Grothendieck's "Long March through Galois theory"

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Note. This short text was originally written as a contribution to the "Grothendieck day" which took place in Utrecht on April 12, 1996. It is brief and informal, and was intended to give the audience some very partial idea of what is contained in Grothendieck's long manuscript "La Longue Marche à travers la Théorie de Galois". The close connections between the ideas expressed there and those in parts of the Esquisse and the letter to Faltings in this volume make it relevant to publish it here.

Alexander Grothendieck wrote the *Long March* between January and June 1981. It consists of about 1600 manuscript pages, and nearly as much again in various addenda and developments. About the first 600 pages, consisting of §§1-37, have been read and edited;* the main body of the *Long March* consists of §§1-53. From the table of contents of §§38-53, it seems that they are mainly devoted to the close study of the ideas expressed in the first part, in the cases where $(g, n) = (0, 3)$ and $(1, 1)$. In 1984, Grothendieck wrote and distributed the *Esquisse d'un Programme*, a 57-page text part of which summarizes, sometimes in a more advanced form, the main themes and problems considered in the *Long March.* Both texts are devoted to raising deep questions and examining various approaches to them, and contain few explicit theorems. In reading the *Long March*, it is essential to keep in mind that the text was written entirely by its author for his own use, and not intended for publication; its new TeX format is misleading, and readers are encouraged to consult the facsimile page at the beginning to have an idea of the form of the original manuscript.

The central objects considered in the *Long March* are the moduli spaces of Riemann surfaces of genus g with n marked points, which we denote by $\mathcal{M}_{g,n}$, and the panoply of objects associated to them: curves and their fundamental groups, the fundamental ("Teichmüller") groups and groupoids of the $\mathcal{M}_{g,n}$, morphisms between the various objects, and principally, their relations with the absolute Galois group $\mathrm{Gal}(\overline{\mathbb{Q}}/\mathbb{Q})$, which we denote here by $\mathbb{\Gamma}_{\mathbb{Q}}$. The type of problems considered can be gathered into two major themes.

1) *Anabelian geometry.* Let X be an algebraic variety defined over a field

* The result of this transcription – the possibility of which was referred to by Grothendieck in the Esquisse as "une compilation de notes pieusement accumulées" – can be obtained by writing to Jean Malgoire at malgoire@math.univ-montp2.fr.

K, and let x be a geometric point of $X_{\overline{K}} = X \otimes \overline{K}$. Then there is a split exact sequence

$$1 \to \hat{\pi}_1(X_{\overline{K}}, x) \to \hat{\pi}_1(X_K, x) \to \mathrm{Gal}(\overline{K}/K) \to 1,$$

where $\hat{\pi}_1(X_{\overline{K}}, x)$ denotes the algebraic fundamental group of X, which is isomorphic to the profinite completion of the topological fundamental group of X, and $\hat{\pi}_1(X_K, x)$ is the *mixed* or *etale* or *arithmetic* fundamental group, isomorphic to the semi-direct product $\hat{\pi}_1(X_{\overline{K}}, x) \rtimes \mathrm{Gal}(\overline{K}/K)$ (see 5.2 of the talk by F. Oort). (Below, we take the liberty of dropping the base point from the notation of the π_1's, but a choice of base point is of course tacitly fixed.) The anabelian question is: *how much information about the isomorphism class of the variety X is contained in the knowledge of the etale fundamental group?* Grothendieck calls varieties which are completely determined by their etale fundamental groups *anabelian varieties*; his "anabelian dream" consists in classifying the anabelian varieties in all dimensions over all fields. Over $\mathbb{Q}$, he expects the moduli spaces $\mathcal{M}_{g,n}$ to be basic anabelian examples. In the particular case of dimension 1 he conjectured that all hyperbolic curves defined over number fields were anabelian varieties, a conjecture which was proved for the punctured projective lines by Nakamura (early 90's), for affine curves by Tamagawa (1995) and finally for all hyperbolic curves defined over number fields by Mochizuki (1996). Mochizuki actually proved this as a consequence of a stronger result, cf. the talk by G. Faltings. Let us remark that there is also a "dimension 0" version of the anabelian conjecture, namely that an infinite finitely generated *field* is determined up to isomorphism by its absolute Galois group. This was proved by Neukirch, Ikeda, Iwasawa and Uchida in the case of global fields, and in full generality by Pop in 1995.

2) *Combinatorial Galois theory.* The other major problem examined in the *Long March* is essentially: can $\Gamma_{\mathbb{Q}}$ be identified/redefined/characterised by considering only its properties related to its role as automorphism group of the algebraic fundamental groups of the $\mathcal{M}_{g,n}$ respecting the natural "stratified" structure of these groups which comes from the corresponding structure on the spaces themselves (i.e. the divisor at infinity of $\mathcal{M}_{g,n}$ is made up of moduli spaces of smaller dimension).

In the remainder of this text we concentrate on the second theme because the first one will be considered in the talk by G. Faltings. We first give a short guide to the notations in the *Long March*. Armed with this, we explain how Grothendieck first phrased the question and gave an initial conjecture, exciting but probably too strong, then changed point of view, gave a much weaker conjecture, and adduced evidence for it. We give some of the most revealing quotations indicating the direction he believed investigations should take; it is our hope that this short text will make it easier to enter into the ideas of the *Long March*, whose form is far from the usual form of a mathematical text. In the appendix to this note, we give a very brief sketch of some ideas of Drinfel'd which turn out, although he had read neither the *Long March* nor the *Esquisse*, to bear an astonishing resemblance to the approach found there. Instead of citing references along the way, we gather at the end of this note a few surveys and articles (mostly preprints) containing all the salient results mentioned here.

Definitions in a simple form. (1) Let $\pi_{g,n}$ denote the abstract group isomorphic to the fundamental group of a closed topological surface of genus g with n punctures. Fix a system $x_1, \ldots, x_g, y_1, \ldots, y_g, l_1, \ldots, l_n$ of generators (called a *basis*) for $\pi_{g,n}$ satisfying the unique relation

$$[x_1, y_1] \cdots [x_g, y_g] l_1 \cdots l_n = 1. \qquad (*)$$

Let a *discrete loop-group* be a group isomorphic to $\pi_{g,n}$, and a profinite loop-group be a profinite group isomorphic to the profinite completion $\hat{\pi}_{g,n}$. A basis of a profinite loop-group is a set of topological generators satisfying (*).

(2) Let π be a profinite loop-group. Define the group $\mathrm{Aut}_{\mathrm{lac}}(\pi)$ to be the group of (continuous) automorphisms of π which "preserve the inertia groups", which means that if $\phi \in \mathrm{Aut}_{\mathrm{lac}}(\pi)$, then there exists $\lambda \in \mathbb{Z}^*$, $\sigma \in S_n$ and $\alpha_1, \ldots, \alpha_n \in \pi$ such that

$$\phi(l_i) = \alpha_i l_{\sigma(i)}^{\lambda} \alpha_i^{-1} \qquad (**)$$

for $i = 1, \ldots, n$. Acting by inner automorphisms, the group π corresponds to a subgroup of $\mathrm{Aut}_{\mathrm{lac}}(\pi)$; let $\mathrm{Autext}_{\mathrm{lac}}(\pi)$ denote the quotient. Following Grothendieck, we write $\hat{\hat{\mathfrak{G}}}(\pi)$ for the group $\mathrm{Aut}_{\mathrm{lac}}(\pi)$ and $\hat{\hat{\mathfrak{T}}}(\pi)$ for $\mathrm{Autext}_{\mathrm{lac}}(\pi)$.

(3) A *discretification* π_0 of a profinite loop-group π is simply the discrete subgroup generated inside π by the elements of some basis of π (i.e. the image of a basis of $\hat{\pi}_{g,n}$ under some isomorphism $\hat{\pi}_{g,n} \xrightarrow{\sim} \pi$). Let $\mathfrak{G}(\pi_0)$ denote the subgroup of $\hat{\hat{\mathfrak{G}}}(\pi)$ fixing π_0. The *prediscretification* $\pi_0^\natural$ associated to π_0 is the orbit of π_0 under the action of the closure of $\mathfrak{G}(\pi_0)$ inside $\hat{\hat{\mathfrak{G}}}(\pi)$. This closure is isomorphic to the profinite completion of $\mathfrak{G}(\pi_0)$ and is written $\hat{\mathfrak{G}}(\pi_0^\natural)$ since it depends only on the prediscretification. Let $\mathfrak{T}(\pi_0)$ and $\hat{\mathfrak{T}}(\pi_0^\natural)$ denote the corresponding subgroups inside $\hat{\hat{\mathfrak{T}}}(\pi)$.

(4) There is a natural "cyclotomic character" $\hat{\hat{\mathfrak{T}}}(\pi) \to \hat{\mathbb{Z}}^*$ given by $\phi \mapsto \lambda$. On the subgroups $\mathfrak{T}(\pi_0)$ and $\hat{\mathfrak{T}}(\pi_0^\natural)$, this character takes only the values ± 1. Let $\hat{\mathfrak{T}}^+(\pi_0^\natural)$ be the kernel of this character in $\hat{\mathfrak{T}}(\pi_0^\natural)$.

Ideas and Conjectures. In §26 of the *Long March*, Grothendieck asks the following rather bold questions.

Question 1. Is $\hat{\mathfrak{T}}^+(\pi_0^\natural)$ independent of the prediscretification $\pi_0^\natural$? Equivalently (since conjugation permutes the discretifications) is $\hat{\mathfrak{T}}^+(\pi_0^\natural)$ normal in $\hat{\hat{\mathfrak{T}}}(\pi)$?

Question 2. Choose a discretification π_0 of π and let $\Sigma = \hat{\mathfrak{T}}^+(\pi_0^\natural)$ and $\mathcal{N}_\Sigma = \mathrm{Norm}_{\hat{\hat{\mathfrak{T}}}(\pi)}(\Sigma)$. Is $\mathcal{N}_\Sigma/\Sigma$ isomorphic to $\mathbb{\Gamma}_{\mathbb{Q}}$?

Question 3. (motivation for the two previous questions) Given a profinite loop-group of type (g,n), does there exist a discretification π_0 such that we can identify term by term the following exact sequences?

$$1 \to \Sigma \to \mathcal{N}_\Sigma \to \mathcal{N}_\Sigma/\Sigma \to 1$$

$$1 \to \pi_1(\mathcal{M}_{g,n,\overline{\mathbb{Q}}}) \to \pi_1(\mathcal{M}_{g,n,\mathbb{Q}}) \to \mathbb{\Gamma}_{\mathbb{Q}} \to 1?$$

If the answer to the first two questions were affirmative, writing $\hat{\mathfrak{T}}^+$ for $\hat{\mathfrak{T}}^+(\pi_0^\natural)$, we would have

$$\hat{\hat{\mathfrak{T}}}(\pi)/\hat{\mathfrak{T}}^+ \simeq \mathbb{\Gamma}_{\mathbb{Q}}.$$

This conjecture is very striking. We can rephrase it a little more simply by choosing $\pi_{g,n}$ as a discretification of $\hat{\pi}_{g,n}$ and writing $\hat{\hat{\mathfrak{T}}}_{g,n}$ for $\hat{\hat{\mathfrak{T}}}(\hat{\pi}_{g,n})$, $\hat{\mathfrak{T}}^+_{g,n}$ for $\hat{\mathfrak{T}}^+(\pi_{g,n})$ and $\mathbb{\Gamma}_{g,n}$ for $\hat{\hat{\mathfrak{T}}}_{g,n}/\hat{\mathfrak{T}}^+_{g,n}$, where by the hypothetical affirmative answer to the first question, the subgroup $\hat{\mathfrak{T}}^+_{g,n}$ is normal. Then we have

Very strong conjecture. *The $\mathbb{\Gamma}_{g,n}$ are isomorphic for all (g,n) such that $2g+n \geq 3$ and they are all isomorphic to $\mathbb{\Gamma}_{\mathbb{Q}}$.* *

In §§27 and 28 Grothendieck asks whether it is possible to prove that the $\mathbb{\Gamma}_{g,n}$ are all isomorphic. §27 deals with fixed g and decreasing n and §28 deals with finite covers. Although he does succeed in deducing these isomorphisms from certain rather strong conjectures, he begins §29 with the remark "The approach in the previous paragraphs seems, after all, very brutal" and comes to the conclusion that

* It follows from Drinfel'd's proof that $GT \neq GT_0$ that this conjecture is wrong for $(0,3)$ (note that $\hat{\mathfrak{T}}^+_{0,3} = 1$), but it is not known to be right or wrong for other (g,n).

> It is possible that $\hat{\hat{\mathfrak{T}}}(\pi)$ is such a wildly pathological group that it will never be possible to say anything reasonable (and true) about the full group, and we will always be obliged to work with smaller subgroups, not too far from the discrete situation (although with some supplementary "arithmetic" aspects du to $\mathbb{I}\Gamma_{\mathbb{Q}}$)! Indeed, there is (for $n \neq 0$) in the group $\hat{\mathfrak{T}}(\pi_0)$ a simplicial structure of successive extensions which will be respected by the outer action of the Galois group, and which must be taken into consideration. It is part of the "structure at ∞" of the moduli spaces $\mathcal{M}_{g,n}$, which even for $n = 0$ is doubtless not trivial, and it is possible that this must be taken into account, in order to put a finger on $\mathbb{I}\Gamma_{\mathbb{Q}}$. (p. 164)

Therefore, at this point, he switches to a different approach: "Without attempting to give a description of $\mathbb{I}\Gamma_{\mathbb{Q}}$ 'inside $\hat{\hat{\mathfrak{T}}}_{g,n}$', we will proceed more inductively, starting from the presence of $\mathbb{I}\Gamma_{\mathbb{Q}}$ (for arithmetico-geometric reasons), and trying to unearth properties of this presence, possibly strong enough to end up giving a purely algebraic characterisation." (p. 165)

From here to the end of the available part of the *Long March*, he proceeds to reexamine the whole of the situation as follows: instead of setting $\mathcal{N}_{g,n}$ to be the normalizer of the subgroup $\hat{\mathfrak{T}}^+(\pi_{g,n})$ inside $\hat{\hat{\mathfrak{T}}}_{g,n}$, he considers the exact sequence

$$1 \to \hat{\pi}_{g,n} \to \hat{\pi}_1(\mathcal{U}_{g,n,\mathbb{Q}}) \to \hat{\pi}_1(\mathcal{M}_{g,n,\mathbb{Q}}) \to 1,$$

where $\mathcal{U}_{g,n}$ is the "universal curve" of type (g,n), which is a scheme over $\mathcal{M}_{g,n}$ (actually isomorphic to the moduli space $\mathcal{M}_{g,n+1}$). Since according to this exact sequence, the elements of $\hat{\pi}_1(\mathcal{U}_{g,n,\mathbb{Q}})$ act on the subgroup $\hat{\pi}_{g,n}$ by conjugation, we obtain a homomorphism $\eta : \hat{\pi}_1(\mathcal{U}_{g,n,\mathbb{Q}}) \hookrightarrow \hat{\hat{\mathfrak{S}}}_{g,n}$; let $M_{g,n}$ denote its image, and let $\mathcal{N}_{g,n}$ be the image of $M_{g,n}$ under the surjection $\hat{\hat{\mathfrak{S}}}_{g,n} \to \hat{\hat{\mathfrak{T}}}_{g,n}$. The image of the subgroup $\pi_1(\mathcal{U}_{g,n,\overline{\mathbb{Q}}})$ under η is exactly $\hat{\mathfrak{S}}(\pi_{g,n})$, so clearly the quotient $\mathbb{I}\Gamma_{g,n} := \mathcal{N}_{g,n}/\hat{\mathfrak{T}}^+(\pi_{g,n})$ has a much better chance of looking like $\mathbb{I}\Gamma_{\mathbb{Q}}$; indeed there is always a surjection $\mathbb{I}\Gamma_{\mathbb{Q}} \to \mathbb{I}\Gamma_{g,n}$. Later, in §33bis Grothendieck notes that he thinks he can prove that $\mathbb{I}\Gamma_{\mathbb{Q}}$ is isomorphic to $\mathbb{I}\Gamma_{0,3}$ using Belyi's theorem, and in §34 and §35 he does so, essentially describing the beginnings of the theory of dessins d'enfants. The argument is simply that firstly, $\hat{\mathfrak{T}}^+_{0,3} = 1$, so $\mathbb{I}\Gamma_{0,3} \subset \hat{\hat{\mathfrak{T}}}_{0,3} = \mathrm{Autext}_{\mathrm{lac}}(\hat{\pi}_{0,3})$, and second that it is easily seen to be a consequence of Belyi's theorem that $\mathbb{I}\Gamma_{\mathbb{Q}}$ acts faithfully on $\pi_{0,3}$. Therefore the surjection $\mathbb{I}\Gamma_{\mathbb{Q}} \to \mathbb{I}\Gamma_{0,3}$ cannot have non-trivial kernel. In §28 we find the conjecture:

Conjecture. Is the natural surjection $\mathbb{I}\Gamma_{\mathbb{Q}} \to \mathbb{I}\Gamma_{g,n}$ an isomorphism for all (g,n) with $2g+n \geq 3$? In other words, are all the $\mathbb{\Gamma}_{g,n}$ isomorphic? Equivalently, is the homomorphism η injective for all such (g,n)?

The sections following §28 are devoted to the behavior of the groups $\mathbb{\Gamma}_{g,n}$ under the natural morphisms of erasing a point (§32) and taking finite covers (§33 bis), but always from the point of view of using only a *certain type* of property of these groups. On p. 179, we read

> We propose to try and make precise the type of properties which will characterize $\mathcal{N}_{g,n}$ inside $\mathcal{N}'_{g,n} = \mathrm{Norm}_{\hat{\mathfrak{T}}_{g,n}}(\hat{\mathfrak{T}}^+_{g,n})$, or $\mathbb{\Gamma}_{g,n}$ inside $\mathbb{\Gamma}'_{g,n} = \mathcal{N}'_{g,n}/\hat{\mathfrak{T}}^+_{g,n}$. We have natural homomorphisms
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> $$\begin{cases} \mathbb{\Gamma}'_{g,n} \to \mathrm{Autext}(\hat{\mathfrak{T}}^+_{g,n}) \\ \mathbb{\Gamma}'_{g,n} \to \mathrm{Autext}(\hat{\mathfrak{T}}^{+!}_{g,n} \end{cases}$$
>
> and I presume that $\mathbb{\Gamma}_{g,n}$ can be described as the inverse image of a suitable closed subgroup of one or the other of these right-hand sides, i.e. that we can describe it in terms of properties of outer operations on $\hat{\mathfrak{T}}^+_{g,n}$ or $\hat{\mathfrak{T}}_{g,n}$...Of course, we could posit conditions on the outer automorphisms (of $\hat{\mathfrak{T}}'^{+!}_{g,n}$, say) which are stable by the successive passages to $\hat{\mathfrak{T}}'^{+!}_{g,n-1}$ etc.... but it is not at all clear that this will suffice to describe the $\mathbb{\Gamma}_{g,n} \subset \mathbb{\Gamma}'_{g,n}$, if only because this condition is empty for the limit case $n=0$ (if $g \geq 2$; or for the cases $g=1$, $n=1$, or $g=0$, $n=3$). It is possible that we have to also use properties of the π_1 of the $\mathcal{M}_{g,n,\overline{\mathbb{Q}}}$ linked to compactification. It is only in the case $(g,n)=(0,3)$ that we need not expect any condition of this kind.

Appendix

In his 1991 article [D], Drinfel'd defined the *Grothendieck-Teichmüller group* $\widehat{GT}$ as a group of transformations of quasi-triangular quasi-Hopf algebras, acting as modifications of the *associativity* and *commutativity constraints*. Let us give a revised but equivalent definition of $\widehat{GT}$ here. Let $\hat{F}_2$ denote the profinite free group on two topological generators x and y. Let θ be the automorphism of $\hat{F}_2$ defined by $\theta(x)=y$ and $\theta(y)=x$. Let ω be the automorphism of $\hat{F}_2$ defined by $\omega(x)=y$ and $\omega(y)=(xy)^{-1}$. Let $K(0,5)$ be the pure mapping class group on 5 strands (a quotient of the pure Artin braid group K_5). This group is generated by five elements $x_{i,i+1}$, $i \in \mathbb{Z}/5\mathbb{Z}$; let ρ be the automorphism of the profinite completion $\hat{K}(0,5)$ given by $\rho(x_{i,i+1}) = x_{i+3,i+4}$. There is an injection $\hat{F}_2 \hookrightarrow \hat{K}(0,5)$ given by $x \mapsto x_{12}$ and $y \mapsto x_{23}$; for all $f \in \hat{F}_2$ let $\tilde{f}$ denote its image in $\hat{K}(0,5)$ by this injection.

Definition. Let $\underline{\widehat{GT}}$ be the set

$$\{(\lambda, f) \in \hat{\mathbb{Z}}^* \times \hat{F}_2' \,|(I) \quad \theta(f)f = 1;$$
$$(II) \;\; \omega^2(fx^m)\omega(fx^m)fx^m = 1 \text{ where } m = (\lambda - 1)/2;$$
$$(III) \; \rho^4(\tilde{f})\rho^3(\tilde{f})\rho^2(\tilde{f})\rho(\tilde{f})\tilde{f} = 1\}.$$

Under a suitable multiplication law, this set can be made into a monoid; let $\widehat{GT}$ be the group of invertible elements of $\underline{\widehat{GT}}$. In his paper Drinfel'd made the following remarks.

(1) There are injections $\widehat{GT} \hookrightarrow \mathrm{Aut}(\hat{B}_n)$ for all $n \geq 2$, where $\hat{B}_n$ denotes the profinite completion of the Artin braid group on n strands B_n. It is easily seen that this action restricts to the pure Artin braid groups $\hat{K}_n$ and passes to the quotients $K(0,n)$ (the pure mapping class groups, isomorphic to the fundamental groups of the moduli spaces $\mathcal{M}_{0,n}$).

(2) Let $\mathcal{B}_{g,n}$ be the set of base points *near infinity* or *tangential base points* on the moduli space $\mathcal{M}_{g,n}$. These base points are simply connected pieces of the neighborhoods of the points of *maximal degeneration* in the Deligne-Mumford compactification of $\mathcal{M}_{g,n}$. When $g = 0$, it is known that the profinite fundamental (Teichmüller) groupoid $\hat{\pi}_1(\mathcal{M}_{0,n}; \mathcal{B}_{0,n})$ can be equipped with the structure of a subgroupoid of a braided tensor category, i.e. with associativity and commutativity constraints. A similar result on the Teichmüller groupoids in all genera seems to be implied by the results in appendix B of [MS].

(3) Putting (1) and (2) together, Drinfel'd conjectures that $\widehat{GT}$ should be the automorphism group of a suitably defined *Teichmüller tower* consisting of the collection of the $\hat{\pi}_1(\mathcal{M}_{g,n}; \mathcal{B}_{g,n})$ linked by natural homomorphisms coming from degeneration of surfaces of type (g,n) (i.e. considering the part at ∞ of $\mathcal{M}_{g,n}$). We were able to prove this statement on the *genus zero* tower, taking as homomorphisms those given by erasing punctures on the n-punctured Riemann sphere.

(4) Drinfel'd indicated, and Ihara proved that there is a homomorphism $\mathrm{Gal}(\overline{\mathbb{Q}}/\mathbb{Q}) \hookrightarrow \widehat{GT}$; using Belyi's theorem it is seen to be injective.

If (3) is true in all genera, then the group $\widehat{GT}$ possesses all the necessary properties to be an automorphism group of the fundamental groupoids of the moduli spaces respecting the natural morphisms between them, which reflects the "simplicial structure of successive extensions" mentioned in Grothendieck's quotation given above. Even more, by acting on the fundamental groupoids based at the base points in the neighborhood of maximal degeneration, $\widehat{GT}$ also respects the "structure at ∞" of the moduli spaces, mentioned in the same quotation. Therefore, particularly if (3) turns out to be true, a precise formulation of Grothendieck's conjecture is given by

Conjecture. $\widehat{GT} = \mathrm{Gal}(\overline{\mathbb{Q}}/\mathbb{Q})$.

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