Let $G$ be a real reductive Lie group and $s$ be a semisimple element in $G$. The problem here is to write the orbital integral of $s$ in terms of the characters $\text{tr} \pi$ of $G$. Such a formula has been obtained by R. Herb in '83 in the case when $G$ is connected, and by D. Renard in '97 when $G/G_0$ is cyclic. From the work of M. Duflo (concrete Plancherel formula for real linear algebraic groups in '83) and also of M. Vergne and Khalgui-Torasso (Poisson-Plancherel formula in '97), one can expect to obtain general inversion formulas using the orbit method. With this point of view, the “descent method” allows us to suppose that $s$ is elliptic. Then we’ll have:

1. to parametrize in terms of coadjoint orbits some subset of the unitary dual of $G$, and to find some “Kirillov’s formulas” for $\text{tr} \pi(e \exp X)$, when $\pi$ is in the subset mentioned above, $e$ is an elliptic element of $G$ and $X$ is an element close to 0 of the Lie algebra of the commutant $G(e)$ of $e$ in $G$ (cf. my paper in the Journal of Lie Theory, '02);

2. to construct a discrete set of invariant integrals on the Lie algebra of $G(e)$ with some associated signs and write a conjecture (“Poisson-orbital formula”) for the sum of the product of these signs with the corresponding invariant integral, such that it formally solves our problem.