows to reduce to the local computation of the paper.

If Y_J is not regular, then the singular locus of $f^{-1}(Y_J)$ is $\bigcup_{J' \subset f_*^{-1}(J), |J'| = |J|+1} Y'_{J'}$ which is not contained in $\bigcup_{i' \notin f_*^{-1}(J)} Y'_{i'}$, so that the singular locus of Y_J is not contained $\bigcup_{i \notin J} Y_i$ either. So we see that Y_J is regular if and only if $Y_J^0 = Y_J \setminus \bigcup_{i \notin J} Y_i$.