Table of maximal eigenspaces

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For each irreducible complex reflection coset $W\phi$ and each $w\phi \in W\phi$ which has a non-trivial $\zeta$-eigenspace $V$ for a root of unity $\zeta$ of order $d$ which is maximal among such eigenspaces, we give the types of $W_d := N_W(V)/C_W(V)$ and of the coset $C_W(V)w\phi$.

Imprimitive groups

The degrees of $G(ne, e, r)$ are $ne, 2ne, \ldots, (r-1)ne$ and $rn$. The codegrees are $0, ne, 2ne, \ldots, (r-2)ne$ when $n > 1$ or $(r-1)e - r$ when $n = 1$.

The coset $G(ne, e, r), t|e$ is defined by the automorphism realized by $s_1^t$ where $s_1$ is the first generator (of order $ne$) of $G(ne, 1, r)$. The only generalized reflection degree with a non-trivial factor is $(rn, \zeta_t^{-1})$. There is a generalized codegree with a non-trivial factor only when $n = 1$, which is $((r-1)e - r, \zeta_t)$.

When $n > 1$ the regular $\zeta$ are such that $\zeta^{-n} = \zeta_t$, in which case $W_d = G(lcm(ne, d), e, gcd(\frac{rn}{d}, e, r))$ where $G(ne, e, 1) = G(n, 1, 1) = \mathbb{Z}/n$.

- For general $d$ we give the result when $e = 1$, the case $G(n, 1, r)$. If we set $d' = \frac{d}{gcd(n, d)}$ then $W_d = G(lcm(n, d), 1, \lfloor \frac{r}{d'} \rfloor)$ and $C_W(V)w$ is of type $G(n, 1, r \mod d')$ (see [Malle, 3C]).

When $n = 1$, we set $d' = \frac{d}{gcd(e, d)}$.

- The regular $\zeta$ are either such that $\zeta^r = \zeta_t$, then $W_d = G(lcm(e, d), e, r)$, or such that $d|(r-1)e$, then $\zeta^r \neq \zeta_t$ then $W_d = G(lcm(e, d), 1, \frac{r-1}{d'})$.

- For general $d$ such that $\zeta^r \neq \zeta_t$ we have $W_d = G(lcm(e, d), 1, \lfloor \frac{r-1}{d'} \rfloor)$ and if we set $m = 1 + ((r-1) \mod d')$ then $C_W(V)w$ is of type $G(e, e, m)s_1^{d'}$ where $\zeta_{d'} = \zeta_t\zeta^{m-r}$ (see [Malle, (5.3), (5.4)]); note that $G(e, e, 1)$ is the trivial group.

Reference

Primitive groups

In the list below, each $W_d$ is given followed by a colon and the list of $d$ for which it is $W_d$. If $\zeta_d$ is not regular the type of the coset $C_W(V)\omega$ is given in square brackets before $d$. In this type, we note $G^i$ a descent of scalars (the product of $i$ copies of $G$ permuted cyclically in the coset).

\[
\begin{align*}
G_1:1,2&\ Z_6:3,6\ Z_4:4 & G_5:1,2,3,6&\ Z_{12}:4,12 \\
G_6:1,2,4&\ Z_{12}:3,6,12 & G_7:1,2,3,4,6,12  \\
G_8:1,2,4&\ Z_{12}:3,6,12\ Z_8:8 & G_9:1,2,4,8&\ Z_{24}:3,6,12,24  \\
G_{10}:1,2,3,4,6,12&\ Z_{24}:8,24 & G_{11}:1,2,3,4,6,8,12,24  \\
G_{12}:1,2&\ Z_6:3,6\ Z_8:4,8 & G_{13}:1,2,4&\ Z_{12}:3,6,12\ Z_8:[A_1 \cdot \zeta_8]^8  \\
G_{14}:1,2,3,6&\ Z_{24}:4,8,12,24 & G_{15}:1,2,3,4,6,12&\ Z_{24}:[A_1 \cdot \zeta_8^{-1}]^8,[A_1 \cdot \zeta_4^3]^24  \\
G_{16}:1,2,5,10&\ Z_{30}:3,6,15,30 \ Z_{20}:4,20 & G_{17}:1,2,4,5,10,20&\ Z_{60}:3,6,12,15,30,60  \\
G_{18}:1,2,3,5,6,10,15,30&\ Z_{60}:4,12,20,60 & G_{19}:1,2,3,4,5,6,10,12,15,20,30,60  \\
G_{20}:1,2,3,6&\ Z_{12}:4,12\ Z_{30}:5,10,15,30 & G_{21}:1,2,3,4,6,12&\ Z_{60}:5,10,15,20,30,60  \\
G_{22}:1,2,4&\ Z_{12}:3,6,12\ Z_{20}:5,10,20 & H_3:1,2&\ Z_6:3,6\ Z_{10}:5,10  \\
G_{24}:1,2&\ Z_6:3,6\ Z_{14}:7,14 & G_{25}:1,3&\ G_5:2,6\ Z_{12}:4,12\ Z_9:9  \\
G_{26}:1,2,3,6&\ Z_{18}:9,18\ Z_{12}:[A_1]4,[A_1 \cdot \zeta_3]12 & G_{27}:1,2,3,6&\ Z_{30}:5,10,15,30\ Z_{12}:[A_1]4,[A_1 \cdot \zeta_3]12  \\
3D_4:G_2:1,2&\ G_4:3,6\ Z_4:12 & 3^3G_3,3,3:G_3,1,2:1,3&\ Z_6:2,6\ Z_3:9  \\
F_1:1,2&\ G_5:3,6\ G_8:4&\ Z_8:8\ Z_{12}:12 & 2F_4: J_2(8):1,2&\ G_{12}:4\ G_8:8&\ Z_6:12\ Z_{12}:24  \\
G_{29}:1,2,4&\ Z_{20}:5,10,20\ Z_{12}:[A_1]3,[A_1]6,[A_1 \cdot \zeta_4^{-1}]12\ Z_8:[2B_2 \cdot \zeta_8^3]8  \\
H_4:1,2&\ G_{20}:3,6\ G_{22}:4&\ G_{16}:5,10\ Z_{12}:12\ Z_{30}:15,30\ Z_{20}:20  \\
G_{31}:1,2,4&\ G_{10}:3,6,12\ Z_{20}:5,10,20&\ G_9:8&\ Z_{24}:24  \\
G_{32}:1,2,3,6&\ G_{10}:4,12\ Z_{30}:5,10,15,30\ Z_{24}:8,24\ Z_{18}:[Z_3]9,[Z_3 \cdot -1]18  \\
G_{33}:1,2&\ G_{20}:3,6\ Z_{10}:5,10\ Z_{18}:9,18&\ G_6:[A_1]4\ Z_{12}:[A_1 \cdot \zeta_3]12  \\
G_{34}:1,2,3,6&\ Z_{42}:7,14,21,42\ G_{10}:[A_1]^4,([A_1 \cdot \zeta_3]^2)12\ Z_{30}:[A_1]5,[A_1]10,[A_1 \cdot \zeta_3^2]15,[A_1 \cdot \zeta_3]30  \\
Z_{24}:[A_1^2]^8,[A_1^2 \cdot \zeta_3^2]24\ Z_{18}:[3^3G_3,3,3]9,[3^3G_3,3,3 \cdot -1]18  \\
E_6:1&\ F_4:2\ G_{25}:3&\ G_8:4\ G_{15}:6\ Z_8:8\ Z_9:9&\ Z_{12}:12\ Z_5:[A_1]5  \\
E_6:2&\ F_4:1\ G_{25}:6&\ G_8:4\ G_5:3\ Z_8:8\ Z_9:18\ Z_{12}:12\ Z_5:[A_1]10  \\
E_7:1,2&\ G_{26}:3,6\ Z_{14}:7,14\ Z_{18}:9,18&\ G_8:[A_1]^4\ Z_{10}:[A_2]5,10\ Z_8:[A_1 \times A_1^2]8\ Z_{12}:[A_1^3]12  \\
E_8:1,2&\ G_{32}:3,6\ G_{31}:4&\ G_{16}:5,10\ G_{10}:12\ Z_{30}:15,30\ Z_{20}:20\ Z_{24}:24\ Z_{14}:[A_1]7,14  \\
Z_{18}:[A_2]9,[2^2A_2]18
\end{align*}
\]

An observation on the table is that in every split case all regular numbers divide a regular degree. Is this clear a priori?