Loren Graham and Jean-Michel Kantor

A Comparison of Two Cultural Approaches to Mathematics: France and Russia, 1890-1930[1]

"... das Wesen der Mathematik liegt gerade in ihrer Freiheit"
Georg Cantor, 1883[2]

Many people would like to know where new scientific ideas come from, and how they arise. This subject is of interest to scientists of course, historians and philosophers of science, psychologists, and many others. In the case of mathematics, new ideas often come in the form of new "mathematical objects:" groups, vector spaces, sets, etc. Some people think these new objects are *invented* in the brains of mathematicians; others believe they are in some sense *discovered*, perhaps in a platonistic world. We think it would be interesting to study how these questions have been confronted by two different groups of mathematicians working on the same problems at the same time, but in two contrasting cultural environments. Will the particular environment influence the way mathematicians in each of the two groups see their work, and perhaps even help them reach conclusions different from those of the other group? We are exploring this issue in the period 1890-1930 in France and Russia on the subject of the birth of the descriptive theory of sets.

In the Russian case we have found that a particular theological view – that of the "Name Worshippers" – played a role in the discussions of the nature of mathematics. The Russian mathematicians influenced by this theological viewpoint believed that they had greater freedom to create mathematical objects than did their French colleagues subscribing to rationalistic and secular principles.

Thus, we are brought to the subject of "religion and science" about which so much has been written.[3] Often that literature has centered on the question of whether religion and science are in "conflict" or "harmony."[4] Answering this question is not our concern, and, in fact, we consider the question simplistic. We could cite episodes in the history of science when many people would say that religion "conflicted" with science (e.g., the cases of Galileo and Darwin) and other episodes when many would say that religion and science seemed to be in harmony (e.g., the cases of Newton and Pascal). Noting these differences, we prefer to look at the context and details of individual cases, without prejudging the issue. We do not consider this article to be an argument either for or against religion.

Our thesis is that in France the predominant secular and rationalist culture acted as a negative influence on accepting infinite sets (in particular, non-denumerable ones) as legitimate mathematical objects, while the mystical religious views of the founders of the Moscow School of Mathematics acted as a positive influence in such acceptance. The leading French mathematicians whose views we will examine are René Baire (1874-1932), Henri Lebesgue (1875-1944), and Emile Borel (1875-1956). The leading Russian mathematicians we will consider are Dmitrii Egorov (1869-1931) and Nikolai Luzin (1883-1950). In the Russian case it is also necessary to look at the influence of Egorov's and Luzin's friend and colleague Pavel Florenskii (1882-1937), who left mathematics for the priesthood. It was the acceptance of transfinite numbers[5] as legitimate mathematical objects by these men, especially Nikolai Luzin, that boosted the development of the theory of functions of a real variable and the birth of the Moscow School of Mathematics, one of the major influences in twentieth century mathematics.

We will start with France, then move to Russia, and then give our conclusions.

Set Theory in Cartesian France

Set theory was first developed, of course, in Germany by George Cantor (1845-1918). Among the contributors who helped its emergence the most important from the point of view of this article is the philosopher, theologian and mathematician Bernard Bolzano (1781-1848), who introduced the word "set" (die Menge) and defended "actual infinite."[6] Cantor's ideas became known almost immediately in France, even if they were not welcomed. Most French mathematicians were not ready for this new mathematics coming from their powerful neighboring country. Charles Hermite (1822-1901) and Paul Appell

(1855-1930), for example, initially opposed the idea of translating Cantor's work into French, and, pressed on the matter, suggested as translator Abbé Joseph Dargent, a Jesuit priest at Saint Sulpice; Hermite observed, "leur tournure philosophique ne sera pas un obstacle pour le traducteur qui connaît Kant"[7] Clearly, Hermite and Appell thought that Cantor's work was more German metaphysics than mathematics. In fact, French philosophers, such as Louis Couturat (1868-1914), earlier a student of Henri Poincaré turned philosopher, and Paul Tannery (1843-1904) were among the first interested in Cantor's ideas. Then they attracted the reluctant attention of mathematicians (such as Jules Tannery, the brother of Paul).

All French mathematicians were not, however, hostile to the same degree to Cantor's set theory, and a few became intrigued by some of its implications even if they remained unwilling to offer full acceptance. Gradually some notions from set theory began to creep into the work of French mathematicians, such as that of Emile Borel (1871-1956), but at first applied to point sets[8] more than abstract set theory itself. Camille Jordan (1838-1922) introduced some set theory in the second edition of his "Cours " at the Ecole Polytechnique around 1885.[9]

The patriarch of French mathematics, Henri Poincaré (1854-1912), might have been expected to be a little more receptive to Cantor, since Poincaré was strongly influenced by Kant, and was a very learned philosopher of science. One can feel Cantor's influence for example in his famous memoir of 1890[10] where he used what would later be called sets of points of measure zero. However, Poincaré relied mostly on intuition and he opposed, sometimes aggressively, the "logicist" current (Russell, Couturat, Peano) whose exponents wanted to reduce mathematics to logic. Couturat even accused Poincaré of adopting nominalism (a philosophy dating to the scholastic period of the middle ages, indirectly connected to the issue of "naming" that will become important to the Russian mathematicians yet to be discussed). Poincaré refused to recognize the transfinite but accepted the law of excluded middle. His rejection of actual infinity, his strong opinions, and his age all put him outside the mainstream of the discussions of set theory.

Meanwhile René Baire (1874-1932), a rigorous mathematician working in the tradition of Cauchy, Borel and Italian mathematicians like Volterra and Gini, was preparing a major twist in the theory of functions which relied on some concepts from the new set theory, those which he found acceptable. Until Baire's work, functions were understood as continuous and differentiable in Euler's sense, at least locally.[11] Baire dared to look at discontinuous functions and he began to classify their mysterious behavior. But the old French establishment of mathematics strongly resisted; Emile Picard (1856-1941), for example, despite his remarkable talent in mathematics and his broad mind on many other subjects, rejected discontinuous functions for many years. In 1898 Picard reacted to Baire's thesis in the following way:

"L'auteur nous paraît avoir une tournure d'esprit favorable à l'étude de ces questions qui sont à la frontière de la mathématique et de la philosophie…."[12]

By connecting Baire's viewpoint with philosophy, not mathematics, Picard was using the key word for the contemptuous critique of set theory. Philosophy was soft and unrigorous, and not a suitable occupation for a mathematician. Later, in 1902, Picard reported on Lebesgue's thesis with similar dismay: "On est parfois saisi de vertige en voyant à quels résultats on arrive quand on abandonne les hypothèses usuelles."[13] And as late as 1905 Picard still rejected discontinuous functions, observing "Natura non facit saltus, nous avons le sentiment, on pourrait dire la croyance, que dans la nature il n'y a pas de place pour la discontinuité."[14]

Emile Borel, on the other hand, was more receptive initially to set theory. Borel, first student, then professor, then Director of the Ecole Normale Superieure, was a central figure in the intellectual and political life of France for more than 50 years. Coming from the rural provinces (Rouerge, in the southwest), he rose very quickly in both mathematical and administrative circles and eventually became a deputy minister of France.[15]

As a young mathematician Borel was captivated by Cantor's work. In his thesis in 1895 he solved a problem of function theory using a result on limit points (what later would be called "Heine-Borel's theorem"), and he was the first to teach a course in set theory at the Ecole Normale Supérieure (Lebesgue, among others, was among the listeners). The same year Borel met Cantor and at first rhapsodized about him. Borel later confessed "J'ai été extrêmement séduit, dès l'âge de 20 ans, par la lecture des travaux de Cantor… . Georg Cantor a apporté dans l'études des mathématiques cet esprit romantique qui est l'un des côtés les plus séduisants de l'âme allemande."[16]

However, as Borel continued his studies little by little his resistance to Cantor began to grow. Part of the reluctance may have come from conversations with his French colleagues, many of whom remained skeptical of the philosophical tenor of Cantor's views. In addition, Borel had a "down-to-earth" approach to mathematics somehow close to his upbringing in rural France. Cantor's transfinite "aleph numbers" were not "down-to-earth."! (as for example normal numbers would be; see endnote 46)

The unease of French mathematicians with set theory was deepened by their observation that the study of Cantor's "alephs" might cause some mental disturbances. Cantor had his first serious attack of depression in 1884, and Baire, who already had some digestive problems, fell badly ill in 1898, as if being punished for his flirtation with the new ideas. Borel, after referring to the illnesses of Cantor and Baire, told his friend Paul Valéry in 1924 that he had to abandon set theory "à cause de la fatigue qu'elle lui imposait et qui lui faisait craindre des troubles sérieux s'il s'obstinait à ce travail "[17] Baire stopped working in 1900, became "neurasthenic," and finally, in 1932, he killed himself.[18]

At the second International Congress of Mathematicians in Paris in 1900 the German mathematician David Hilbert (1862-1943) presented a set of unsolved problems in his field that remains well-known to the present day.[19] The essence of the first problem that Hilbert posed was the mystery of the continuum. The Continuum Hypothesis asked whether any non-denumerable subset of the real line (the continuum) had the power (the cardinal) of the continuum. This question had been in Cantor's mind since the eighties; and he developed at that time a strategic approach to it that consisted in looking carefully at more and more complex subsets of the real line, an approach that was to lead to Descriptive Set Theory. The problem had a strong geometrical aspect, although there had been efforts in the past century (e.g., Weierstrass, Dedekind[20]) to "arithmetize the continuum."

Henri Lebesgue (1875-1941) had an entirely geometrical approach to all mathematics, but became interested in functions, seeing deep connnections with geometry: "Il y a entre la théorie générale des fonctions de variables réelles et la géométrie pure des liens que je sens étroits, encore qu'ils restent un peu mystérieux pour moi."[21] Lebesgue from 1894 to 1897 was at the Ecole Normale Supérieure where his master and friend was Borel. Courageously picking up the study of functions of a real variable at about the time that Borel was losing his nerve, Lebesgue moved into opposition to the old school (Jordan, Darboux), which was reluctant to study the "monsters" appearing in the field. The fruitful approach which Lebesgue took was to compare the classification of functions constructed by Baire[22] with the recently developed measure theory of Borel.[23]

The Axiom of Choice and the French Five Letters

On September 26, 1904, the German mathematician Ernst Zermelo (1871-1953) wrote to David Hilbert and told him that he had developed a "Beweis, dass jede Menge wohlgeordnet warden kann"[24]. In the proof he used what would later be called the Axiom of Choice (AC): "For any family of non-empty sets there exists a mapping which associates to each of these sets a common element." The statement of Zermelo was a thunderclap, due in part to its simplicity. As Lebesgue observed, "Zermelo arriva, et ce fut la bagarre!"[25] Zermelo's proclamation stimulated a large debate that lasted for more than ten years. The starting point of the controversy was the exchange of five letters among French mathematicians in 1905.[26]

Jacques Hadamard (1865-1963) was the only one of the four French mathematicians participating in the exchange not to oppose completely Zermelo's axiom. He took a very personal approach, saying that "la notion de correspondance qui peut etre décrite est en dehors des mathématiques, relève du domaine de la psychologie et est relative à une propriété de notre esprit"[27] Needless to say, this opinion only increased the suspicion of his critics.

Lebesgue was strictly opposed to the use of the Axiom of Choice. He maintained that "Raisonner sur un ensemble c'est raisonner sur des objets pris dans un sac C sur lesquels on sait seulement qu'ils ont une propriété B en commun n'appartenant pas aux autres objets de C. On ne sait par conséquent rien qui permette d'aborder la définition de l'ordre des éléments; on ne sait même pas les distinguer"[28]

It became clear that the use of the Axiom of Choice was leading to new mathematical paradoxes, soon called antinomies, demonstrating their philosophical importance. Already mathematicians were wrestling with Burali-Forti's seeming paradox of 1897[29] and Russell's paradoxes of 1902-1903[30]; now with the appearance of the Axiom of Choice they found themselves facing

Richard's paradox about the definition of a number that cannot be defined (!)[31] and even geometrical paradoxes. Such problems stirred the opposition of Borel and Lebesgue, called later by the intuitionist school of the Dutch mathematician L. E. J. Brouwer (1881-1966) "the French preintuitionists."[32]

The discussions among Baire, Lebesgue, Borel, and Hadamard became very intricate, mixing philosophy, linguistics, and psychology. The debates lasted for many years without any agreement, and were featured in numerous articles in journals with rather large distributions. In 1912 Hadamard, in correspondence with Borel, even made a connection between the veracity of the Axiom of Choice and the existence of Brownian motion![33]

The central issue was clear in Lebesgue's letter of 1905. He showed that the Axiom of Choice is equivalent to the principle of well-ordering and then asked, "Peut -on s'assurer de l'existence d'un être mathématique sans le définir? Définir veut toujours dire nommer une propriété du défini."[34] Therefore it was clear that the ontological status of mathematical objects was at stake.

Emile Picard, a strict opponent of set theory, was ironical but correct in his summary of the situation:

> Ces spéculations sur l'infini forment un chapitre tout nouveau dans la science mathématique de ces dernières années, mais il faut reconnaître que ce chapitre n'est pas exempt de paradoxes. C'est ainsi que l'on a pu définir certains nombres appartenant et n'appartenant pas tout à la fois à des ensembles déterminés. Toutes les difficultés de ce genre résultent de ce

> qu'on ne s'entend pas sur le mot existence. Certains adeptes de la théorie des ensembles sont des scolastiques, qui auraient aimé à discuter les preuves de l'existence de Dieu, avec Saint-Anselme et son contradicteur, le moine de Noirmoutiers Gaunilon."[35]

(Here Picard was raising the issue of religion in order to discredit set theory; on the contrary, the Russian mathematicians to be discussed later in this article would raise religion in order to strengthen set theory).

It is strange and striking that Baire and Borel, who had in their recent work shown scientific courage by using the transfinites of set theory, now in the public discussion (or "fight" according to Lebesgue), contradicted what was implicit in their earlier views. This contradiction has been noticed also by Gregory Moore (loc.cit.) when restricted to measure of sets and to the work of Lebesgue and Borel, and later Sierpinski; the contradiction developed more strongly in the coming years, to the extent that Baire and Borel not only rejected the Axiom of Choice[36] but the use of transfinite numbers as well. As late as 1908 Borel still opposed the use of non-denumerable infinities.[37]

This genuine contradiction calls for an explanation. Why did prominent French mathematicians, not only those like Picard who had never wavered in their opposition to set theory, but also those like Baire and Borel who had flirted with some aspects of it, move toward greater and greater resistance?

In our opinion French mathematicians were being influenced by a negative feedback coming from the cultural and philosophical milieu of their country.

This interpretation is strengthened by comparing the reception of set theory in France to that in other countries.[38] In France, in our view, three different influences were at work, sometimes in opposite directions, and mediated through close connections (even sometimes family connections) between mathematicians and philosophers and the closed, centralized educational system, especially that of the Ecole Normale Supérieure and the Ecole Polytechnique. Those three influences, briefly described below, are Descartes, positivism, and Pascal.

First of all, France is the country of Descartes, the true protector of French thinkers and scientists. In the period we are examining here there was even a strong identification with Descartes among the radical left, of which Borel was an active member.[39] According to Descartes one must

> … diviser chacune des difficultés que j'examinerais, en autant de parcelles qu'il se pourrait, et qu'il serait requis pour les mieux résoudre. Š conduire par ordre mes pensées, en commençant par les objets les plus simples et les plus aisés à connaître, pour monter peu à peu, comme par degrés, jusque la connaissance des plus composés; et supposant même de l'ordre entre ceux qui ne se précèdent point naturellement les uns les autres.[40]

Any problem should be decomposed into its simple components, and thought means clarity. The same principle of analysis and expression was taught to every school-child in France with the statement of the seventeenth-century literary critic

Boileau "Ce qui se conçoit bien s'énonce clairement, et les mots pour le dire arrivent aisément. "[41]

One can see the strength of Descartes' influence in the numerous articles and speeches in his honor given during the period we are examining. For example, on the three hundredth anniversary of Descartes' death, Emile Picard, the outspoken opponent of set theory, proclaimed "J'ai toujours eu, comme il convient, un respect infini pour Descartes . . . . Il faut juger Descartes savant sur l'orientation toute nouvelle donnée à la science par ses intuitions géniales et par une méthode."[42]

Descartes was also an influential representative of the view that mathematics was the universal and least biased form of knowledge. Borel was defending this view when he remarked during the discussions of set theory "Dès lors qu'il s'agit de mathématiques et non de philosophie, le désaccord ne peut provenir que d'un malentendu."[43] Borel and other French mathematicians wanted to keep, to the maximum degree possible, philosophical questions outside of mathematical ones. The Russian mathematicians we will be examining wanted to integrate philosophical, indeed religious, issues with mathematics.

A second important influence among French mathematicians was positivism. The end of the nineteenth century saw the triumph of August Comte's positivism not only at the Sorbonne but throughout French education by means of the reform of the educational system in 1902.[44] For Comte once science liberates itself from all metaphysical influences and enters the "positive stage" its goal is no longer a metaphysical quest for truth or a rational theory purporting to represent reality. Instead, science is composed of laws (correlations of observable facts) which can be used by the scientist without pronouncements on the nature of reality.

Pascal's influence, although less important than Cartesianism or positivism, and sometimes in contradiction with them, can be seen for example in the distinction between "definition de noms et definition de choses" ("the definition of names and the definition of things"). Also Pascal was fighting Descartes's belief in "final causes." According to Pascal there is no absolute truth, just geometrical clarity (he understood Geometry in a very broad way; for example, he called the theory of probability which he created "Géométrie du hazard," aleae geometria) The importance of geometry would give strength to Lebesgue and Borel in fighting against rules without geometric foundations as expressed by logicians like Russell or Couturat.[45]

These various influences, sometimes contradictory, warned French mathematicians not to mix psychology or philosophy (not to speak of religion!) with mathematics but instead to restrict mathematical notions to those for which a clear definition as well as a clear "representation in the mind" could be found. Borel abandoned set theory and concentrated on specific concrete issues such as the existence of normal numbers.[46] Lebesgue restricted himself to "effective sets" precisely defined in an explicit way. But Lebesgue was still attracted to the geometric mysteries of the continuum, and it is moving to see how these contradictions forced him into a mistake he could have avoided only by using transfinite indices.[47] This mistake left an opening for the Russian mathematicians

Suslin and Luzin, who welcomed set theory, to correct the mistake twelve years later.

Russian Mysticism and Mathematics

Dmitrii Fedorovich Egorov (1869-1931) and Nikolai Nikolaevich Luzin (1883-1850) were the founders of the Moscow School of Mathematics.[48] In the first years of the twentieth century Luzin studied mathematics in Moscow University under Egorov along with another person who was influential in forming the ideas of the Moscow School, Pavel Florenskii (1882-1937). In their mature and professionally active years all three of these men – Egorov, Florenskii and Luzin -- were deeply religious.[49] One of them, Florenskii, abandoned mathematics for religious studies, disappointing his teachers, and became a priest. Egorov and Luzin went on to become outstanding mathematicians who helped create an explosion of mathematical research in Moscow in the nineteen twenties and early thirties. Florenskii and Egorov would eventually be arrested by the Communist authorities, accused of mixing mathematics and religion, and subsequently die in prison.[50] (It is one of the cruel ironies of history that the Communist charge that Florenskii and Egorov mixed mathematics and religion was correct, although, contrary to the assumption of the Communists, the mixture was amazingly fruitful to the field of mathematics.) The third -- Luzin -- narrowly escaped imprisonment, even though he was put on “trial” for ideological deviations.[51] Of these three it was Florenskii, the priest, who most prominently developed a new ideology of mathematics and religion that played a role in the pioneering mathematics work of Egorov, Luzin, and their students.

From 1905 to 1908 Luzin underwent a psychological crisis so severe that several times he contemplated suicide.[52] One precipitating event in Russia was the unsuccessful Revolution of 1905, a moment which sobered many leftwing members of the intelligentsia like Luzin who had often talked romantically of their hopes for a "revolution" without comprehending the violence that revolutions typically bring. 1905 was a year of truth. Luzin was shaken not only by the shedding of blood, but also by personally witnessing poverty. He was particularly shocked to see poor women resort to prostitution in order obtain food.[53]

Luzin possessed a tender, somewhat naïve personality, and he was not prepared for the pain he saw around him during and immediately after the abortive revolution. In an effort to get him out of his spiritual crisis his teacher Egorov sent him abroad in December, 1905, but the trip did not solve Luzin's spiritual and intellectual problems. His faith in science and mathematics had collapsed. He was totally without a purpose in life. In despair on May 1, 1906, he wrote Florenskii from Paris:[54]

> You found me a mere child at the University, knowing nothing. I don't know how it happened, but I cannot be satisfied any more with the analytic functions and Taylor series . . . . To see the misery of people, to

> see the torment of life . . . -- this is an unbearable sight. . . . I cannot live by science alone . . . . I have nothing, no worldview, and no education.

In a long correspondence and in numerous meetings in a monastery town outside Moscow (Sergeev Posad, Zagorsk), Florenskii, a devout believer, supplied Luzin with a new worldview. This worldview was one that combined both religion and mathematics and, as we will see, gave the desperate Luzin reason to believe that he could renew his mathematical research while at the same time serving moral and religious purposes

Florenskii thought that much of the nineteenth century had been a disaster from the standpoint of philosophy, religion, and ethics, and that the particular type of mathematics that reigned during that century was one of the important causes of this misfortune. The governing mathematical principle of the nineteenth century which Florenskii saw as responsible for ethical decline was "continuity," the belief that all phenomena pass from one state to another smoothly. In substitution of this "false" principle of continuity Florenskii proposed its opposite, discontinuity, which he saw as morally and religiously superior.

Florenskii was well aware that discussions of the antinomy of continuity-discontinuity were very old, dating back to the Greeks, and perhaps discussed most famously in Zeno's paradoxes, but Florenskii believed that the issue had a new relevance because the nineteenth century was, according to him, the unfortunate apogee in faith in continuity; indeed, he wrote that in the nineteenth century "the cementing idea of continuity brought everything together in one gigantic monolith."[55]

The mathematical approach which created this monolith, continued Florenskii, was infinitesimal calculus. This method became all-powerful because calculus was at the heart of the physical sciences through Newtonian mechanics. One of the results of its seeming omnipotence was that mathematicians concentrated on continuous functions, since all differentiable functions are continuous. Florenskii believed that as a result mathematicians and philosophers tended to ignore those problems that could not be analyzed by calculus, the discontinuous phenomena.

Believing that continuous functions were "deterministic"[56], Florenskii saw the expansion of the philosophy of determinism throughout psychology, sociology, and religion as the destructive result of a temporary emphasis in mathematics. Thus he held nineteenth century mathematics responsible for the erosion of earlier beliefs in freedom of will, religious autonomy, and redemption.

Florenskii thought that the field that was "guilty" of the glaring overestimation of continuity -- mathematics -- was destined to lead thinkers out of the blind alley which it had created. In the 1880's Georg Cantor, founder of set theory, had analyzed "continuum" as merely a set among possible other sets, and had therefore deprived the concept of its metaphysical, dogmatic power.[57] (The first terms from set theory were introduced at Moscow University in 1900-01, the year Florenskii entered mathematical studies there, and Florenskii quickly became aware of the event.[58]) Now the road was open, maintained Florenskii, to restore discontinuity to its rightful place in one's worldview. Florenskii called for "the dawn of a new discontinuous worldview"[59] and challenged his mathematician colleagues like Luzin and Egorov to foster this new approach, one which would, he thought, combine mathematics, religion, and philosophy.

Although Florenskii became a priest in the Russian Orthodox Church, his theological views were unconventional. Indeed, he supported a viewpoint that was condemned by the officials of the Church as a "heresy." We must briefly look at the history of this controversy within the Russian Orthodox Church in order to understand how it became linked to mathematics.

In the years 1907-1917 the world of Russian Orthodoxy, the state religion, was shaken by a theological struggle. A polemic developed between two groups of religious believers, the Imiaslavtsy ("Worshippers of the Name" or "Nominalists") and the "Imiabortsy" ("Anti-Nominalists").[60] The dispute was rooted in an ancient question over how humans can worship an unknowable deity. If God is in principle beyond the comprehension of mortals (and holy scripture contains many such assertions[61]), how, in complete ignorance of His nature, can human beings worship Him? What does one worship? The most common response given to this dilemma throughout religious history was the resort to symbols: icons, names, rituals, music, relics, scents, tastes, art, architecture, literature. Symbolism is the use of a perceptible object or activity to represent to the mind the semblance of something which is not shown but realized by association with it.[62]

In 1907 a monk of the Orthodox Church, Ilarion, who had earlier spent years in a Russian monastery in Mt. Athos in Greece, published a book entitled *In the Mountains of the Caucasus* which seized on an existing symbolic tradition in Orthodox liturgy, especially the chanting of the "Jesus Prayer," (Iisusovaia molitva) and raised it to a new prominence.[63] In the Jesus Prayer the religious believer chants the names of Christ and God over and over again, hundreds of times, until his whole body reaches a state of religious ecstasy in which even the beating of his heart, in addition to his breathing cycle, is supposedly in tune with the chanted words "Christ" and "God." (a state vividly described by J. D. Salinger in *Franny and Zooey*)[64] According to Ilarion the worshipper achieves a state of unity with God through the rhythmic pronouncing of His name. And this demonstrates, said Ilarion, that the name of God is holy in itself, that the name of God *is* God ("Imia Bozhie est' sam Bog").[65]

At first this book was well received by many Russians interested in religious thought. Ilarion's views became very popular among the hundreds of Russian monks in Mt. Athos, and gradually spread its influence elsewhere. But the highest officials in Russian Orthodoxy, in St. Petersburg and Moscow, soon began to consider the book not just as a description of the significance of prayer but as a theological assertion. For many of them the adherents of Ilarion's beliefs were heretics, even pagan pantheists, because they allegedly confused the symbols of God with God Himself. On May 18,1913, the Holy Synod in St. Petersburg condemned the Name Worshippers; soon thereafter the Russian Navy, with the approval of Tsar Nikolai II and the patriarch of Constantinople (under whose jurisdiction the monasteries of Mt. Athos came), sent several gunboats to Mt. Athos to bring the rebellious monks forcibly to heel. Over 600 unrepentant monks were flushed out of their cells with fire hoses and brought under guard to Odessa. In later detentions, the number grew to approximately 1000.[66] The dissidents strongly protested their treatment and obtained promises of further investigation and reconsideration. The Name Worshippers had some defenders in high places (including Florenskii, now a professor at the Moscow Theological Academy), and the tsar himself seemed to be of two minds on the question.[67]

With the advent of World War I the issue receded into the background, but until the end of the tsarist regime the adherents of the "heresy" were forbidden to return to Mt. Athos or to reside in major cities like St. Petersburg and Moscow. The most fervent of them retreated to monasteries, including ones with which Florenskii was very familiar, where they continued to practice and propagate their variant of the faith.

After the Bolshevik Revolution in October/November 1917 the Name Worshippers, now living all over rural Russia, were more successful than most other religious believers in continuing their practices out of view of Soviet political authorities, who were trying to suppress religion. After all, the Name Worshippers had already been defined as heretics and excluded from the established churches.[68] But in secret they continued their faith, and as a result of the fact that they were out of view they were not compromised by association with the Bolsheviks, as some of the established Church leaders soon became. The dissidents claimed to be representatives of the undefiled "true faith," increasing their popularity with some religious opponents of the new Communist regime. (Amazingly, as late as 1983 rumors spread about the secret existence in the Soviet Union of followers of the dissident faith of Name Worshipping[69], and some people have asserted that descendants of the sect members still practice their faith in the Caucasus today, outlasting their Soviet oppressors.[70] And in Moscow recent publications illustrate that some of the ideas of Name Worshipping are still attractive to intellectuals[71]).

At the time of the Bolshevik Revolution Pavel Florenskii was living in a monastery town near Moscow and he was close religiously and intellectually to the Name Worshipper dissidents. He communicated their ideas to Luzin and Egorov and he translated them into mathematical parlance. In the early nineteen twenties there was a "name worshipper circle" (imeslavcheskii kruzhok) in Moscow where the ideas of the religious dissidents and the concepts of mathematics were brought together. Participants in the circle included fifteen or sixteen philosophers, mathematicians, and religious thinkers. Sometimes the circle met at Egorov's apartment and at several of these meetings Florenskii presented papers.[72] Here he expounded the view that "the point where divine and human energy meet is 'the symbol,' which is greater than itself."[73] To Florenskii religious and mathematical symbols can attain full autonomy.

Florenskii saw that the Name Worshippers had raised the issue of "naming" to a new prominence. To name something was to give birth to a new entity. Florenskii was convinced that mathematics was a product of the free creativity of human beings and that it had a religious significance. Humans could exercise Free Will and put in perspective mathematics and philosophy. The famous sentence of Georg Cantor (cited in the original on the first page of this article) that “The essence of mathematics lies precisely in its freedom” clearly had a strong appeal to Florenskii. Mathematicians could create beings (sets) by just naming them.

For example, defining the set of numbers such that their squares are less than 2, and naming it “A”, and analogously the set of numbers such that their squares are larger than 2, and naming it “B”, was immediately bringing into existence (essentially the Eudoxus-Cauchy construction) the real number $\sqrt{2}$.

The development of set theory was to Florenskii a brilliant example of how naming and classifying can bring mathematical breakthroughs. A “set” was simply a naming of entities according to an arbitrary mental system, not a recognition of types of real material objects. When a mathematician created a “set” by naming it he was giving birth to a new mathematical being. The naming of sets was a mathematical act, just as the naming of God was a religious one, according to the Name Worshippers. A new form of mathematics was coming, said Florenskii, and it would rescue mankind from the materialistic, deterministic modes of analysis so common in the nineteenth century. And, indeed, set theory, new insights on continuous and discontinuous phenomena like the development under the name “Arithmology” of the discovery in 1897 by Hensel of the p-adic numbers (which strongly impressed Egorov, Luzin, and Florenskii, and their followers), and discontinuous functions -- became hallmarks of the Moscow School of Mathematics.

The idea that "naming" is an act of creation goes back very far in religious and mythological thought. The claim has been made that the Egyptian god Ptah created with his tongue that which he conceived.[74] In Genesis we are told that God said "Let there be light: and there was light." In other words, He named light first, and then He created it. Names are words, and the first verse in the Gospel according to St. John states: "In the beginning was the Word, and the Word was with God, and the word was God." In the Jewish mystical tradition of the Kabbala (Book of Creation, Zohar) there is a belief in creation through emanation, and the name of God is considered holy.[75]

The connection between the religious dissidents in Russia who emphasized the importance of the names of Jesus and God and the new trends in Moscow mathematics went beyond the suggestions and implications so far discussed. There was a linguistic direct connection. The Moscow mathematicians Luzin and Egorov were in close communication with French mathematicians with similar concerns. As noted earlier, Henri Lebesgue introduced in 1904 the concept of "effective sets," by which he meant sets which could be constructed without using Zermelo's Axiom of Choice.[76] He spoke of "naming a set" ("nommer un ensemble")[77] and such a set was then often called a "named set" ("ensemble nommé", loc.cit.). The Russian equivalent was "imennoe mnozhestvo". Thus the root word "imia" ("name") occurred in the Russian language in both the mathematical terms for the new types of sets, and the religious trend of "imiaslavie" ("Name Worshipping"). And, indeed, much of Luzin's work on set theory involved the study of effective sets ("named sets")[78].

To Florenskii this meant that both religion and mathematics were moving in the same direction.

Roger Cooke of the University of Vermont studied Luzin's personal notes in the archives in Moscow and noted that Luzin

> frequently studied the concept of a 'nameable' object and its relationship to the attempted catalog of the flora and fauna of analysis found in the Baire classification. To Luzin the continuum conjecture was merely one aspect of the general problem of naming the set of countable ordinal numbers; he seems to have believed that if one could name this set in the sense of Lebesgue, it would not be difficult to settle the question of its cardinality. . . . Luzin was trying very hard to name all the countable ordinals. . . . [79]

At one point Luzin wrote in his notes "Everything seems to be a daydream, playing with symbols, which however, yield great things."[80] At another moment Luzin scribbled: "nommer, c'est avoir individu."[81]

Both the French and the Russian mathematicians were wrestling with the problem of what is a mathematical object, what is allowed and what should be a good definition of such. As noted earlier, Lebesgue wrote to Borel in 1905 "Peut –on s'assurer de l'existence d'un être mathématique sans le definir?"[82] ("Is it possible to convince yourself of the existence of a mathematical being without defining it?") To Florenskii the question was the analogue of "Is it possible to convince yourself of the existence of God without defining Him?" The answer for Florenskii and later for Egorov and Luzin was that the act of naming in itself gave the object existence. Thus "naming" became the key to both religion and mathematics. The Name Worshippers gave existence to God by worshipping His name, and mathematicians gave existence to sets by naming them.

The circle of eager students which formed around Luzin at about the time of the beginning of World War I and continued throughout the early twenties was known as "Lusitania." An indirect hint of the place of religion in the concerns of the Lusitanians is given by the description of the group by one of its original members, M. A. Lavrent'ev (1900-1980).[83] According to Lavrent'ev (later a significant mathematician and the founder of the "Academic City" in Novosibirsk) the Lusitanians acknowledged two leaders: "God-the-father" Egorov and "God-the-son" Luzin. It was Luzin who told the young Lusitanians: "Egorov is the chief of our society," and "our discoveries belong to Egorov." Students in the society were given the monastic titles of "novices." Noting Lavrent'ev's description of the group, Esther Phillips wrote, "There was clearly a strong sense of belonging to an inner circle or a secret order."[84] All the principals and novices went to Egorov's home three times a year: Easter, Christmas, and his name-day. Among the Lusitanians there was intense camaraderie inspired by Luzin, who was described as extroverted and theatrical, and who evoked real devotion among students and colleagues. Egorov, on the other hand, the senior member, was more reserved and formal. According to Lavrent'ev, Luzin's chief assistants in managing Lusitania were three students, each with his own function:

Pavel Aleksandrov was the creator, Pavel Uryson the keeper, and Viacheslav Stepanov the herald of the mysteries of Lusitania.

The Moscow School of Mathematics gave birth to a new field, the descriptive theory of sets. The date of birth of this field can be given precisely; it was the day in 1917 when one of Luzin's students, Mikhail Suslin (1894-1919)[85] rushed into the office of his thesis advisor (Luzin) to show him a mistake he had found in a seminal article published in 1905 by the great French mathematician Henri Lebesgue. Fortunately there was an eye-witness to the scene: Waclaw Sierpinski, a young Polish student of Luzin's. Sierpinski later recalled, "M. Lusin treated very seriously this young student who was claiming that he had found a mistake in the paper of such an eminent scientist."[86]

It took a few months for Suslin and Luzin to understand the deep structure behind this problem. As Sierpinski continued, "In order to construct an example of a B-measurable (B for Borel) plane set with a non-B-measurable projection, Suslin created a whole new class of sets which he called analytic sets (A-sets)."[87] Suslin noted that the projection of an intersection of sets does not always coincide with the intersection of their projections. He then went further, with the help of Luzin, and, using non-denumerable cardinals, introduced a new class of sets called projective sets.[88]

Conclusions

We believe that our study of French and Russian developments in set theory and the theory of functions points strongly toward the importance of cultural factors in the process and creation of mathematics – in the French case, Descartes, positivism, and Pascal; in the Russian case, mystical religious beliefs, particularly those of the Name Worshipping movement. As a result the French and Russians followed different approaches.

We realize that intellectual causation of the type we are describing can never be proved, only made plausible and hopefully persuasive. However, we think that our interpretation has grown stronger as the evidence for the rationalistic scruples of the French and the religious mysticism of the Russians has accumulated. We further believe that a comparative study of the type we are presenting produces more credible evidence for the influence of society on the development of science than can an example in which only one society is being considered. The juxtaposition of the French and Russian mathematicians working on the same problems at the same time highlights the differences in their viewpoints. And the Russian case is particularly interesting because it indicates that religion may have played, at least temporarily, a positive function in science.

The influence of religion in mathematics is not a new topic, but instead stretches back at least to Pythagoras. Hermann Weyl noted in his analysis of the Greek golden age of mathematics,

> Aside from the fact that mathematics is the necessary instrument of natural science, purely mathematical inquiry in itself, according to the conviction of many great thinkers, by its special character, its certainty

> and stringency, lifts the human mind into closer proximity with the divine than is attainable through any other medium. Mathematics is the science of the infinite, its goal the symbolic comprehension of the infinite with human, that is finite, means. It is the great achievement of the Greeks to have made the contrast between the finite and the infinite fruitful for the cognition of reality. Coming from the Orient, the religious intuition of the infinite, the apeiron, takes hold of the Greek soul. This tension between the finite and the infinite and its conciliation now become the driving motive of Greek investigation.[89]

In our view the rationalistic commitments of the French mathematicians in our story were an inhibiting factor in their acceptance of transfinite numbers, and the religious interests of the Russians were a promoting factor, but we do not see this relationship as necessary or inherent. What we are presenting is not an argument about the essential relationship of religion and science, but a particular interpretation of that relationship advanced at a specific time and place.

Although Luzin was very close to a number of leading French mathematicians and cited his debt to them, his worldview was different. In their study of set theory the French wanted to identify philosophical, mathematical and psychological components clearly and keep them separate. On the contrary, Luzin and some of his friends believed that mathematics was linked to religion, but they could not be explicit about these links because of the hostile Soviet environment after the Revolution. They knew that they could easily get into trouble with the authorities if the views discussed in the meetings of the "imiaslavie circle" ("name-worshipping circle") became known[90]. Eventually Luzin and his friends were caught and persecuted, but only after they had made mathematical breakthroughs. In the nineteen twenties Luzin's religious and philosophical approach helped stimulate in him a profound mathematical originality. He and his students created a new field: the descriptive theory of sets.

One of the leading French mathematicians in this story, Henri Lebesgue, finally acknowledged that it was exactly "philosophy" – what he and his French colleagues tried to avoid in mathematics – that helped Luzin make his innovations. Lebesgue wrote in a preface to Luzin's 1930 book published in Paris that with Luzin "exigences mathématiques et exigences philosophiques sont constamment associées, on peut meme dire fondues."[91] Lebesgue admitted that this approach helped Luzin and his students to find a concept he had not seen. Once his eyes were opened, Lebesgue was astounded by the fruitfulness of the Russian approach. In open wonderment he declared, "M. Lusin examine les questions d'un point de vue philosophique et aboutit ainsi à des résultats mathématiques : originalité sans précédent !"[92]

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[2] "The essence of mathematics lies precisely in its freedom."

[3] There are hundreds of books on religion and science. A few that deserve notice are: John Hedley Brook, *Science and Religion: Some Historical Perspectives* (Cambridge: Cambridge University Press, 1991); Edward J. Larson, *Trial and Error: The American Controversy Over Creation and Evolution* (Oxford: Oxford University Press, 1989); Michael Ruse, *Can a Darwinian be a Christian? The relationship between science and religion* (Cambridge: Cambridge University Press, 2001); Frederick Gregory, *Nature Lost? Natural science and the German theological traditions of the nineteenth century* (Cambridge: Harvard University Press, 1992); Max Jammer, *Einstein and Religion: physics and theology* (Princeton: Princeton University Press, 1999); Louis Châtellier, *Les espaces infinis et le silence de Dieu: science et religion XVIe-XIXe siècle* (Paris: Aubier, 2003); C. Chevalley, *Pascal, contingence et probabilities* (Paris: PUF, 1995); T. Levy,

Figures de l'infinites: les mathématiques au regard des cultures (Paris: Seuil, 1987); P. P. Gaidenko and V. N. Katanosov (eds), *Nauka, filosofiia, religiia: v poiskakh obshchego znamenatelia* (Moscow: IF RAN, 2003); Stefan Blinderhöfer, *Naturwissenschaftlicher Weltzugang und der Überschuss der Schöpfungsperspektive* (Frankfurt: Lang, 2000). An older book still of value is: Charles Coulston Gillispie, *Genesis and Geology: A study in the relations of scientific thought, natural theology, and social opinion in Great Britain, 1790-1850* (Cambridge: Harvard University Press, 1951, 1969).

[4] The classic statement of the conflict thesis is Andrew Dickson White, *A History of the Warfare of Science with Theology in Christendom* (New York: Appleton, 1896; Buffalo: Prometheus Books, 1993). A book which revised the "warfare" model is: David C. Lindberg and Ronald L. Numbers (eds.), *God and Nature: Historical Perspectives on the Encounter Between Science and Christianity* (Berkeley: University of California Press, 1986).

[5]Transfinite numbers, closely connected with philosophical and even religious considerations about "the Absolute," were introduced by Cantor in 1882. He justified their introduction by the freedom of mathematicians to define new beings (see the quotation on the first page of this article, under the title). Also, see J. Ferreiros, *Labyrinth of thought: a history of set theory and its role in modern mathematics* (Boston: Birkhauser, 1999); Walter Purkert, "Georg Cantor und die Antinomien der

Mengenlehre," *Bulletin de la Société mathématique de Belgique*, 1986, 38: pp. 313-327; and W. Purkert, *Georg Cantor*, 1845-1918 (Basel, Boston: Birkhauser, 1987).

[6] B. Bolzano, *Les Paradoxes de l'infini* (Paris : Seuil, 1993) ; J. Sebestik, *Logique et mathématique chez Bernard Bolzano* (Paris : Vrin, 1992).

[7] "Their philosophical turn of mind will not be an obstacle for a translator who knows Kant," Charles Hermite, "Lettres à G. Mittag-Leffler publiées et annotées par Pierre Dugac,'' *Cah. Sém. Hist. Math.,* 1980, 5:49-285.

[8] Limit point, derived set of points.

[9] C. Jordan, *Cours d'analyse de l'École polytechnique* (Paris : Gauthier-Villars, 1893-1896), 2e éd.

[10]H. Poincaré, "Sur le problème des trois corps et les équations de la dynamique," *Acta Math.*, 1890, XIII :1-270. See thesis in preparation by A. Robadey.

[11] For the history of the theory of functions see from the same Luzin who will be one of our main characters: N.Luzin, "Function," (in two parts) *American Math. Monthly*, 1988, 105 (No. 1): 59-67; and 1988, 105 (No. 3): 263-270, (translated by A. Schenitzer from the *Great Soviet Encyclopaedia* of 1934) or, more recently, Fyodor A Medvedev, *Scenes from the history of real functions* (translated by Roger Cooke) (Boston: Birkhauser, 1991).

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[12] "The author seems to have a turn of mind favoring those questions which are on the border-line between mathematics and philosophy, " quoted in Hélène Gispert (ed.), *La France mathématique; la Société mathématique de France (1872-1914)* (Paris: Société Française d'Histoire des Sciences et des Techniques/Société Mathématique de France, 1991).

[13] One is struck by vertigo when one sees the result that comes from abandoning usual hypotheses," quoted p.284, *loc. cit.*.

[14] "Nature does not make jumps; we have the feeling, one can even say the belief, that in nature there is no place for discontinuity, ¨ *La science moderne et son état actuel*, (Paris : Flammarion, 1905), Ch.II, "Sciences mathématiques et astronomie".

[15] See P. Guiraldenq, *Emile Borel, 1871-1956: E'espace et le temps d'une view sur deux siècles* (St-Affrique: l'Imprimerie du Progrès, 1999).

[16] "I was completely seduced when I was twenty. Georg Cantor brought to the study of mathematics that romantic spirit which is one of the most attractive aspects of the German soul, ¨ E. Borel, *Œuvres complètes* (Éditions du Centre national de la recherche scientifique, 1972), Vol. IV, p. 2101.

[17] "because of the fatigue it caused him, which made him fear and foresee in himself

serious illness if he persisted in that work" Valery, *Cahiers*, quoted by P. Dugac, *Histoire de l'analyse* (Paris: Vuibert, 2003); P. Valery, *Cahiers*, Vol. I (Paris: Gallimard, 1973), p. 797.

[18] See the letter received by his brother, p. 313, in P.Dugac, ¨Notes et documents sur la vie et l'œuvre de René Baire,¨ *Arch. Hist. Ex. Sc.,* 1976, 15 : 297-383.

[19] D. Hilbert, ¨Sur les problèmes futurs des Mathématiques¨ (translated by L. Laugel), *Congrès. intern. des math*. (Paris: Compte-rendu du deuxieme congres international des mathematicians: CIM, 1900), 58-114, English version : F. Browder (ed.) *Mathematical Developments Arising from Hilbert Problems* (Providence: American Mathematical Society, 1976). Shortly before the mathematics congress there was an international congress of philosophy in which the mathematicians Hadamard, Borel, and Poincare participated.

[20] R. Dedekind, *Stetigkeit und irrationale Zahlen* (Braunschweig: Vieweg, 1872).

[21] "There are connections which I feel to be very close between the general theory of functions of a real variable and pure geometry, but they remain a little mysterious to me." Introduction inédite à la "Notice sur les travaux scientifiques de M. Henri Lebesgue de 1922," unpublished *Oeuvres complètes* (Genève : L'Enseignement mathématique, 1972-73), v. 1, p. 90.

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[22] R. Baire, *Œuvres scientifiques* (ed. by P. Lelong with help of P. Dugac) (Paris : Gauthiers-Villars, 1990).

[23] See a detailed analysis of Borel and Lebesgue's work on measure theory in G. H. Moore, "Lebesgue's measure problem and Zermelo's axiom of choice: the mathematical effects of a philosophical dispute," *History and Philosophy of Science: Selected Papers, Ann. New York Acad. Sci.,* 412 (New York: New York Academy of Sciences, 1983).

[24]"A proof that any set can be well-ordered", given an order with the same properties as the set of positive integers. E. Zermelo, "Beweis, dass jede Menge wohlgeordnet warden kann (Aus einem an Herrn Hilbert gerichteten Briefe," *Mathematische Annalen*, 1904, 59: 514-516. See also G. H. Moore, *Zermelo's Axiom of Choice: Its origin, development and influence* (Berlin: Springer, 1990).

[25] "Zermelo arrived and the fight began." H. Lebesgue, *Œuvres scientifiques* (Genève: L'enseignement mathématique, 1972), p. 294.

[26] J. Hadamard, R. Baire, H. Lebesgue, E. Borel, "Cinq lettres sur la theorie des ensembles," *Bulletin de la Societe Mathematique de France,* 1905, 23:261-273.

[27] "The question of what is a *correspondence that can be described* is a matter of psychology and relates to a property of the mind outside the domain of mathematics," *loc. cit.* Letter I.

[28] "To analyze a set is to analyze objects in a bag; we know only that the objects in the bag have a property 'B' in common that others do not have. Consequently, we do not know anything which allows us to define any order among them. One does not even know how to distinguish them," letter of Lebesgue to Borel, February 1905, in H. Lebesgue, *Les lendemains de l'intégrale: Lettres à Emile Borel* (Paris: Vuibert, 2004).

[29] C. Burali-Forti, ¨Una questione sui numeri transfiniti,¨ *Rendiconti del Circolo Matematico di Palermo* , 1897, 11 :154-164.

[30] B. Russell, "On some difficulties in the theory of transfinite numbers and order types," *London Mathematical Society*,1906,4:29-53.

[31] J. Richard, "Lettre à Monsieur le rédacteur," *Revue générale des Sciences*, 12 (June 1905), p. 12. For all three references above see also J. Ferreiros, *Labyrinth of thought: a history of set theory and its role in modern mathematics (*Boston: Birkhauser, 1999*).*

[32] See, for example, A.Heyting, *Les Fondements des mathématiques, Intuitionnisme,*

Théorie de la démonstration (Paris : Gauthier-Villars, 1955). A thorough philosophical discussion is due to J.Bouveresse. See "Sur les sens du mot platonisme dans l'expression 'platonisme mathématique', ¨ Conférence du 19 novembre 1998 à l'Université de Genève, Société romande de philosophie, groupe genevois http://un2sg4.unige.ch/athena/bouveresse/bou_plat.html).

[33] Quoted by E. Borel in "L'infini mathématique et la réalité", *Rev. du mois 1914,* 18,.71-84. See *: Emile Borel: Philosophe et homme d'action, Pages choisies et présentées par M. Fréchet* (Paris: Gauthier-Villars, 1967), p. 183.

[34] "Can we convince ourselves of the existence of a mathematical being without defining it? To define always means naming a characteristic property of what is being defined."

[35] "These speculations about infinity are a completely new chapter in the history of mathematics of recent years, but it is necessary to recognize that this chapter does not escape paradoxes. Thus, one can define certain numbers that belong, and at the same time do not belong, to specific sets. All problems of this type are caused by a lack of agreement on what existence means. Some believers in set theory are scholastics who would have loved to discuss the proofs of the existence of God with Saint Anselme and his opponent Augilon, the monk of Noirmoutiers." E. Picard, *La science moderne et son etat* (Paris : E. Flammarion, 1909), extract from chapter II. See Anselme de Cantorbery,

*Proslogion Allocution sur l'existence de Dieu suivi de sa réfutation par Gaunilon et de la réponse d'Anselme (*Paris : Flammarion, 1993).

[36] E.Borel, *Atti del IV Congresso Internazionale dei Matematici* : Roma, 6-11 April 1908.

[37]The same rejection is maintained through all prefaces of later editions (4 of them) of his famous *Leçons sur la théorie des fonctions (Éléments et principes de la théorie des ensembles: applications à la théorie des fonctions*). 3. éd. (Paris: Gauthier-Villars ; (Collection de monographies sur la théorie des fonctions) until 1950.

[38] See Moore G. *loc.cit* for more positive reactions in England, Germany, Italy and United States.

[39] The tradition of democracy and importance of education were revived during the Dreyfus controversy in which many friends of Borel participated.

[40] "to divide each of the difficulties under examination into as many parts as possible, and as might be necessary for its adequate solution. To conduct my thoughts in such order that, by commencing with objects the simplest and easiest to know, I might ascend little by little, and, as it were, step by step, to the knowledge of the more complex, assigning in thought a certain order even to those objects which in their own nature do not stand in a

relation of antecedence and sequence." René Descartes, *Discours de la méthode pour bien conduire sa raison, et chercher la vérité dans les sciences* (Paris : chez Michel Bobin et Nicolas Le gras, 1668).

[41]"Anything that is understood well can be expressed clearly, and the words to tell them come easily" . Nicolas Boileau-despréaux, *Les quatre poétiques / d'Aristote, d'Horace, de Vida, de Despréaux* (Paris : Saillant et Nyon-Desaint, 1771).

[42] "One must judge Descartes on the completely new orientation he gave to science by his genius-like intuitions and by his method." E. Picard, *Une édition nouvelle du discourse de la méthode de Descartes* (Paris : Gauthier-Villars, 1934).

[43] "As long as we are dealing with mathematics and not philosophy, disagreement can only stem from misunderstanding."

[44] See B. Belhoste, "L'enseignement secondaire français et les sciences au début du XX° siècle : La réforme de 1902 des plans d'études et des programmes,¨ *Rev. Hist. Sci.*, 1990, XLIII/4 :371-400.

[45] For a thorough analysis of Pascal's differences with Descartes on the nature of mathematics,see Catherine Chevalley, *Contingence et probabilities (*Paris: P.U.F., 1995).

[46] Normal numbers are numbers say between 0 and 1, which, if written in decimal expansion, present each sequence of n letters with frequency as expected for "random" numbers. They were introduced by Borel (*Leçons sur la théorie des fonctions* [Paris: Gauthier-Villars, 1914], p. 197). Absolutely normal numbers are numbers which have the analogous property on any basis. Although the absolutely normals are of measure one, it is a difficult question to give explicit ("effective" was the word used then) constructions. For references and recent progress see V. Becher and S. Figueira, "An example of a computable absolutely normal number," *Theoretical computer science,* 2002, 270: 948-958.

[47]See Jean-Michel Kantor, "Dans le gouffre du continu," publication pending.

[48] Allen Shields, "Years Ago: Luzin and Egorov," *The Mathematical Intelligencer* , 1987, 9: 24-27; Smilka Zdravkovska and Peter L. Duren (eds.), *Golden Years of Moscow Mathematics* (Providence: American Mathematical Society, 1993); Alexander Vucinich, "Mathematics and Dialectics in the Soviet Union: The Pre-Stalin Period," *Historia Matematica* , 1999, 26: 107-24; P. S. Aleksandrov, *Matematika v SSSR za 15 let* (Moscow: Gostekhizdat, 1932).

[49]As a priest, it is hardly necessary to prove the importance of religion to Florenskii; his best known published statement of faith is probably to be found in his *Stolp i utverzhdenie istiny* (Moscow: Put', 1914), published in English as *The Pillar and Ground of the Truth* (trans. by Boris Jakim, introduction by Richard F. Gustafson)

(Princeton: Princeton University Press, 1997). The deep religiosity of Egorov is described in S. S. Demidov, "Professor Moskovskogo universiteta Dmitrii Fedorovich Egorov i imiaslavie v Rossii v pervoi treti XX stoletiia," *Istoriko-matematicheskie issledovaniia*, 1999, 39 (2nd series, No. 4): 137 and passim. Luzin's conversion by Florenskii to a religious view is described in various sources, including Charles Ford, "The Influence of P. A. Florensky on N. N. Luzin," *Historia Mathematica*, 1998, 25:332-339. Also see Ford, "Dmitrii Egorov: Mathematics and Religion in Moscow," *The Mathematical Intelligencer,* 1991. 13: 24-30. Ford has done very valuable work on this topic, and we are grateful for his assistance, but we do not agree with him that Egorov's deep religious views and mathematical conceptions were disconnected.

[50] Florenskii was first arrested in 1928, then released, then arrested again in 1933 and sentenced to ten years in labor camps in Siberia. He was executed on August 8, 1937. In 1956 he was rehabilitated and has slowly gained attention since then as a philosopher of language and culture, a theologian, and, most recently, as an influence on Russian mathematics. See Richard Gustafson, "Introduction," *The Pillar and Ground of the Truth*, (Princeton: Princeton University Press), 1997, pp. ix-xxiii. Egorov was rebuked by the Communist Party in 1929, arrested in 1930, and sent to prison. There he went on a hunger strike. Just before his death he was taken under guard to a hospital in Kazan; he died on September 10, 1931. We are told that he died in the arms of the wife of the mathematician N.G. Chebotaryov, who was a doctor in the hospital. Chebotaryov's son G. N. Chebotaryov wrote, "On umer na maminykh rukakh . . ." ("He died in my mother's arms . . .") in G. N. Chebotaryov, "Iz vospominanii ob ottse," in Iu. B. Ermolaev (ed.)

Nikolai Grigor'evich Chebotarev (Kazan: Izdatel'stvo Kazanskogo universiteta, 1994), p. 56.

[51]S. S. Demidov and V. D. Esakov, "'Delo akademika N. N. Luzina' v svete stalinskoi reformy sovetskoi nauki," *Istoriko-matematicheskie issledovania*, 1999, 39 (2nd Series, No . 4): 156-170. S. S. Demidov and B. V. Levshin (eds.), *Delo akademika Nikolaia Nikolaevicha Luzina* (Sankt Peterburg: Russkii khristianskii gumanitarnyi institute, 1999); A. P. Iushkevich, "Delo akademika N. N. Luzina," *Vestnik AN SSSR,* 1989, 4: 102-13; Alexey Levin, "Anatomy of a Public Campaign: 'Academician Luzin's Case' in Soviet Political History," *Slavic Review*, 1990, 49: 211-52; A. N. Bogoliubov and N. M. Rozhenko, "Opyt' 'vnedreniia' dialektiki v matematiku v kontse 20-kh-nachale 30-kh godov," *Voprosy filosofii* , 1991, 9: 32-43..

[52]On July 15, 1908, Luzin wrote Florenskii: "Two times I was very close to suicide – then I came here . . . looking to talk with you, and both times I felt as if I had leaned on a 'pillar' . . . . I owe my interest in life to you." Charles Ford, "The Influence of P. A. Florensky on N. N. Luzin," *Historia Mathematica*, 1998, 25:338.

[53] *Ibid.*

[54]"Perepiska N. N. Luzina s P. A. Florenskim," *Istoriko-matematicheskie issledovaniia*, 1989, 31:136.

[55]Florenskii, *op. cit.*, p. 160. For a philosophical discussion of discontinuity, especially in the Russian context, see M. D. Akhundov, *Problema preryvnosti i neprerynosti prostranstva i vremeni*, (Moscow: Nauka, 1974). Also, J. Bouveresse, *Weyl, Wittgenstien, et le problème du continu,* in J.-M. Slanskis and H. Sinaceur (eds.) *Le Labyrinthe du continu* (Paris, Berlin, New York: Springer-Verlang, 1992).

[56] P. A. Florenskii, "Vvedenie k dissertatsii 'Ideia preryvnosti kak element mirosozertsaniia'", *Istoriko-matematicheskie issledovaniia*, 1986, 30:170.

[57] On Cantor see Joseph W. Dauben, *Georg Cantor: his mathematics and philosophy of the infinite*, (Cambridge: Harvard University Press, 1979)..

[58] F. A. Medvedev, "O kurse lektsii B. K. Mlodzeevskogo po teorii funktsii deistvitel'nogo peremennogo, prochitannykh osen'iu 1902 g. v Moskovskom universitete," *Istoriko-matematicheskie issledovaniia*, 1986, 30:130-147.

[59]Florenskii, "Vvedenie k dissertatsii . . .," op. cit., pp. 159-176.

[60]*Imiaslavie: antologiia* (Faktorial Press: Moscow, 2002); also see O. L. Solomina and A. M. Khitrov, *Zabytye stranitsy russkogo imiaslaviia: sbornik dokumentov i publikatsii po afonskim sobytiiam 1910-1913 gg.* (Palomnik: Moscow, 2001); Iu. Rasskazov, *Sekrety imen: ot imiaslavii do filosofii iazyka (Labirint: Moscow, 2000); Ilarion, Sviashchennaia taina tserkvi: vvedenie v istoriiu I problematiku imiaslavskikh sporov*,

(Aleteiia, St. Petersburg, 2002); Georges Florovsky, *Ways of Russian Theology*, Part Two, vol. 6, in Richard S. Haugh (ed.), The *Collected Works of Georges Florovsky* (trans. Robert L. Nichols), (Buchervertriebsanstalt: Belmont, Massachusetts, 1987), p. 376; Tom Dykstra, "Heresy on Mt. Athos: Conflict over the Name of God Among Russian Monks and Hierarchs, 1912-1914," master's essay, St. Vladimir's Seminary, New York, 1988; Antoine Niviere, "Les moines onomatodoxes et l'intelligentsia Russe," *Cahiers du Monde russe et soviétique,* 1988, 29 (2): 181-194; Scott M. Kenworthy, "Church, State, and Society in Late-Imperial Russia: The Imiaslavie Controversy," paper given at American Association for the Advancement of Slavic Studies National Convention, Pittsburgh, November 2002. Although the debates over Name Worshipping became particularly intense in the first third of the twentieth century, the roots of the controversy go far back in the history of Eastern Orthodoxy, and can be found in some of the writings and sayings of Basil the Great, John Chrysostom, and other Church figures. See Georges Florovsky, *op. cit*.

[61]For example, in Philipp 4:7, "and the peace of God, which surpasses all understanding, will guard your hearts and minds through Christ Jesus."

[62] Both mathematics and religion make heavy, but very different, uses of symbols. And in both the question of the autonomy of the symbols arises. Although Florian Cajori does not directly confront this issue, see his useful *A History of Mathematical Notations* (The Open Court Publishing Company: Chicago, 1929); also, Paul D. L. Avis, *God and the*

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creative imagination: metaphor, symbol, and myth in religion and theology (Routledge: New York, 1999).

[63]The full name of the book was *In the mountains of the Caucasus: a conversation between two elder ascetics concerning the inner union of our hearts with the Lord through the prayer of Jesus Christ; or the spiritual activity of contemporary hermits, composed by the hermit of the Caucasus mountains, the monk Ilarion (Na gorakh Kavkaza, beseda dvukh startsev podvishnikov o vnutrennem edinenii s Gospodom nashikh serdets chrez molitvu Iisus Khristova ili dukhovnaia deiatel'nost' soveremennykh pustinnikov, sostavil pustynnozhitel' Kavkavskikh gor skhimonakh Ilarion*), first edition, Batalpashinsk, 1907; second corrected and expanded edition, 1910; third edition, Kievskaia Perchersskaia Lavra, 1912). The roots of the Jesus Prayer go back centuries; the start of the tradition is often cited as Apostle Paul's instructions to the faithful to "pray without ceasing" (1 Thess. 5.17). The tradition exists in both Catholic and Orthodox liturgies, and there is an extensive literature on it. In Russia the Jesus Prayer acquired a special prominence, especially after the publication of the folklore classic *The Way of the Pilgrim* in Kazan in 1884. The book popularized the prayer and was translated into many languages.

[64] Salinger has Franny observing to her incredulous friend Lane, "Well, the starets tells him about the Jesus Prayer first of all. . . . If you keep saying that prayer over and over again – you only have to just do it with your <u>lips</u> at first – then eventually what happens, the prayer becomes self-active. Something <u>happens</u> after a while. I don't know what, but

something happens, and the words get synchronized with the person's heartbeats, and then you're actually praying without ceasing. Which has a really tremendous, mystical effect on your whole outlook. I mean that's the whole point of it, more or less. I mean you do it to purify your whole outlook and get an absolutely new conception of what everything's about." J. D. Salinger, *Franny and Zooey*, (Little Brown and Company: Boston, 1961), pp. 36-37.

[65]A. F. Losev, "Imiaslavie," http://www.ccel.org/contrib/ru/Other/Losev/ONOMATOD.HTM, p. 3; E. S. Polshchuk (ed.), *Imiaslavie: antologiia*, (Faktorial Press, Moscow, 2002), p. 490.

[66]*Imiaslavie,* pp. 479-518.

[67] According to Vladimir Gubanov the Holy Synod was urging the tsar to squelch the heresy before it split the faith and the nation, but the monk Grigory Rasputin, close to the court, was defending the Name Worshippers. The tsar evidently hesitated but in the end gave in to the Synod. Vladimir Gubanov, *Tsar' Nikolai II i novye mucheniki: prorochestva, chudesa, otkrytiia i molitvy: dokumenty*, (St. Petersburg: Obshchestvo svt. Vasilliia Velikogo, 2000). Tom Dykstra also has written that Rasputin may have supported the Name Worshippers in his "Heresy on Mt. Athos: Conflict over the Name of God Among Russian Monks and Hierarchs, 1912-1914," master's essay, St. Vladimir's Seminary, New York, 1988.

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[68]S. S. Demidov, "Professor Moskovskogo universiteta Dmitrii Fedorovich Egorov i imeslavie v Rossii v pervoi treti XX stoletiia," *Istoriko-matematicheskie issledovaniia*, 1999, 39:129-130.

[69] *Imiaslavie: antologiia*, p. 513.

[70] Ep. Ilarion (Alfeev), *Sviashchennaia taina tserkvi*, vol. 2, http://st-jhouse.narod.ru/biblio/texts.htm.

[71] See the recent essay by A. N. Parshin, well-known mathematician, pupil of Igor Shafarevich and corresponding member in the department of mathematics of the Russian Academy of Sciences: "Svet i Slovo (k filosofii imeni)," *Imiaslavie: antologiia*, pp. 529-544.

[72] *Imiaslavie : antologiia*, p. 513.

[73] http://st-jhouse.narod.ru/biblio/texts.htm.

[74]For a modern translation of Memphite Theology see Marshall Clagett, *Ancient Egyptian Science*, Vol. I, (American Philosophical Society: Philadelphia, 1989), pp. 305-312, 595-602. We are grateful to John Murdoch for this suggestion.

[75] Gershom Scholem, Major *Trends in Jewish Mysticism* (New York: Schocken Books, 1995).

[76] Later, after the intervention of Zermelo and also Richard's letter, the term "effective" or "denumerable" will be used for example, by Borel: See Annex and J. Hadamard, R. Baire, H. Lebesgue, and E. Borel, "Cinq lettres sur la théorie des ensembles," *Bulletin de la Société Mathématique de France*, 1905, 33(4): 261-273.

[77] Henri Lebesgue, "Contribution a l'étude des correspondances de M. Zermelo," *Bulletin de la Société mathématique de France*, 1907, 35:227-237, especially pp. 228, 236.

[78] Henri Lebesgue, "Contribution a l'étude des correspondances de M. Zermelo," pp. 227-237, especially pp. 228, 236.

[79] Roger Cooke, "N. N. Luzin on the Problems of Set Theory" unpublished draft, January, 1990, pp. 1-2.

[80] Cooke, *op. cit.*, p. 7.

[81] Cooke, *op. cit.*, citing Archive of the Academy of Sciences of the USSR, fond 606, op. 1, ed. khr. 34.

[82] See note 33.

[83] M. A. Lavrent'ev, "Nikolai Nikolaevich Luzin," *Russian Mathematical Surveys*, 1974, 29 (5):173-178, and *Uspekhi Mathematicheskikh Nauk* 1974, 29 (5):177-182.

[84] Esther R. Phillips, "Nicolai Nicolaevich Luzin and the Moscow School of the Theory of Functions," *Historia Mathematica*, 1978, 5:p. 293.

[85] For more information on Suslin, who died tragically young, see: V. I. Igoshin, "A short biography of Mikhail Yakovlevich Suslin, " *Russian Mathematical Surveys* 51(3):371-383.

[86] Waclaw Sierpinski, *Les ensembles projectifs et analytiques*, (Gauthier-Villars: Paris, 1950).

[87] Ibid.

[88] Yiannis N. Moschovakis, *Descriptive set theory*, (North-Holland: Amsterdam/New York, 1980).

[89]Weyl, *God and the Universe: The Open World* (New Haven: Yale University Press, 1932), p. 8.

[90] While we know that Egorov was a leader of this circle, we have no concrete evidence that Luzin was or ever attended meetings of it. We do know that Luzin was a friend of Father Florenskii, that he was familiar with imiaslavie (the Name Worshipping

movement), and that in his mathematical research he put great emphasis on "naming." Luzin was more cautious than Egorov, and probably made more of an attempt to conceal his religious views from the Soviet authorities.

[91]"mathematical exigencies and philosophical exigencies are constantly associated, one can even say fused." Henri Lebesgue, "Préface," in Nicolas Lusin, *Leçons sur les ensembles analytiques et leurs applications* (Gauthier-Villars: Paris, 1930), p. xi.

[92]"M. Luzin examines questions from a philosophical point of view and ends up with mathematical results. This is an originality without precedent!" Henri Lebesgue, *op. cit.* ("Préface"), p. ix.